IOSR Journal Of Applied Physics (IOSR-JAP)
e-ISSN: 2278-4861.Volume 15, Issue 5 Ser. I (Sept. – October. 2023), PP 35-41
www.Iosrjournals.Org

Mathematical Model Of Doppler Shifts

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Abstract
A mathematical model is proposed for the quantitative analysis of Doppler Shifts. In the model, four equations: Equation of Light Speed, Equation of Position, Vision of Light and Vision of Object are established and correlated together such that the wavelength, frequency and light speed of various Doppler Shifts including Axial Doppler Shift, Acceleration Doppler Shift and Transverse Doppler Shift can be calculated. In general, the mathematical analyses agree very well with the observations.

Keywords: *Doppler Effect, Doppler Shift, Axial Shift, Acceleration Shift, Transverse Shift, Equation of Light Speed, Vision of Light, Redshift, BlueShift, Cosmological Redshift.*

Date of Submission: 24-09-2023 Date of Acceptance: 04-10-2023

I. Doppler Effect

The frequency of a wave-like signal such as sound or light is dependent on the relative movement of the sender and the receiver. This phenomenon is known as the "Doppler Effect" [1]. Conventionally, it is understood that when a source of light is moving toward the observer (reference point), each successive photon is emitted from a position closer to the previous one. In other words, the wavelength between two subsequent photons is smaller, which causes an increase in the frequency and a shift in the wavelength to the blue end of the spectrum. This is commonly known as "Blue Shift". Conversely, if the source of light is moving away from the observer (reference point), each photon is emitted from a position farther from the previous photon, resulting in long wavelengths between the two subsequent photons. This causes a reduction in the frequency and a shift in wavelength toward the red end of the spectrum, which is known as "Redshift" (Fig. 1).

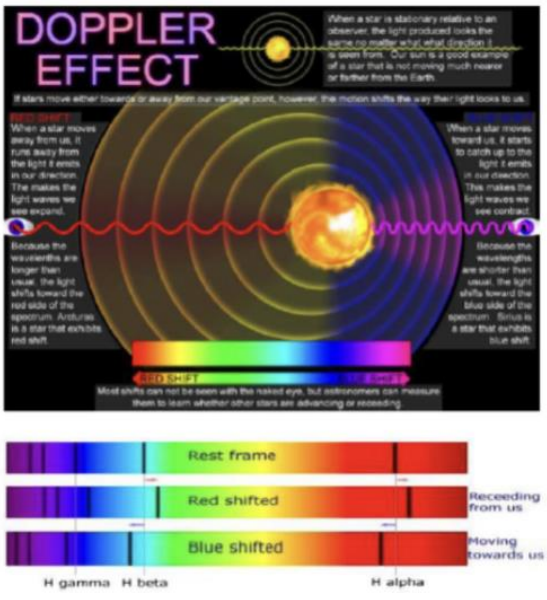

Fig. 1 Doppler Effect and Blueshift & Redshift phenomena.

Like most scientists including Einstein, I first thought that the Blueshift, Redshift and the Doppler Effect could only exist in the Non-Inertia Transformation process [2] as that in sound propagation. I therefore believe that a photon emitted from a light source travels in space at a constant Absolute Light Speed 3×10^8 m/s can also be observed at its origin in Absolute Space System, without any influence from its light source, as noted in my previous publication [2]. This, however, is in conflict with my logical thinking. It is hard to believe

that a ball-like particle thrown out of the window of a train will not follow the train. The concept that photon emission is a Non-Inertia Transformation has bothered me for quite some time until I developed the Acceleration Doppler Effect based on Photon Inertia Transformation to solve the problem.

The Doppler Effect can be proved easily in the Non-Inertia Transformation Process with the signal source traveling at a constant speed [2] either toward or away from the observer such as that of sound propagation. However, photon emission from the light source is an Inertia Transformation Process [3]. According to Equation of Light Speed [4], Normal Light Speed (also wavelength and frequency) observed at the reference point can change with: (1) Absolute Light Speed which is dependent on the local gravitational field and aging of the universe, and (2) Inertia Light Speed which is dependent on the relative speed and direction between light source and reference point. On one hand, the change of Absolute Light Speed can cause Cosmological Redshift and Gravitational Redshift. On the other hand, the variation of Inertia Light Speed can result in Doppler Shifts including Axial Doppler Shift, Acceleration Doppler Shift and Transverse Doppler Shift. Acceleration Doppler Shift can also be used to derive and interpret Cosmological Redshift and Hubble's Law, however, where the acceleration energy coming from remains a mystery.

