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Reactive Electrodynamics of Unclosed Electromagnetic Systems

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Abstract: *Even quasi-static fields can carry momentum, and this would appear to contradict a general theorem that the total momentum of a closed system is constant if its center of mass is at rest. In this case, there must be some other (hidden) momentum and reactive (hidden) force that cancel the electromagnetic momentum and the electromagnetic force. In unclosed systems with changing electric current the electromagnetic force equals to the so-called self-force by means of which the unclosed conductor acts on itself. An attempt to connect the hidden force with the self-force is the basic peculiarity of this work.*

Key Word*: Hidden momentum, Self-force, Conservation of momentum, Alternative electric current*

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I. Introduction

After deep, long and repeated scientific debates, the deficit force necessary to maintain the linear and angular momentum conservation laws in classical electrodynamics is found [1]. This is the so called hidden force which is the time derivative of the hidden momentum proportional to the Pointing vector. A body the center of mass of which is at rest can carry a nonzero mechanical momentum, called hidden momentum [2]. It is this force that we call the reactive force by means of which the body repulses, formally speaking, from the field. In real examples, like the Tamm's capacitor [3], the magnitude of the repelling force is so small that it is difficult to observe [1].

There exists another side of the problem. In magnetostatics, it relates to applicability of Newton's third law to magnetic interaction of unclosed electric currents [4]. These currents generate magnetic field, which exert forces back on the currents which generated them. For a closed loop, the sum of all the forces, including self-forces, remains zero and the momentum in the closed system conserves [5]. The self-force and self-torque are found to be significant [6,7]. To find out relation between the self-force and the electromagnetic force is the subject of this work.

II. Electromagnetic force and electromagnetic induction

One can appeal to electromagnetic induction in order to understand what happens. Fig. 1 shows a capacitor located in a uniform changing magnetic field.

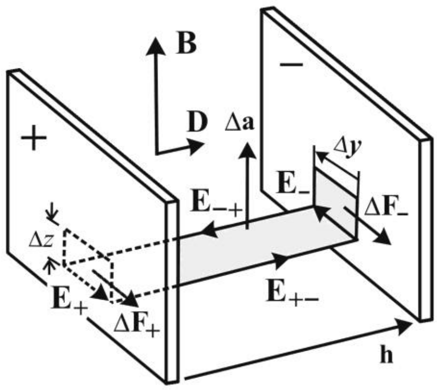

Fig. 1. A charged capacitor in the presence of an external uniform magnetic field.

According to Faraday's law, the changing magnetic field will induce an electric field

$$\oint (\mathbf{E}d\mathbf{l}) = -\frac{d}{dt}\int (\mathbf{B}d\mathbf{a}), \tag{1}$$

that means

$$(\mathbf{E}_{+-}\mathbf{h}) + (\mathbf{E}_{-}\Delta\mathbf{y}) - (\mathbf{E}_{-+}\mathbf{h}) - (\mathbf{E}_{+}\Delta\mathbf{y}) = -\frac{d}{dt}(\mathbf{B}[\mathbf{h} \times \Delta\mathbf{y}]) \cdot \tag{2}$$

The components $\mathbf{E}_{+-}$ and $\mathbf{E}_{-+}$ do not contribute to the force, so the net force on the volume $\Delta v = h\Delta y\Delta z$ is

$$\Delta\mathbf{F}_{em} = \Delta\mathbf{F}_{-} + \Delta\mathbf{F}_{+} = \sigma\frac{dB}{dt}\Delta v\mathbf{e}_{y}, \tag{3}$$

where $\mathbf{e}_y$ is the unit vector directed along the coordinate axis Y.

For the parallel-plate capacitor the displacement field D is equal to the surface charge density $\sigma = D$, therefore the density electromagnetic force $\mathbf{f}_{em} = d\mathbf{F}_{em}/dv$ can be rewritten in the form

$$\mathbf{f}_{em} = [\mathbf{D} \times \frac{d\mathbf{B}}{dt}], \tag{4}$$

and equals to the time derivative of the linear momentum density for the static electric field inductance **D**.

No force is exerted on a source of the changing magnetic field by the electric field of the charged capacitor. The center of mass of the capacitor and source of the magnetic field is certainly at rest, so if there is momentum in the field, there must therefore some compensating non-electromagnetic momentum elsewhere, called hidden momentum, for which

$$\mathbf{f}_{r} = -[\frac{d\mathbf{D}}{dt} \times \mathbf{B}], \tag{5}$$

equivalent to the reactive force density: $\mathbf{f}_{em} + \mathbf{f}_{r} = 0$.

