# **Supersymmetric E<sub>6</sub> Models with Low Intermediate Scales**

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**Abstract:** We propose supersymmetric  $E_6$  models with intermediate Left-right symmetry as a result of spontaneous compactification of  $E_8$  theory in a ten dimensional space. We show that much lower value of Left-right symmetry breaking scale and consistent unification scale can be achieved in presence of some appropriate light multiplets resulting from spontaneous compactification of higher dimensions at the Planck scale. In the model we could successfully lower the intermediate Left-right symmetry breaking scale  $M_R$  up to  $10^5$  GeV. With such a lower value of  $M_R$ , we can easily accommodate low scale leptogenesis in tune with gravitino constraint. The model can also predict desired value of neutrino mass that can be tested at LHC.

Keywords: Exceptional groups, Left-right symmetry, Renormalization group equation, Supersymmetry.

### I. Introduction

Exceptional groups play a key role in particle physics i.e.  $E_3 = SU(3) \otimes SU(2), E_4 = SU(5), E_5 =$ SO(10), E<sub>6</sub> and E<sub>8</sub> are the known non-abelian gauge groups of unification models. Specifically the Grand-Unification Theories (GUT) based on an E<sub>6</sub> gauge group turn out to be very promising candidates for unification that have no gauge anomalies.  $E_6$  [1] is the only exceptional Lie group that has complex representations and therefore the only exceptional group that can be used as a GUT in four dimensions. Further its Super symmetric (SUSY) version is inspired by  $E_8 \otimes E'_8$  string theory [2], which is the theory of everything. In the present paper, we consider super-symmetric  $E_6$  model with two possible intermediate symmetries, with an attempt to correlate it with the well-known cosmological problem of matter-antimatter asymmetry through the possibility of Leptogenesis [3]. The discovery of neutrino masses makes leptogenesis a very attractive scenario for explaining the puzzle of the baryon asymmetry of the universe. In its simplest version, Leptogenesis is dominated by the CP violating interactions of the lightest heavy Majorana neutrinos, thus relate the observed baryon asymmetry in the Universe to the low-energy neutrino data. The original GUT-scale leptogenesis scenario, however, runs into certain difficulties within super symmetric models. In particular, potential problem arises from the over closure of the universe by the thermally produced gravitinos. To avoid overproduction of gravitinos, the reheat temperature of the Universe should be lower than  $10^9 - 10^6$  GeV. This implies that the heavy Majorana neutrinos should accordingly have masses as low as  $10^9$  GeV. Since leptogenesis takes place just below the scale of Left-right symmetry breaking, one has to search for a model with low scale Left-right symmetry breaking. On the other hand, when the mass scale of the right-handed neutrinos is low, it has been shown that sufficient amount of baryon asymmetry in the universe can be generated through Resonant Leptogenesis [4]. In tune with the above requirements, in the present paper, we propose super symmetric  $E_6$  models with two possible breaking chains, such that much lower value of Left-right symmetry breaking scale and a consistent unification scale can be achieved in presence of some additional light Higgs multiplets. We address the issues of neutrino mass, low-scale leptogenesis consistent with the gravitino constraint, manifest unification of gauge couplings through renormalizable interactions, which can be testable at the Tevatron, LHC or ILC. The paper is organized as follows. In the next section, we discuss the models along with the patterns of symmetry breaking.

Section–3 is devoted to obtain the mass scales at different stages through the Renormalization Group calculations including the one-loop beta function. We shall then conclude in the last section with a remark on the possibility of a light neutrino and leptogenesis.