II. Equation of Light Speed

When a photon emitted from a light source, due to the Vision of Light and Photon Inertia Transformation [35], it undergoes two separate motions: ejection motion which gives Absolute Light Speed and inertia motion which provides Inertia Light Speed.

Light speed **C'** (Normal Light Speed) observed at a reference point is the vector summation of Absolute Light Speed C observed at light source (3×10^8 m/s dependent on the local gravitational field and aging of the universe) and the speed of light source **V** observed at the reference point (Inertia Light Speed)**.** This is known as **"**Equation of Light Speed" [5] [4].

$$\mathbf{C' = C + V}$$

Equation of Light Speed holds at the time of photon emission, no matter the reference points. It also works for a constant speed light source at anytime with the reference point at light origin or its inertia system.

III. Doppler Shift

Fig. 2 is a schematic drawing of one direction Doppler Shifts (Axial and Acceleration Doppler Shifts) based on Absolute Space System at light origin (reference point). Because the star is far away from earth, both earth and light origin are nearly stationary to each other, therefore earth can also be considered as a reference point for the same Absolute Space System. As a result, all the measurement observed on earth is the same as that observed at the light origin in the same Absolute Space System.

The light source (star) can either move toward or away from the observer on earth. Assuming it takes time t for a photon traveling from the light origin to earth. S is the distance between the light source and the light origin at time t, V_o is the speed of the light source (star) at the beginning, V_t is the speed of the light source (star) at time t, and a is the constant acceleration of the light source (star) in time t. P is the distance of the photon traveling from the light origin to earth in time t, C' is the light speed observed at the light origin or earth, and D is the distance between the light source (star) and the photon when the photon reaches earth at time t. Also λ_1 is the wavelength, ν_1 is the frequency and C_1 is the light speed of the photon observed on earth. With the above notations, Blueshift and Redshift caused by Doppler Effects can be studied as follows:

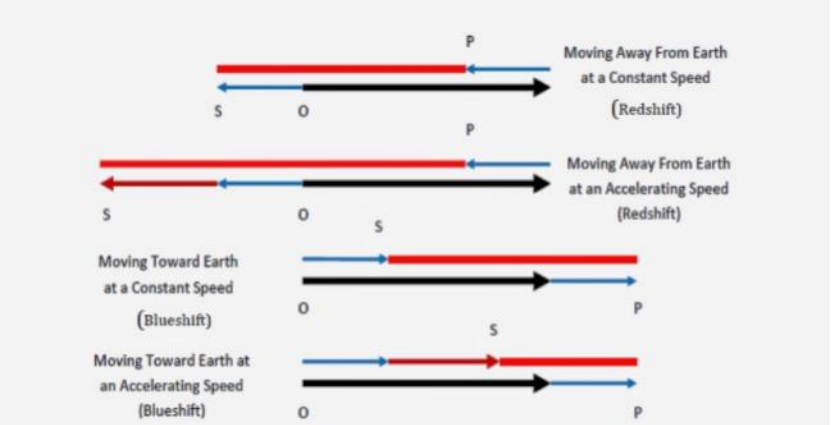

Fig. 2 Redshift and Blueshift caused by Axial Doppler and Acceleration Doppler Effects.

First, the distance vectors between light origin, light source (star) and photon can be correlated to each others as follows (equation of positions):

OS = **S** = Distance vector from light origin to light source (star) = Movement of light source (star) away from light origin.

SP = **D** = Distance vector from light source (star) to photon = Vision of light observed from light source (star).

OP = **P** = Distance vector from light origin to photon = Vision of light observed from light origin and ground (P_t= Distance vector from light origin to earth).

OP = **OS** + **SP**

Also,

P = **S** + **D**

D = **P** – **S**

In addition, according to Equation of Light Speed, when photon separate from the light source (star), the speed of photon observed at the light origin **C'** is equal to the vector summation of light speed observed at the light source (star) **C** (Absolute Light Speed $3x10^8$ m/s) and the speed of the light source (star) observed at the light origin $\mathbf{V_0}$.

