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Solution of MHD Effect on Transient Free Convection Flow past a Vertical Plate with Variable Temperature and Chemical Reaction of First Order.

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Abstract: *The first order chemical reaction effects on transient free convective flow of a viscous incompressible flow past a vertical plate with mass transfer in the presence of magnetic field is considered. Solutions of the governing equations are obtained by the Laplace transform method and are presented graphically for the different values of physical parameters. It is observed that magnetic field parameter and chemical reaction parameter influence the velocity and concentration profiles significantly.*

Key words: *Chemical Reaction, Free Convection, MHD, Variable Temperature*

I. Introduction

Heat and mass transfer with chemical reaction occurs almost in many branches of science and engineering. There are two types of Chemical reactions, homogeneous and heterogeneous. In a mixed system, a reaction is said to be homogeneous if it takes place uniformly within the solution and heterogeneous if it takes place at an interface. In most chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be first-order if the rate of the reaction is directly proportional to concentration. In many chemical engineering processes, chemical reaction occurs between a foreign mass and the fluid. This type of chemical reaction may change the temperature and the heat content of the fluid and may affect the free convection process. However if the presence of such foreign mass is very low then we can assume the first order chemical .These processes takes place in numerous applications such as food processing, polymer production, manufacturing of ceramics.

Further due to some important industrial and engineering applications such as liquid metal cooling in nuclear reactors, magnetic control of molten iron flow in steel industry etc., magneto-convection has also been gaining considerable attention amongst researchers. Hence combined study will surely enhance the already developed areas further for more complex studies.

Exact solutions of free convection flow past a vertical plate in free convective flow was first obtained by Soundalgekar [14] and the same problem with mass transfer effect was considered by Soundalgekar and Akolkar [15]. Das *et. al.* [3] studied the effects of mass transfer on free convection flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. They also studied the transient free convection flow past infinite vertical plate with periodic temperature [5]. Effect of mass transfer on the flow past an infinite vertical oscillating plate with constant heat flux was studied by Soundalgekar *et. al.* [17].

The effects of mass transfer on free convection flow past a semi-infinite vertical isothermal plate was first studied by Gebhart and Pera [10] and the effects of mass transfer on the flow past an impulsively started infinite vertical plate with variable temperature was studied by Soundalgekar *et. al.* [16]. Muthucumaraswamy *et. al.* [12] considered the effects of mass transfer on impulsively started infinite vertical plate with variable temperature and uniform mass flux. All of them considered the fact that free convection current caused by temperature differences is also caused by the differences in concentration or material constitution as suggested by Gebhart [9].

Das *et. al.* [4] considered the effects of mass transfer on flow past an impulsively started vertical plate and Muthucumaraswamy and Meenakshisundaram [13] studied the chemical reaction effects on vertical oscillating plate with variable temperature and chemical reaction. Deka and Neog [6], [7] considered the combined effects of thermal radiation and chemical reaction on free convection flow past a vertical plate in porous medium and with MHD respectively.

In a very recent paper Neog [8] studied the effect of MHD with chemical reaction on electrically conducting fluid past oscillating plate. Chaudhary and Jain [2] studied the magneto hydrodynamic transient heat and mass transfer flow by free convection past a vertical plate, when the temperature of the plate oscillates in time about a constant mean temperature and the plate is embedded in a porous medium. They extended the work of Das et.al. [3], which include the effects of mass transfer, magnetic field and porous medium.

Although many authors studied mass transfer with or without chemical reaction in flow past oscillating vertical plate by considering different surface conditions but the study on the effects of magnetic field on free convection heat and mass transfer in the presence of transverse magnetic field and chemical reaction with variable temperature has not been found in literature and hence the motivation to undertake this study. It is therefore proposed to study the effects of chemical reaction on hydro magnetic flow past a vertical plate with variable temperature under the assumption of first order chemical reaction.