### II. The Model

In the present paper, we take an  $E_6$  gauge theory coupled with N =1 SUSY in four dimension. This  $E_6$  gauge model may be viewed as a remnant of supersymmetric  $E_8$  group in a ten dimensional theory with compact six dimensional coset space ( $G_2/SU(3)$ ). It has been shown in [5] that, as a result of Coset Space Dimensional Reduction one can obtain a  $E_6$  model with Higgs  $\{27+\overline{27}+650\}$  [6]. In the conventional superstring inspired  $E_6$  models [2], the Higgs sector is confined to only 27 and  $\overline{27}$ . However here we shall take an additional Higgs belonging to  $\{650\}$  representation of  $E_6$  to lower the left-right symmetry breaking scale. To allow low scale left-right symmetry breaking scale we take account of contribution to beta function from light chiral multiplets belonging to  $\{650\}$  representation of  $E_6$  We shall now consider the symmetry breaking pattern from  $E_6$  to low energy as given by the following two models.

Model -I:

 $\begin{array}{c} E_{6} \otimes SUSY \xrightarrow{M_{P}} SU(2)_{L} \otimes SU(2)_{R} \otimes SU(4)_{C} \otimes U(1)_{\Psi} (G_{2241}) \otimes SUSY \\ \xrightarrow{M_{U}} SU(2)_{L} \otimes SU(2)_{R} \otimes SU(3)_{C} \otimes U(1)_{B-L} \otimes U(1)_{\Psi} (G_{22311}) \otimes SUSY \\ \xrightarrow{M_{R}} SU(2)_{L} \otimes SU(3)_{C} \otimes U(1)_{y} \otimes U(1)_{\chi} (G_{2311}) \otimes SUSY \\ \xrightarrow{M_{S}=M_{\chi}} SU(2)_{L} \otimes SU(3)_{C} \otimes U(1)_{y} (G_{231}) \\ \xrightarrow{M_{Z}} SU(2)_{L} \otimes SU(3)_{C} \otimes U(1)_{0} (G_{31}) \end{array}$ (1)

In the above breaking channel, the first step of symmetry breaking takes place at Planks scale. Here the exceptional gauge group  $E_6$  is broken down to the Left – Right Pati-Salam group  $(G_{422})$  extended by an additional  $U(1)_{\Psi}$  by the vacuum expectation value (VeV) of  $(1,1,1)_0$  ( $G_{4221\Psi}$ ) contained in 54<sub>0</sub> representation of SO(10)  $\otimes U(1)_{\chi} \subset 650$  of  $E_6$ . In the first step, the effect of 54<sub>0</sub> containing  $G_{224}$ -singlets breaks SO(10) keeping D-parity intact [6]. Then in the second step, the SU(4) symmetry is broken down to SU(3)<sub>C</sub> $\otimes U(1)_{B-L}$  by the VeV of  $(1,1,15)_0 \subset 45_0$  and  $210_0 \subset 650$ , near the Gut scale  $M_U$ . Here we may note that within the mass scale  $M_U$  and  $M_P$  there is a surrogate SUSY GUT [7] based on ( $G_{2241}$ )  $\otimes$  SUSY which resolves the proton decay as well as the doublet-triplet splitting problem of SUSY GUTs, with a Planck scale unification in a more natural way. This is also expected, as  $E_6$  may be viewed as remnant of  $E_8$  superstring theory at the Planck scale. In the next step the SU(2)<sub>R</sub> $\otimes U(1)_{\Psi} \otimes U(1)_{B-L}$  symmetry is broken down to  $U(1)_y \otimes U(1)_\chi$  symmetry at the  $M_R$  scale, by the VeV of  $(1,2,4)_{1/2} \oplus (1,2,4)_{-1/2}$  of  $16_{1/2} \oplus \overline{16}_{-1/2}$  (SO(10)  $\otimes U(1)_{\chi}$ )  $\subset 27 \oplus \overline{27}$  representation of  $E_6$ . Then the  $U(1)_{\chi}$  symmetry is spontaneously broken down at the super symmetry scale  $M_S$  which is assumed to occur at the TeV scale. This is achieved by the  $G_{2241}$  singlet S(1,1,1)<sub>2</sub> contained in the {27} representation. Finally the electroweak symmetry breaking is achieved by the VeV of the bi-doublet (2,2,1)\_{-1} \subset 10\_{-1} \subset 27, at  $M_Z$ .