III. Electromagnetic force and electromagnetic induction

Now we are in a position to consider a general case. Fig 2 shows two plates joined by a conductor J with changing electric current of density **j**.

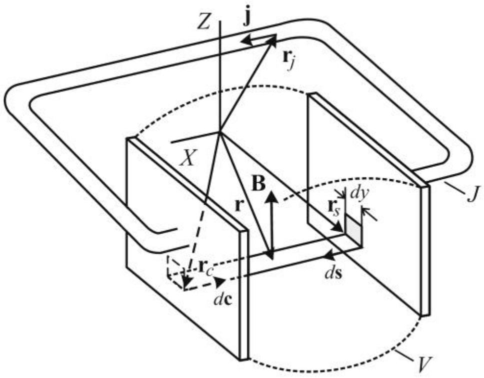

Fig. 2. Two conducting plates in a circuit of alternative electric current.

In a conductor, the displacement current is extremely weak, the magnetic field acts on ordinary currents, exerting on them a net force

$$\mathbf{F} = \int_{J} [\mathbf{j}(\mathbf{r}) \times \mathbf{B}(\mathbf{r})] d^3 r_j, \tag{6}$$

where the integration sweeps over the conductor including the conducting plates. Using the Maxwell equation

$$[\nabla \times \mathbf{B}(\mathbf{r})] = \mu_0 \mathbf{j}(\mathbf{r}), \tag{7}$$

one can obtain.

$$\mathbf{F} = \frac{1}{\mu_0}\int_{J} (\nabla(\frac{B^2(\mathbf{r})}{2}) - (\nabla\mathbf{B}(\mathbf{r}))\mathbf{B}(\mathbf{r})) d^3 r_j \quad . \tag{8}$$

Changing the integration over the volume of the conductor by integration over the surface, one can write

$$\mathbf{F}=\frac{1}{\mu_0}\oint_J (d\mathbf{s}_J\cdot\frac{B^2(\mathbf{r})}{2}-(d\mathbf{s}_J\cdot\mathbf{B}(\mathbf{r}))\mathbf{B}(\mathbf{r}))\,. \tag{9}$$

The magnetic field distribution on the surface of the conductor is identically determined by the field distribution in space. That is why

$$\mathbf{F}=\frac{1}{\mu_0}\int_V [[\nabla\times\mathbf{B}(\mathbf{r})]\times\mathbf{B}(\mathbf{r})]d^3r\,, \tag{10}$$

with integrating over all space outside the conductor.

One may use the Biot-Savart law to describe the magnetic field produced by electric currents in the conductor, for which

$$[\nabla\times\mathbf{B}(\mathbf{r})]=-\frac{\mu_0}{4\pi}\int_J \Delta\frac{\mathbf{j}(\mathbf{r}_j)}{|\mathbf{r}-\mathbf{r}_j|}d^3r_j\,. \tag{11}$$

Using the formal expression for the Dirac delta-function [8]

$$\Delta\frac{\mathbf{j}(\mathbf{r}_j)}{|\mathbf{r}-\mathbf{r}_j|}=-4\pi\mathbf{j}(\mathbf{r}_j)\delta(\mathbf{r}-\mathbf{r}_j)\,, \tag{12}$$

in internal integrals in (10) and changing order of the integration, one can write

$$\mathbf{F}=\frac{\mu_0}{4\pi}\int_J d^3r'_j\mathbf{j}(\mathbf{r}'_j)\int_V\frac{(\mathbf{j}(\mathbf{r})(\mathbf{r}-\mathbf{r}'_j))}{|\mathbf{r}-\mathbf{r}'_j|^3}d^3r\,. \tag{13}$$

With the help of the gradient theorem and taking into account that the divergence of the vector potential is chosen to be zero, one obtains

$$\mathbf{F}=-\int_S \mathbf{A}(\mathbf{r}_s)(d\mathbf{s}_s\mathbf{j}(\mathbf{r}_s))-\int_C \mathbf{A}(\mathbf{r}_c)(d\mathbf{s}_c\mathbf{j}(\mathbf{r}_c))\,, \tag{14}$$

for the surface elements bound the outside the volume J (Fig 2). Here

$$\mathbf{A}(\mathbf{r})=\frac{\mu_0}{4\pi}\int_J\frac{\mathbf{j}(\mathbf{r}'_j)}{|\mathbf{r}-\mathbf{r}'_j|}d^3r'_j\,, \tag{15}$$

is the vector potential in the point **r**.