II. Mathematical Analysis

An unsteady natural convection flow of a viscous incompressible electrically conducting fluid past an infinite vertical plate is considered here. To visualize the flow pattern a Cartesian co-ordinate system is considered where x'-axis is taken along the infinite vertical plate, the y'-axis is normal to the plate and fluid fills the region $y' \geq 0$. Initially, the fluid and the plate are kept at the same constant temperature T'_∞ and species concentration C'_∞. At time $t' > 0$, the plate is assumed to be moving continuously in its own plane with a uniform velocity U_0 and at the same time the plate temperature is raised linearly with time and the level of species concentration is raised to C'_w . A magnetic field of uniform strength B_0 is applied normal to the plate. It is assumed that the magnetic Reynolds number is very small and the induced magnetic field is negligible in comparison to the transverse magnetic field. It is also assumed that the effect of viscous dissipation is negligible in the energy equation and the level of species concentration is very low so the Soret and Dufour effects are negligible.

As the plate is infinite in extent so the derivatives of all the flow variables with respect to x' vanish and they can be assumed to be functions of y' and t' only. Thus the motion is one dimensional with only non-zero vertical velocity component u', varying with y' and t' only. Due to one dimensional nature, the equation of continuity is trivially satisfied.

Under the above assumptions and following Boussinesq approximation, the unsteady flow field is governed by the following set of equations:

$$\frac{\partial u'}{\partial t'} = g\beta(T'-T'_\infty) + g\beta^*(C'-C'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho}u' \tag{1}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K_1 C' \tag{3}$$

The following initial and boundary conditions are considered here.

$$\left.\begin{array}{ll} u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \quad for\ all\ y'\ and\ t' \leq 0 & \\ \left.\begin{array}{l} u' = U_0, \quad T' = T'_\infty + (T'_w - T'_\infty)At', \quad C' = C'_w \quad at\ y' = 0 \\ u' \to 0, \quad T' \to T'_\infty, \quad C' \to C'_\infty \quad as\ y' \to \infty \end{array}\right\}, t > 0 & \end{array}\right\} \tag{4}$$

Where, $A = \frac{u_0^2}{\nu}$.

Now, to reduce the above equations in non-dimensional form we introduce the following non-dimensional quantities.

$$\left.\begin{array}{l} u = \frac{u'}{U_0},\ t = \frac{t' U_0^2}{\nu},\ y = \frac{y' U_0}{\nu},\ \theta = \frac{T'-T'_\infty}{T'_w-T'_\infty}, \\ \phi = \frac{C'-C'_\infty}{C'_w-C'_\infty}, \quad Gr = \frac{g\beta\nu\,(T'_w-T'_\infty)}{U_0^3}, \\ Gm = \frac{g\beta^*\nu\,(C'_w-C'_\infty)}{U_0^3},\ \Pr = \frac{\mu C_p}{\kappa}, \\ R = \frac{\nu K_1}{U_0^2}, \quad Sc = \frac{\nu}{D}, \quad M = \frac{\sigma B_0^2 \upsilon}{\rho U_0^2} \end{array}\right\} \tag{5}$$

Thus, with the introduction of these non dimensional quantities (5), the equations (1), (2) and (3) are reduced to the following forms.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\,\theta + Gm\,\phi - Mu \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{7}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} - R\phi \tag{8}$$

And the initial and boundary conditions (4) are as follows:

$$\left.\begin{array}{llll} u = 0, & \theta = 0, & \phi = 0, & \text{for all } y \text{ and } t \le 0 \\ u = 1, & \theta = t, & \phi = 1 & \text{at } y = 0 \\ u \to 0, & \theta \to 0, & \phi \to 0 & \text{as } y \to \infty \end{array}\right\} \begin{array}{l} \\ \left.\right\}, \ t > 0 \end{array} \tag{9}$$