## Model-II:

Here the SUSY  $E_6$  symmetry is broken down to the Left-right symmetry  $SU(2)_L \otimes SU(2)_R \otimes SU(3)_C \otimes U(1)_{B-L}$  extended by an additional  $U(1)_{\Psi}$ , i.e.  $(G_{22311})$ , by the vacuum expectation value (VeV) of  $(1,1,15)_0(G_{2241}) \subset 210_0 (SO(10) \otimes U(1)_{\Psi}) \subset 650$ , near the Planck scale  $M_P$ . In the next step, as in the previous model, the  $SU(2)_R \otimes U(1)_{\Psi} \otimes U(1)_{B-L}$  symmetry is spontaneously broken to  $U(1)_y \otimes U(1)_{\chi}$  symmetry at the  $M_R$  scale. This  $U(1)_{\chi}$  symmetry is broken spontaneously at a mass scale  $M_{\chi}$  of the order of  $10^3$  GeV, by the  $G_{2311}$  singlet  $(1,1)_{0,2}$  contained in the  $\{27\}$  representation. This may provide a heavy neutral Z-boson along with the conventional Z-boson. Finally the electro-weak symmetry breaking occurs at  $(M_Z)$ . For simplicity, we consider that the Supersymmetry scale  $(M_S)$  lie at the electroweak symmetry breaking scale  $(M_Z)$ .

### III. Gauge Coupling Unification through Renormalization Group Analysis.

Now we discuss the Renormalization Group equations, including one-loop beta function contributions to calculate the corresponding mass scales at different stages. In the minimal supersymmetric models, it has been observed that, the inter mediate scale  $M_R$  is very close to the Grand Unification scale  $M_U$ ~10<sup>16</sup> GeV at the one-loop level, which is inconsistent with the accommodation of leptogenesis in the model. In the present case, we show that the result can be better and consistent with leptogenesis, if some additional light multiples are taken into account. As has been mentioned before, to implement leptogenesis, the intermediate mass scale (where the Left-Right symmetry breaks) must be much lower ( $M_R < 10^9 \text{GeV}$ ). Therefore to lower the  $M_R$  scale consistently, we consider some appropriate additional light multiplets, belonging to {650} representation of  $E_6$  which get their respective masses at the  $M_R$  scale. The effect of these multiplets will be visible through renormalization group equation, which we shall discuss in detail for Model-II and Model-II respectively.

We shall now consider the Renormalization Group (R.G.) equations involved at different mass scales involved in the symmetry breaking pattern of Model-I. Between the mass scales  $M_Z$  and  $M_S$  the R.G. equations run as,

$$\begin{aligned} \alpha_{i}^{-1}(M_{Z}) &= \alpha_{i}^{-1}(M) + \frac{bi}{2\pi} \{ \ln\left(\frac{M}{M_{Z}}\right) \} \\ &\Rightarrow \alpha_{i}^{-1}(M) = \alpha_{i}^{-1}(M_{Z}) - \frac{bi}{2\pi} \left[ \ln(10) \log_{10}(M) - \ln(M_{Z}) \right] \end{aligned}$$
(3)

where,  $\alpha_i = (g_i^2/4\pi)$ ,  $g_i$  being the coupling constant for the corresponding gauge interaction. Here 'i' stands for  $SU(3)_C$ ,  $SU(2)_L$  and  $U(1)_Y$  gauge groups and  $b_i$  is the one loop beta function. The formula for non-super symmetric beta function values involved in one-loop calculations for SU(N) group are given as:

bi = 
$$\frac{-11}{3}$$
 N +  $\frac{4}{3}$  N<sub>g</sub> +  $\frac{1}{6}$   $\sum$  Ti<sup>2</sup>. (4)