The expression (14) is absolutely equivalent to that for the self-force [9]. In the known form [6], not convenient for calculations, this force, by means of which an unclosed electric conductor with an alternative current acts on itself, may be rewritten as

$$\mathbf{F}=\frac{\mu_0}{4\pi}\int_J d^3r_j\int_J\frac{[\mathbf{j}(\mathbf{r}_j)\times[\mathbf{j}(\mathbf{r}'_j)\times(\mathbf{r}_j-\mathbf{r}'_j))}{|\mathbf{r}_j-\mathbf{r}'_j|^3}d^3r'_j\,. \tag{16}$$

All that remains is to demonstrate that this force can be essential. To do this, one may consider the unclosed electric contour of zero thickness in which alternative current flows. Unfortunately, this example is studied qualitatively only [10], thought the calculations (16) are simple and yield

$$F=-\frac{\mu_0 I^2}{2\pi}\ln\frac{\delta((a^2+(b-\delta)^2)^{1/2}+a)}{(b-\delta)((a^2+\delta^2)^{1/2}+a)}, \tag{17}$$

and logarithmically diverges at small δ (Fig. 3). Without such an example, the qualitative consideration [10] and the so-called EM-drive propulsion [11] seem to be inaccurate.

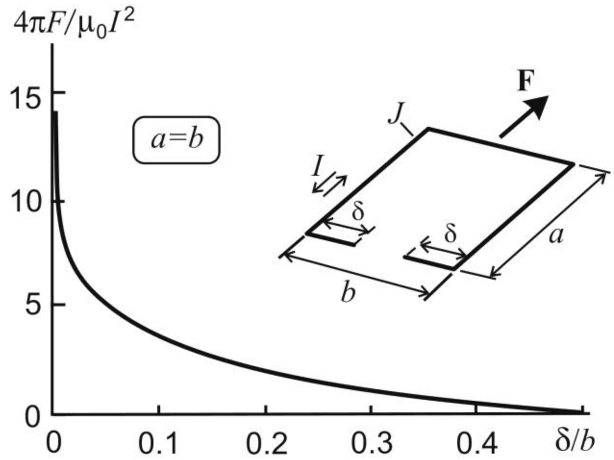

Fig. 3. Behavior of the self-force for the unclosed infinite thin conductor *J* with the alternative current *I*.

IV. Conclusion

In quasi-static approximation, the electromagnetic force is exactly equal to the self-force exerted on an unclosed electric circuit by the magnetic field produced by the circuit. In comparison with the example with a charged condenser located in the changing magnetic field, the self-force is essential and can be experimentally investigated using ordinary laboratory equipment. Moreover, when conductor is infinitely thin the self-force equal to the electromagnetic force diverges. This may be a reason for consideration of new propulsion methods based on violating Newton's third law in ordinary form. Meanwhile, the total momentum conserves, so that the hidden momentum compensates the electromagnetic momentum, and the electromagnetic force canceled by the reactive force, in the other words, by time derivative of the hidden linear momentum.

References

[1]. Graham GM, Lahoz DG. Observation of static electromagnetic angular momentum in vacuo. Nature. 1980; 285(5761): 154–155
[2]. Bubson D, Reynolds SP, Bjokquist R, Griffits DJ. Hidden momentum, field momentum, and electromagnetic impulse. American Journal of Physics. 2009; 77(9): 826-833
[3]. Tamm IE. Fundamentals of the theory of electricity, Moscow: Mir Publisher, 1979.
[4]. Graneau N. The finite size of the metallic current element. Physics Letters A. 1990; 147(2-3): 92-96.
[5]. Gerasimov SA Self-interaction and vector potential in magnetostatics. Physica Scripta. 1997; 56(5): 462-464.
[6]. Cavalleri G, Bettoni G, Tonni E, Spavieri G. Experimental proof of standard electrodynamics of measuring the self-force on a part of current loop. Physical Review E. 1998; 58(2): 2505-2517.
[7]. Gerasimov SA, Gorokhovikov SL, Grigorian MA. A special feature of unipolar rotation. Technical Physics Letters. 2005; 31(1): 79-80.
[8]. Jackson JD. Classical electrodynamics, New-York: John Wiley, 1998.
[9]. Gerasimov SA Interaction of current elements, conservation of momentum and pseudo-self-action in magnetostatics. Applied Physics and Mathematics. 2014; 3(1): 3-6.
[10]. Ivtsenkov GG. Reactive and support-less movement: electromagnetic propulsion. Galilian Electrodynamics. 2017; 27(1): 3-9.
[11]. Wu CW. Comments on theoretical foundation of EM drive. Acta Astronautica. 2018; 144(1): 214-215.

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