Solutions of the equations (6), (7) and (8) subject to the initial and boundary conditions (9) are obtained by Laplace transform technique. Thus with the help of Abramowitz and Stegum [1] and Hetnarski's [11] algorithm for inverse Laplace transform we have obtained the solutions as follows:

$$\theta(y,t) = \left(t + \frac{y^2 Pr}{2}\right) erfc\left(\frac{y\sqrt{Pr}}{2\sqrt{t}}\right) - \frac{y\sqrt{t\,Pr}}{\sqrt{\pi}} e^{-\frac{y^2 Pr}{4t}} \tag{10}$$

$$\phi(y,t) = \frac{1}{2}\left\{e^{-y\sqrt{ScR}} erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{Rt}\right) + e^{y\sqrt{ScR}} erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{Rt}\right)\right\} \tag{11}$$

$$\begin{aligned} u(y,t) = {} & \frac{G_4}{2}\left\{e^{-y\sqrt{M}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{Mt}\right) + e^{y\sqrt{M}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{Mt}\right)\right\} \\ & + \frac{yG_1}{4b\sqrt{M}}\left\{e^{-y\sqrt{M}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{Mt}\right) - e^{y\sqrt{M}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{Mt}\right)\right\} \\ & + \frac{G_1}{2b^2}\left[e^{bt}\left(\left\{e^{-y\sqrt{c}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{ct}\right) + e^{y\sqrt{c}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{ct}\right)\right\}\right.\right. \\ & \left.\left. - \left\{e^{-y\sqrt{b\,\mathrm{Pr}}} erfc\left(\frac{y\sqrt{\mathrm{Pr}}}{2\sqrt{t}} - \sqrt{bt}\right) + e^{y\sqrt{b\,\mathrm{Pr}}} erfc\left(\frac{y\sqrt{\mathrm{Pr}}}{2\sqrt{t}} + \sqrt{bt}\right)\right\}\right)\right] \\ & + \frac{G_2}{2d}\left[e^{dt}\left(\left\{e^{-y\sqrt{k}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{kt}\right) + e^{y\sqrt{k}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{kt}\right)\right\}\right.\right. \\ & \left. - \left\{e^{-y\sqrt{hSc}} erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{ht}\right) + e^{y\sqrt{hSc}} erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{ht}\right)\right\}\right) \\ & \left. + \left\{e^{-y\sqrt{ScR}} erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{Rt}\right) + e^{y\sqrt{ScR}} erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{Rt}\right)\right\}\right] \\ & + \frac{G_1}{b}\left[\left(t + \frac{y^2\,\mathrm{Pr}}{2}\right) erfc\left(\frac{y\sqrt{\mathrm{Pr}}}{2\sqrt{t}}\right) - \frac{y\sqrt{t\,\mathrm{Pr}}}{\sqrt{\pi}} e^{-\frac{y^2\,\mathrm{Pr}}{4t}}\right] + \frac{G_3}{2} erfc\left(\frac{y\sqrt{\mathrm{Pr}}}{2\sqrt{t}}\right) \end{aligned} \tag{12}$$

Here, the following symbols are used in the above solutions:

$$\left.\begin{aligned}
&b = \frac{M}{\Pr - 1}, c = M + b, d = \frac{RSc - M}{1 - Sc}, h = R + d, \\
&k = M + d, \quad G_1 = \frac{Gr}{\Pr - 1}, G_2 = \frac{Gm}{Sc - 1}, \\
&G_3 = 1 - \frac{G_2}{d} - \frac{G_1}{b^2}, \quad G_4 = G_3 - \frac{tG_1}{b}
\end{aligned}\right\} \qquad (13)$$

III. Results and Discussion

In order to know the influence of different physical parameters viz., chemical reaction parameter (R), Schmidt number (S_C), thermal Grashof number(G_r), mass Grashof number(G_m) , Hartmann number (M) , Prandtl number(P_r) and time on the physical flow situation .Computations are carried out for vertical velocity, temperature and concentration and they are presented graphically in figure.