Here  $N_g$  is the number of generation,  $T_i$  is the contribution from Higgs. The electro-weak symmetry breaks by the VeV of the bi-doublet  $\phi(2,2,1)_{.1}$  [under  $G_{2241}$ ] and also the Left-handed doublet  $H_L(2,1,4)_{1/2} \subset 16_{1/2} \subset 27$ . Therefore the beta function value for this stage are given as

$$\begin{pmatrix} \mathbf{b}_{y} \\ \mathbf{b}_{21} \\ \mathbf{b}_{3c} \end{pmatrix} = \begin{pmatrix} \frac{21}{5} \\ -3 \\ -7 \end{pmatrix}$$
 (5)

At the  $M_{\text{S}}$  scale the MSSM particles get mass, therefore the formula for beta function values gets changed to,

$$b_i^s = -3N + 2Ng + \Sigma Ti$$
, (for i=SU(3)<sub>C</sub>, SU(2)<sub>L</sub>,U(1)<sub>y</sub>) (6)

For the mass scale lying between Minimal Supper Symmetry breaking scale and the intermediate Left-Right Symmetry breaking Mass scale ( $M_S < M < M_R$ ), the minimal super symmetric standard model one-loop beta function coefficients are given as:

$$\begin{pmatrix} b_y \\ b_{2l} \\ b_{3c} \end{pmatrix} = \begin{pmatrix} \frac{33}{5} \\ 1 \\ -3 \end{pmatrix}$$
(7)

The corresponding R.G. equations is given as,

$$\alpha_i^{-1}(\mathbf{M}) = \alpha_i^{-1}(\mathbf{M}_{\mathrm{S}}) - \frac{b_i^2}{2\pi} \left[ \ln(10) \log_{10}(\mathbf{M}) - \ln(\mathbf{M}_{\mathrm{S}}) \right]$$
(8)

Then for the mass scale between the intermediate Left-Right symmetry breaking scale and GUT scale  $(M_R < M < M_{GUT})$ , the renormalization group equations run as:

$$\alpha_{i}^{-1}(M) = \alpha_{i}^{-1}(M_{R}) - \frac{b_{i}^{s}}{2\pi} [\ln(10) \log_{10}(M) - \ln(M_{R})],$$
  
Here i = SU(3)<sub>C</sub>, SU(2)<sub>L</sub>=SU(2)<sub>R</sub>, U(1)<sub>B-L</sub>. (9)

Now we shall consider some additional light multiplets, which get their respective masses at the intermediate Left-Right Symmetry breaking mass scale  $(M_R)$  as,

- Two copies of  $\sigma(1,1,3)_{-1/3,1} \oplus \overline{\sigma}(1,1,\overline{3})_{1/3,1} (G_{22311}) \subset 10_1 \subset 27$
- ▶ One copy of  $\eta(1,1,3)_{-2/3,0} \oplus \overline{\eta}(1,1,\overline{3})_{2/3,0}(G_{22311}) \subset 45_0 \subset 650$
- Two copies of  $H_L(2,1,1)_{1/2,-3} \oplus \overline{H}_R(1,2,1)_{-1/2,-3}$  and  $\overline{H}_L(2,1,1)_{-1/2,3} \oplus H_R(1,2,1)_{1/2,3}$  (G<sub>22311</sub>)  $\subset 144_{-3} \oplus \overline{144}_3 \subset 650$  of  $E_6$  (10)

So, all these extra light multiplets contribute to the beta function value for the mass scale greater than  $M_R$ . Therefore the one loop beta function contributions between mass scales  $M_R$  and  $M_U$  are given as,

$$\begin{pmatrix} b_{B-L} \\ b_{2L=2R} \\ b_{3c} \end{pmatrix} = \begin{pmatrix} 18 \\ 3 \\ 0 \end{pmatrix}$$
(11)

Finally in between the Grand Unification Mass scale  $M_U$  and Plank scale ( $M_U < M < M_P$ ), the R.G. equation runs as:

$$\alpha_{i}^{-1}(M) = \alpha_{i}^{-1}(M_{U}) - \frac{b_{i}^{2}}{2\pi} [In(10) \log_{10}(M) - In(M_{U})]$$
(For i=  
SU(4)<sub>C</sub>,SU(2)<sub>L</sub>=SU(2)<sub>R</sub> and  $U(1)_{\Psi}$ ) (12)