$\mathbf{C'} = \mathbf{C} + \mathbf{V_0}$

Therefore,

$\mathbf{OP} = \mathbf{P} = \mathbf{C'}t = \mathbf{C}t + \mathbf{V_0}t = P\mathbf{s}$

$\mathbf{OS} = \mathbf{S} = \mathbf{V_0}t + \frac{1}{2}\,\mathbf{a}t^2 = S\mathbf{s}$

Also,

$\mathbf{D} = \mathbf{P} - \mathbf{S} = (P - S)\mathbf{s}$

Where t is the traveling time of photon from light origin to earth, a is the acceleration of light source (star), **s** is the unit vector toward earth, D is the distance between light source and earth (photon position at time t), S is the distance between light origin and light source and P is the distance between light origin and earth (photon position at time t).

Furthermore, the wavelength, light speed and frequency of the photon generated at the light source (star) and observed on earth can be calculated as follows:

$\lambda_1 = D/\nu t$

$C_1 = P/t$

$\nu_1 = C_1/\lambda_1$

Where λ_1, C_1 and ν_1 are the wavelength, light speed and frequency of the photon generated at the light source (star) and observed on earth. λ, C and ν are the wavelength, light speed and frequency of the photon generated and observed at the light source (star), which assuming are the same as that generated and observed on earth.

IV. Axial Doppler Shift

When the light source (star) either moves toward or away from the observer (reference point) on earth at a constant speed ($V_o = V_t$ and a = 0), Blueshift and Redshift can be observed respectively. This is called "Axial Doppler Shift" ("Axial Doppler Effect") [6]. A detailed analysis is discussed as follows:

Blueshift

In case the light source (star) moves toward the observer (reference point) at a constant speed,

$S = V_o t$

$P = Ct + V_o t$

$D = P - S = Ct$

Therefore,

$\lambda_1 = D/\nu t = Ct/\nu t = C/\nu$

$\lambda_1 = \lambda$

$C_1 = P/t = (Ct + V_o t)/t = C + V_o$

$C_1 > C$

$\nu_1 = C_1/\lambda_1 = (C + V_o)/\lambda$

$\nu_1 = (1 + V_o/C)\,\nu$

$\nu_1 > \nu$

When the light source (star) moves toward earth (reference point) at a constant speed, the wavelength maintains unchanged, but both frequency and light speed become bigger, such that Blueshift can be observed on earth.

Redshift

In case the light source (star) moves away from earth (reference point) at a constant speed,

$S = – V_o t$

$P = Ct – V_o t$

$D = P – S = Ct$

Therefore,

$\lambda_1 = D/\nu t = Ct/\nu t = C/\nu$

$\lambda_1 = \lambda$

$C_1 = P/t = (Ct – V_o t)/t = C - V_o$

$C_1 < C$

$\nu_1 = C_1/\lambda_1 = (C – V_o)/ \lambda$

$\nu_1 = (1 – V_o/C) \nu$

$\nu_1 < \nu$

When the light source (star) moves away from earth (reference point) at a constant speed, the wavelength maintains unchanged, but both frequency and light speed become smaller, such that Redshift can be observed on earth.

V. Acceleration Doppler Shift

When the light source (star) either moves toward or away from the observer (reference point) on earth at a constant acceleration speed ($V_o \neq V_t$ and $a \neq 0$), Blueshift and Redshift can be observed respectively. This is called "Acceleration Doppler Shift" ("Acceleration Doppler Effect") [3]. A detailed analysis is discussed as follows:

Blueshift

In case the light source (star) moving toward the observer (reference point) on earth at a constant acceleration speed,

$S = V_o t + ½ at^2$

$P = Ct + V_o t$

$D = P – S = Ct – ½ at^2$

Therefore,

$\lambda_1 = D/\nu t = (Ct – ½ at^2)/\nu t = (C – ½ at)/\nu$

$\lambda_1 < \lambda$

$C_1 = P/t = (Ct + V_o t)/t = C + V_o$

$C_1 > C$

$\nu_1 = C_1/\lambda_1$

$\nu_1 = ((C + V_o)/(C – ½ at)) \nu$

$\nu_1 > \nu$

When the light source (star) moves toward earth (reference point) at a constant acceleration speed, the wavelength becomes smaller, but both the frequency and light speed become bigger, such that Blueshift can be observed on earth.