In figure-1 concentration profiles are presented for different values of Schmidt number (Sc) and chemical reaction parameter (*R*). From the figure it is observed that increase of Schmidt number (Sc) and chemical reaction parameter (*R*) lead to the decrease in concentration of the species.

Figure-2(a) represents the temperature profiles for different values of Prandtl number (*Pr*) and figure-2(b) represents temperature profiles at different times *t*. Since temperature is considered as time dependent, therefore, figure 2(a) & 2(b) clearly reflects the situation. It is further observed that temperature decreases with the increase of *Pr*.

Figures-3(a) & 3(b) represents the velocity Profiles for different values of parameters *Gr*, *Gm*, *R* and *M*, *Sc* and *Pr*. Influence of *Gr, Gm* and *R* are shown in figure 3(a) for fixed values of *M*, *Sc* and *Pr*. It is clear from the figure -3(a) that velocity increases with the increase of *Gr* and *Gm* but decreases with the increases of *R*. In figure 3(b) effects of *M* and *Sc* are presented for some fixed values of *t, Gr, Gm, Pr* and *R*. It is clear from the figure 3(b) that velocity decreases when *M* and *Sc* increase.

IV. Figures

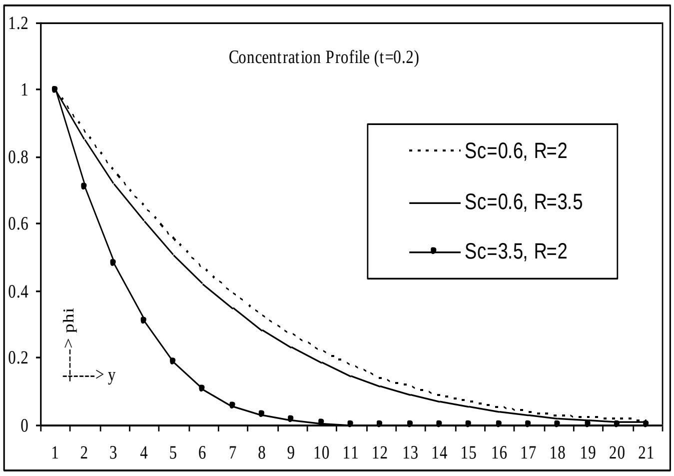

Figure-1: Concentration Profile (Effect of Chemical reaction parameter and Schmidt number).

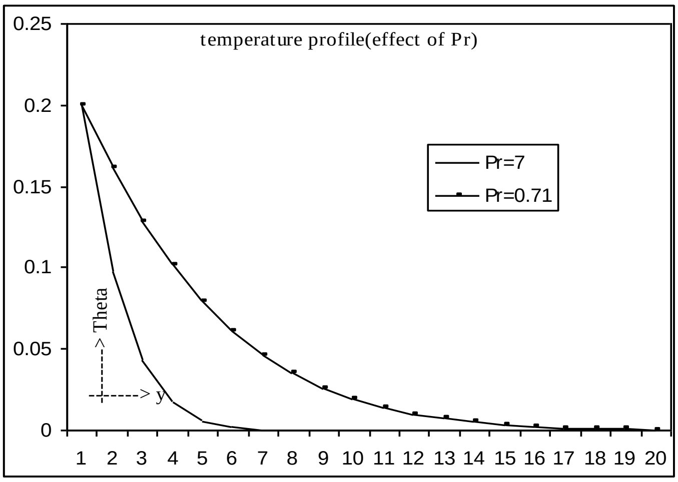

Figure-2(a): Temperature Profile (Effect of Pr)