(15)

The beta function contributions in this stage are given as: (b, b)

$$\begin{pmatrix} b_{\psi} \\ b_{2L=2R} \\ b_{4_c} \end{pmatrix} = \begin{pmatrix} 11.3 \\ 9 \\ 4 \end{pmatrix}$$
(13)

Using the R.G. equations (3), (8), (9) and(12), we can observe the running couplings of SU(4)<sub>C</sub>, SU(2)<sub>L</sub> and SU(2)<sub>R</sub> unify at a very high scale close to the Plank scale (M<sub>P</sub>) $\approx$ 10<sup>19</sup>GeV. We assume the total unification of all the forces at that mass scale (M<sub>P</sub>). Therefore the gauge coupling for U(1)<sub>\P</sub> run down from that scale to the M<sub>R</sub> scale (with the beta function value 11.3). Then at M<sub>R</sub> the values of the gauge coupling constant for U(1)<sub>\(\Cap(R)</sub> [8]] is determined by the linear combination of values of coupling constant of groups U(1)<sub>\(\Pri)</sub>, SU(4)<sub>C</sub> and SU(2)<sub>R</sub>. At the M<sub>R</sub> scale, the SU(2)<sub>R</sub> $\otimes$ U(1)<sub>\(\Pri)</sub> $\otimes$ U(1)<sub>B-L</sub> symmetry is broken to U(1)<sub>\(\Cap(R)</sub>  $\otimes$ U(1)<sub>\(\Cap(R)</sub> symmetry by the VeVs of (1,2,4)<sub>-1/2</sub>H<sub>R</sub>  $\oplus$  (1,2,4)<sub>1/2</sub>H<sub>R</sub>. The gauge bosons associated with diagonal SU(4) generator is  $T_4^{15} = T_{B-L}$ , the diagonal SU(2)<sub>R</sub> generator  $T_R^3$  and the U(1)<sub>\(\Pri)</sub> generator T<sub>\(\Pri)</sub> are related by the Higgs bosons to create two light mass less gauge bosons associated with U(1)<sub>\(\Pri)</sub> and U(1)<sub>\(\Pri)</sub>.

$$U(1)_{B-L} \bigotimes U(1)T_4^3 \bigotimes U(1)_{\Psi} \longrightarrow U(1)_Y \bigotimes U(1)_{\chi}$$
(14)

Therefore at M<sub>R</sub>, we can write,  $\chi = T_{\psi} + T_R^3 - C_{12}^2 Y$ 

Where  $C_{12} = \cos \theta_{12}$ , for  $\theta_{12}$ , is the mixing angle such that,  $\tan \theta_{12} = \frac{g_{2R}}{g_{B-L}}$ . Here Y and  $\chi$  are not normalized, such that the normalized charges are,  $Y \implies (Y/N_Y)$  and  $\chi \implies (\chi/N_\chi)$ , where  $N_Y^2 = 3/5$ ,  $N_\chi^2 = 7-2C_{12}^2 + (\frac{5}{2})C_{12}^4$ . Thus at  $M_R$  we can write that,

$$\alpha_{B-L}^{-1} = \frac{5}{2} \alpha_Y^{-1} - \frac{3}{2} \alpha_{2L}^{-1}$$
(16)

Now the value of the gauge coupling constant of  $U(1)_\chi$  at  $M_R$  scale, can be calculated from the following equation.

$$\alpha_{\chi}^{-1} = \frac{1}{N_{\chi}^{2}} \left[ 6 \, \alpha_{\psi}^{-1} + \frac{1}{\alpha_{B-L} + \alpha_{2R}} \right] \tag{17}$$