Redshift

In case the light source (star) moving away from the observer (reference point) on earth at a constant acceleration speed,

$S = – (V_o t + ½ at^2)$

$P = Ct – V_o t$

$D = P – S = Ct + ½ at^2$

Therefore,

$\lambda_1 = D/\nu t = (Ct + ½ at^2)/\nu t = (C + ½ at)/\nu$

$\lambda_1 > \lambda$

$C_1 = P/t = (Ct – V_o t)/t = C – V_o$

$C_1 < C$

$\nu_1 = C_1/\lambda_1$

$\nu_1 = ((C – V_o)/(C + ½ at)) \nu$

$\nu_1 < \nu$

When the light source (star) moves away from earth (reference point) at a constant acceleration speed, the wavelength becomes bigger, but both the frequency and light speed become smaller, such that Redshift can be observed on earth.

Mathematical Model of Doppler Shifts

To better analyze Doppler Shifts, a general mathematical model as illustrated in Fig. 3 can be applied which includes Equation of Light Speed **C' = V + C**, Equation of position **P = S + D**, Vision of photon **P = C't = Vt + Ct** and Vision of light source **S = Vt + ½ at²**, as well as their correlations to the wavelength $\lambda_1 = D/vt$, Light Speed $C_1 = P/t$ and Frequency $\nu_1 = C_1/\lambda_1$ of Doppler Shifts observed on earth. This mathematical model can be used for the analyses of various Doppler Shifts including Axial, Acceleration and Transverse Doppler Shifts.

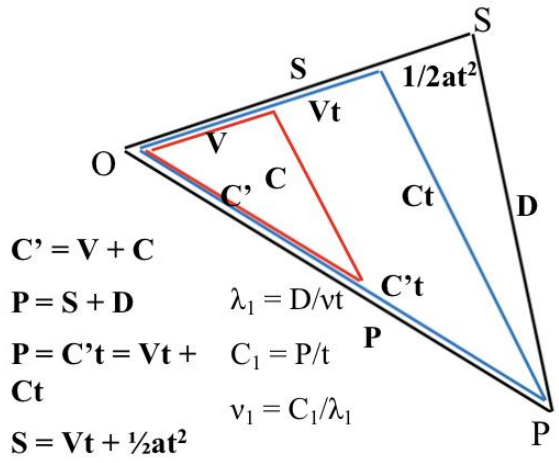

Fig. 3 Equation of Light Speed C' = V + C, Equation of position P = S + D, Vision of photon P = C't = Vt + Ct and Vision of light source $S = Vt + ½ at^2$. Their correlations to the wavelength $\lambda_1 = D/vt$, Light Speed $C_1 = P/t$ and Frequency $\nu_1 = C_1/\lambda_1$ of Doppler Shifts.

Transverse Doppler Shift

When the light source (star) moves at a constant speed in the transverse direction to the observer, both Blueshift and Redshift can be observed. This is called "Transverse Doppler Shift" ("Transverse Doppler Effect") [7]. A detailed analysis is discussed as follows:

Fig. 4 shows the correlations between Normal Light Speed **C'**, Absolute Light Speed **C** and Inertia Light Speed **V** during Transverse Doppler Shift. Where **C'** is the light speed observed at the reference point (light origin or earth), **C** is the light speed observed at the light source and **V** is the speed of the light source observed at the reference point.

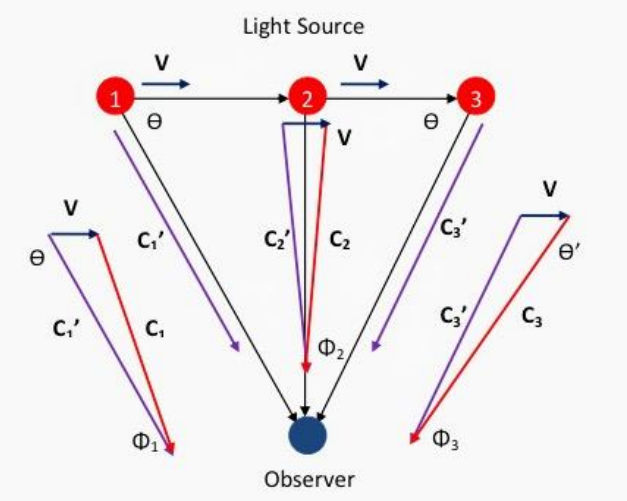

Fig. 4 Transverse Doppler Shift and Equation of Light Speed.