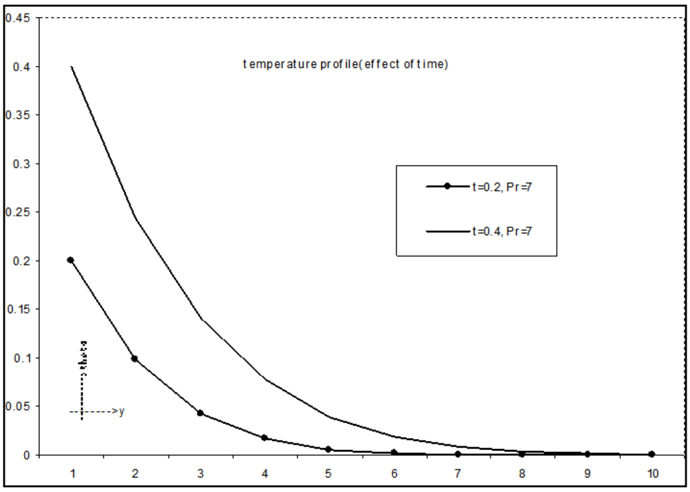

Figure-2(b): Temperature Profile (Effect of time)

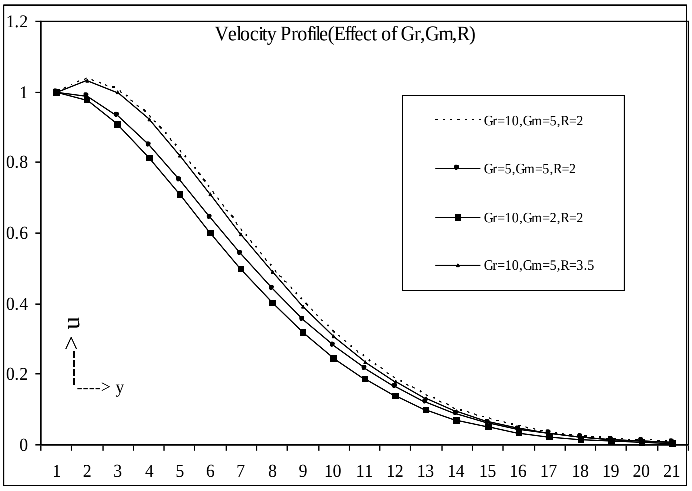

Figure-3(a): Velocity Profile (Effect of Gr, Gm and R)

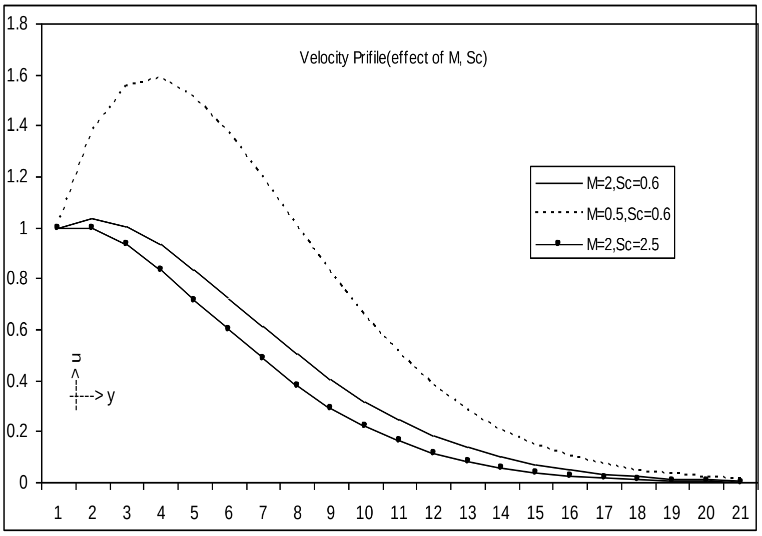

Figure-3b: Velocity Profile (Effect of M and Sc)

V. Conclusions

An exact analysis is performed to study the influence of chemically reacting hydro- magnetic flow past a vertical plate with variable temperature and chemical reaction. Exact solutions of equations are obtained by Laplace transform technique. Some of the important conclusions of the study are as follows:

- Concentration decreases as *Sc* and *R* increase.
- Velocity increases with increasing *Gr*, *Gm* and decreasing *M* and *R*.
- Also increase in *Sc and R* lead to decrease in velocity.

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