Here,  $N_{\chi}^2$  is the normalization constant and is related to the cosine of the mixing angle C<sub>12</sub>, as has been mentioned before. In between the mass scales M<sub>R</sub> and M<sub>S</sub>, the gauge coupling of U(1)<sub> $\chi$ </sub> runs down according to the renormalization group equation

$$\alpha_{\chi}^{-1}(M) = \alpha_{\chi}^{-1}(M_{\rm R}) + \frac{b\chi}{2\pi} \{ \ln (M_{\rm R}) - \ln(M) \}$$
(18)

Here  $b\chi$  is the beta function value for U(1) $\chi$ , which also varies with the cosine of the mixing angle C<sub>12</sub> as in the formula bellow

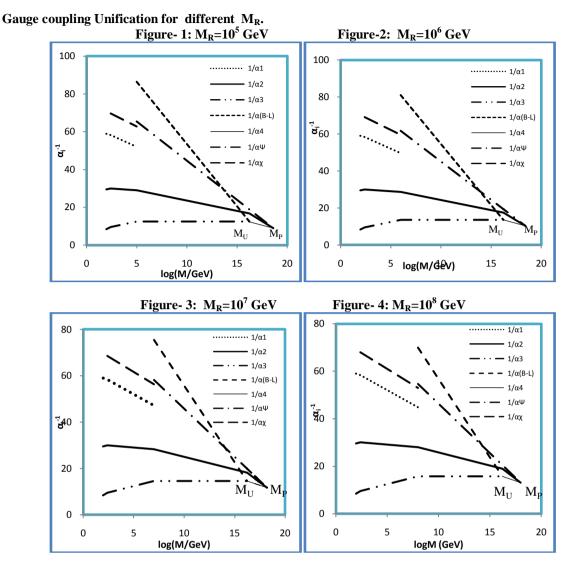
$$b\chi = 6 + \Sigma T_{\chi}^2 = 6 + \frac{1}{N_{\chi}^2} \Sigma \chi_i^2$$
(19)

Where  $\chi_i$  can be calculated by equation (15). The values of  $\chi_i^2$ ,  $N_{\chi}$  and  $b_{\chi}$  for different  $M_R$  scale are noted in the Table-1.

M <sub>R</sub> (GeV)	$C_{12}^2$	$\sum \chi_i^2 = \chi_{\phi}^2 + \chi_S^2$	$N_{\chi}^2$	b <sub>χ</sub>
10 <sup>5</sup>	0.2518	8.5598	6.602	7.296533
$10^{6}$	0.2619	8.4478	6.5905	7.296529
107	0.2731	8.5283	6.578	7.29648
10 <sup>8</sup>	0.2857	8.5101	6.5645	7.29637

Table-1: Beta function value of  $U(1)_{\mu}$  corresponding to different  $M_{\rm B}$ 

Using the evolution equations (3), (8), (9), (12) and (18) along with the values of  $b_{\chi}$  (as given in Table - 1), we obtain the mass scales  $M_U$  and  $M_P$  graphically for  $M_R$  in the range  $10^5$ -  $10^8$  GeV. Here we have used the input values of Standard Model coupling measured on the Z-pole at LEP as  $\alpha_1(M_Z) = 0.016947$ ,  $\alpha_2(M_Z) = 0.033813$  and  $\alpha_3(M_Z) = 0.1187$  to calculate the couplings. We observe that consistent  $M_U$  and  $M_P$  are obtained even with low  $M_R$  as given in Figure 1-4.



From the graph, we have noted down the values of coupling constants and masses at the unification scale corresponding to the different values of the intermediate scales ( $M_R$ ). The result is given in Table-2.