Position 1 – The beginning position of transverse
The angle between **V** and **C'** is Θ, and the angle between **C'** and **C** is Φ (extremely small, Cos Φ = 1).
Because
C' = **C** + **V**
Therefore,
C' = C Cos Φ + V Cos Θ
Also,
Cos Φ = 1
Therefore,
C' = C + V Cos Θ
Because
$C_1 = C'$
Therefore,
$C_1 > C$
Also,
D = Ct
$\lambda_1 = D/\nu t = Ct/\nu t$
Therefore,
$\lambda_1 = \lambda$
And
$\nu_1 = C_1/\lambda_1 = (C + V \text{ Cos } \Theta)/\lambda$
Therefore,
$\nu_1 > \nu$

Because ν_1 the frequency of the photon generated from the light source (star) at position 1 observed on earth is larger than ν the frequency of the photon generated and observed at the light source (star), same as that of the photon generated and observed on earth, therefore, blue shift can be observed at position 1 on earth.

Position 2 – The closest position between light source and observer
The angle between **V** and **C'** is 90^0, and the angle between **C'** and **C** is Φ (extremely small, Cos Φ = 1).
Because
C' = **C** + **V**
Therefore,
C' = C Cos Φ + V Cos 90^0
Also,
Cos Φ = 1
Cos $90^0 = 0$
Therefore,
C' = C
Also,
$C_1 = C'$
Therefore,
$C_1 = C$
Because
D = Ct
$\lambda_1 = D/\nu t = Ct/\nu t$
Therefore,
$\lambda_1 = \lambda$
And
$\nu_1 = C_1/\lambda_1 = C/\lambda$
Therefore,
$\nu_1 = \nu$
Because ν_1 the frequency of the photon generated from the light source (star) at position 2 observed on earth is equal to ν the frequency of the photon generated and observed at the light source (star), as the same as that of the photon generated and observed on earth, therefore, zero shift can be observed at position 2 on earth.

Position 3 – The ending position of transverse
The angle between **V** and **C'** is Θ, and the angle between **C'** and **C** is Φ (extremely small, Cos Φ = 1).
Because
C' = **C** + **V**

Therefore,
$C' = C \cos \Phi - V \cos \Theta$
Also,
$\cos \Phi = 1$
Therefore,
$C' = C - V \cos \Theta$
Because
$C_1 = C'$
Therefore,
$C_1 < C$
Also,
$D = Ct$
$\lambda_1 = D/\nu t = Ct/\nu t$
Therefore,
$\lambda_1 = \lambda$
And
$\nu_1 = C_1/\lambda_1 = \{C - V \cos \Theta)/\lambda$
Therefore,
$\nu_1 < \nu$

Because ν_1 the frequency of the photon generated from the light source (star) at position 3 observed on earth is smaller than ν the frequency of the photon generated and observed at the light source (star), same as that of the photon generated and observed on earth, therefore, red shift can be observed at position 3 on earth.

As a result, for a light source (star) in transverse motion to earth, blue shift should first be observed at the beginning stage, then zero shift will show up in the middle stage, and red shift will come out at the end stage. (This is the revision of my previous publication [8]).

VI. Conclusion

A mathematical model is proposed for the quantitative analysis of Doppler Shifts. The model includes four equations: Equation of Light Speed $\mathbf{C' = V + C}$, Equation of position $\mathbf{P = S + D}$, Vision of photon $\mathbf{P = C't = Vt + Ct}$ and Vision of light source $\mathbf{S = Vt + \frac{1}{2} at^2}$, as well as their correlations to the wavelength $\lambda_1 = D/\nu t$, Light Speed $C_1 = P/t$ and Frequency $\nu_1 = C_1/\lambda_1$ of Doppler Shifts. This mathematical model can be used for various Doppler Shifts including Axial, Acceleration and Transverse Doppler Shifts. In general, the mathematical analyses agree very well with the observations.

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