Table 2. What scales $W_0$ , $W_p$ and $w_p$ for unreferent values of $W_R$ .					
M	<sub>R</sub> in (GeV)	M <sub>U</sub> in (GeV)	$M_P$ in (GeV)	$\alpha_{\rm p}^{-1}$	
	$10^{5}$	$10^{16.229}$	$10^{18.61}$	8.899	
	$10^{6}$	$10^{16.2285}$	$10^{18.405}$	10.307	
	107	$10^{16.2285}$	$10^{18.205}$	11.7	
	$10^{8}$	$10^{16.2285}$	$10^{18.005}$	13.093	

Table-2 : Mass scales  $M_U$ ,  $M_P$  and  $\propto_n^{-1}$  for different values of  $M_R$ .

Similarly, we can achieve a low intermediate scale with Model-II( as given in eq. (2)) and a consistent Planck scale unification with an intermediate  $G_{22311}$  symmetry, when the contributions from some light multiplets are taken into account. Now we consider the R.G. equations at different mass scales as has been done in Model-I. Between the mass scales  $M_{Z=S}$  to  $M_{\chi}$  the RNGE are given as,

$$\alpha_{i}^{-1}(M_{Z}) = \alpha_{i}^{-1}(M_{\chi}) + \frac{bi}{2\pi} \{ \ln\left(\frac{M}{M_{Z}}\right) \}, \ i = 3c, 2L, Y$$
(20)

Where the one loop beta function  $b_i$  includes the supersymmetric contributions as has been given in eq. (7). Then in the similar manner, as in the previous model, we can have Renormalization Group equations for subsequent stages of symmetry breaking. Between the mass scale  $M_{\chi}$  to  $M_R$  we follow the same evolution equations. Now we consider some appropriate additional light multiplets, belonging to {650} and {27} representation of  $E_6$  which get their respective masses at the  $M_R$  scale in order to achieve low intermediate symmetry. We take the following light multiplets,

(21)

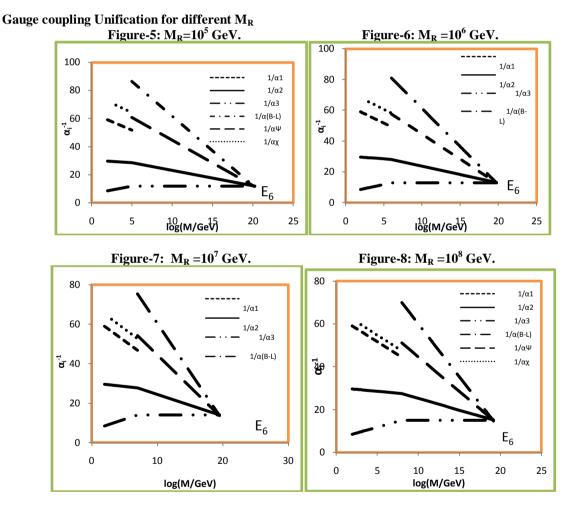
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- $\succ$  η(1,1,6)-2/3,0 (G<sub>22311</sub>) ⊂(1,1,20')<sub>0</sub>(G<sub>2241</sub>) ⊂ 54<sub>0</sub>⊂650
- $\blacktriangleright$   $\overline{\sigma}$  (1,1,3)-1/3,-1(G<sub>22311</sub>) ⊂ (1,1,6)-1⊂10 -1⊂27
- $\triangleright$  φ(2,2,1)<sub>0,-1</sub> (G<sub>22311</sub>) ⊂10<sub>-1</sub>⊂27 of E<sub>6</sub>

All these extra light multiplets contribute to the beta function values. Therefore the one loop beta function contributions  $M_R$  to  $M_P$  are given as,

$$\begin{pmatrix} b_{B-L} \\ b_{2L=2R} \\ b_{3c} \\ b_{\psi} \end{pmatrix} = \begin{pmatrix} 13.5 \\ 3 \\ 0 \\ 8.833 \end{pmatrix}$$
(22)

Using the evolution equations we have done a graphical analysis in a similar manner as has been done in the Model-1. It is observed that the running couplings of  $SU(3)_C$ ,  $SU(2)_L$ ,  $SU(2)_R$  and  $U(1)_{B-L}$  get unified at a very high scale close to the Plank scale  $(M_P)\approx 10^{19}$ GeV, for an allowed value of  $M_R$  in the range  $10^5$ - $10^9$  GeV. Here the gauge coupling for  $U(1)_{\Psi}$  run down from  $M_P$  to the  $M_R$  scale with beta function value 8.83. The corresponding graphs are given in Figure: 5-8.

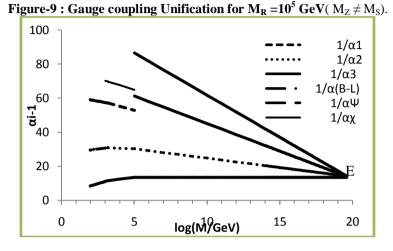


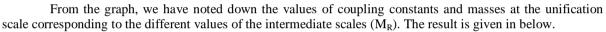
From the graph, we have noted down the values of coupling constants and masses at the unification scale corresponding to the different values of the intermediate scales ( $M_R$ ). The result is given in Table-3.

**Table-3 : Values of**  $M_P$  and  $\propto_p^{-1}$  for different  $M_R(M_Z = M_S)$ 

	- P	
M <sub>R</sub> in GeV)	$M_P$ in (GeV)	$\alpha_p^{-1}$
10 <sup>5</sup>	$10^{20.1}$	11.78
$10^{6}$	$10^{19.76}$	12.9
$10^{7}$	$10^{19.41}$	14.04
$10^{8}$	10 <sup>19.1</sup>	15.09
109	10 <sup>18.76</sup>	16.2

Here we note that, this model does not allow  $M_R$  scale to lie below  $10^5$ GeV, as the unification scale is shown to be high (~ $10^{20}$  GeV.). Thus it puts a lower bound on  $M_R$  as compared to Model-I. We have also investigated for the case  $M_Z \neq M_S$  and  $M_S = M_\chi$  to lie at TeV scale, as has been done in Model-1. For the sake of completeness, we give the graph for evolution of couplings, for  $M_R=10^5$  GeV ( $M_Z \neq M_S$ ) in Figure-9. It is observed that, for  $M_S$  at the TeV scale, the situation improves with consistent unification scale.





**Table-4 : Values of**  $M_P$  and  $\propto_P^{-1}$  for different  $M_R$  ( $M_Z \neq M_S$ ).

Tuble 4. Values of hip and sp tot affected hig $(m_Z \neq m_S)$				
]	M <sub>R</sub> in (GeV)	$M_P$ in (GeV)	$\propto_{p}^{-1}$	
	$10^{5}$	10 <sup>19.68</sup>	13.83	
	$10^{6}$	$10^{19.34}$	14.94	
	107	$10^{19.03}$	15.99	
	$10^{8}$	10 <sup>18.67</sup>	17.14	
	10 <sup>9</sup>	$10^{18.35}$	18.22	

#### IV. Discussion

We have considered super symmetric  $E_6$  models [10] with two possible intermediate scales, in which we can have a low Left-right symmetry breaking scale  $M_R$  of the order of  $10^5$ - $10^8$  GeV. In the given models, where we explore the possibility of low  $M_R$ , it is difficult to achieve consistent unification with minimal particle contents. This is achieved by introducing additional light multiplets belonging to {650} and {27} representation of  $E_6$ . Further the model can also predict a light left handed neutrino [11] through the double seesaw and type III seesaw mechanism, with the presence of the singlet S (1,1,1)<sub>2</sub>. The mass can be obtained in the sub-eV range, even for low  $M_R$  without fine-tuning of the Yukawa coupling. As far as leptogenesis is concerned, it allows resonant leptogenesis through the decay of the singlet fermions, which are the superposition of S(1,1,1)<sub>2</sub> and the right handed neutrino. These states lie below  $10^8$  GeV constrained by the low  $M_R$  to generate lepton asymmetry, consistent with the gravitino constraint. Thus the supersymmetric  $E_6$  models with low intermediate scales can have nice phenomenological implication and can be testable in the future colliders.

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