# A New Version of The Grand Unified Theory 

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#### Abstract

In this paper a new version of the Grand Unified Theory GUT is presented. By using the Schwarzschild solution of the gravitational equation by Einstein and using a Riemann-Christoffel tensor together with Christoffel symbols of general relativity, a useful potential function is obtained. By combining the Schwarzschild solution with quaternions, a solution is obtained of the extended t-parameter Einstein energy tensor. This leads to information about conservation of energy and the sum of forces equal to zero. The mass term of the Schwarzschild solution can be divided into several similar masses of physics like gravitation, electromagnetic force, weak interaction and strong interaction. By using the Dynamic Relativistic Laplace Equation (DRLE) in rectangular coordinates this equation has a complex solution similar to a light wave. Similar calculations have been derived with spherical coordinates giving similar solutions for different systems like galaxies, planets, molecules, atoms and atomic nuclei. Many examples of these different systems are shown here by using DRLE. These calculations can be further verified by using a 5-dimentional ultrasphere and calculations using geodetic lines, giving information about flat space time and linear space.


Keywords : Astrophysics, Theory of Relativity, Theory of Everything, Universal Formula, Atomic, Nuclear and Molecular physics, Linear Space and Flat Space Time.

## I. Introduction

A Grand Unified Theory (GUT) is a model in particle physics in which at high energy, the three gauge interactionsof the Standard Model, which are the electromagnetic, weak and strong interactions or forces, are merged into one single force. This unified interaction has only one unified coupling constant, and could maybe be realized in nature during the "grand unification period" during Big Bang in the early universe. Several such theories have been proposed during many years, but none has been generally accepted and managed experimental examination satisfactory. What the world is waiting for is a formula, which includes the gravitation too, named "theory of everything" which will show theoretical - and experimental confidence in all parts.In this paper a new version of Grand Unified Theory is derived, which includes gravitation, electromagnetic - , weak and strong interactions and also dark energy.

This very paper starts by using the gravitation equation by Einstein, where Riemann - Christoffel tensor together with Christoffel symbols of general relativity, are used. At these calculations a useful potential Schwarzschild solution is derived, which together with using quaternions, gives an extended t-parameter energy tensor by Einstein. This energy tensor gives information about conservation of energy and that the sum of forces in a system which are equal to zero. A key point in this paper is the division of several similar masses in the Schwarzschild equation from gravitation, electromagnetic force, weak interaction and strong interaction together with their different coupling constants. By using the Dynamic Relativistic Laplace Equation (DRLE) in rectangular coordinates, an equation similar to a light wave is obtained in the complex solution. These calculations have been derived with spherical coordinates for different systems like galaxies, planets, molecules, atoms and nuclei. Many examples of these different systems are shown here by using DRLE. These calculations can be further verified by using a 5-dimentional ultrasphere and calculations using geodetic lines. An extension of the Schwarzschild solution is also derived giving information of the total rest energy and kinetic energies of all forces in a system. Several different proofs of these ideas, have been derived here to manifest this to basic physics. Calculations using geodetic lines, have also giving information about flat space time and linear space.

## II. The Schwarzschild Solution Of The Einstein Gravitational Equations

A. For the law of gravitation by Einstein choose :
$\mathrm{G}_{\mu \mathrm{v}}=0$
where $\mathrm{G}_{\mu \mathrm{v}}$ is the contracted Riemann-Christoffel tensor, Eddington (1):
$\mathrm{G}_{\mu \nu}=\Gamma^{\alpha}{ }_{\mu \rho} \Gamma^{\rho}{ }_{\alpha \nu}-\Gamma_{\mu \nu}^{\alpha} \Gamma^{\rho}{ }_{\alpha \rho}+\left(\partial \Gamma_{\mu \rho}^{\rho} \partial \mathrm{x}_{\nu}\right)-\left(\partial \Gamma_{\mu \nu}^{\rho} / \partial \mathrm{x}_{\rho}\right)$
By using $\Gamma_{\mu \rho}^{\rho}=\left(\partial(\log \sqrt{ }(-\mathrm{g})) / \partial \mathrm{x}_{\mu}\right)$ we can simplify it and with alternations of dummy
suffixes which leads to :
$\mathrm{G}_{\mu \nu}=-\left(\partial \Gamma^{\alpha}{ }_{\mu \nu} /\left(\partial \mathrm{x}_{\alpha}\right)+\Gamma^{\beta}{ }_{\mu \alpha} \quad \Gamma^{\alpha}{ }_{\nu \beta}+\partial^{2} /\left(\partial \mathrm{x}_{\mu} \partial \mathrm{x}_{\mathrm{n}} \mathrm{u}\right)(\log \sqrt{ }(-\mathrm{g}))-\Gamma^{\alpha}{ }_{\mu \nu}\left(\partial(\log \sqrt{ }(-\mathrm{g})) / \partial \mathrm{x}_{\alpha}\right)\right.$ (3)
B. By usingChristoffel symbols of general relativity, will lead to the following expressions:
$\Gamma_{111}^{1}=(1 / 2) \lambda \quad$ ( 4 )
$\Gamma_{12}^{2}=(1 / r) \quad$ (5)
$\begin{aligned} \Gamma_{13}^{3} & =(1 / \mathrm{r}) \\ \Gamma^{4} & =(1 / 2) v\end{aligned}$
$\Gamma^{4}{ }_{14}=(1 / 2) v^{\prime}$
$\Gamma_{22}^{1}=-v \exp (-\lambda) \quad$ (8)
$\Gamma_{23}^{12}=\cot \theta$
$\Gamma^{1}{ }_{33}=-v \sin ^{2} \theta \exp (-\lambda)$
$\Gamma_{33}^{3}=-\sin \theta \cos \theta$
$\Gamma^{1}{ }_{44}=(1 / 2) v^{\prime} \exp (v-\lambda)$
C. We are now introducing the $G_{\mu \nu}$ factors into the Einstein gravitational equations
leading to:
$G_{11}=(1 / 2) v^{\prime \prime}-(1 / 4) \lambda^{\prime} v^{\prime}+(1 / 4) v^{\prime 2}-\left(\lambda^{\prime} / r\right)=0$
$G_{22}=\exp (-\lambda)\left(1+(1 / 2) r\left(v^{\prime}-\lambda^{\prime}\right)\right)-1=0$
$G_{33}=\sin ^{2} \theta\left(\exp (-\lambda)\left(1+(1 / 2) r\left(v^{\prime}-\lambda^{\prime}\right)\right)-1=0\right.$
$G_{44}=\exp (v-\lambda)\left(-(1 / 2) v^{\prime \prime}+(1 / 4) \lambda^{\prime} v^{\prime}-(1 / 4) v^{\prime 2}-\left(v^{\prime} / r\right)\right)=0$
$\mathbf{G}_{12}=\mathbf{0}$
From the first and the fourth equation we have $\lambda^{\prime}=-v^{\prime}$.
Now $\lambda$ and $v$ will vanish when $\mathrm{r} \rightarrow \infty$, which requires that, according to Eddington ( 1 ) and Dodson and Poston ( 2 ):
$\lambda=-v$
Then the second equation ( 14 ) becomes :
$\exp v\left(1+r v^{\prime}\right)=1$
By substituting $\exp v=\gamma$ then
$\gamma+\mathbf{r} \gamma=\mathbf{1}$
and from this by integration we have :
$\gamma=1$ - ( $2 \mathrm{~m} / \mathrm{r}$ )
which is the Schwarzschild potential equation in gravitational units where 2 m is an integration constant according to Schwarzschild ( 3 ) and Eddington ( 1 ).

## III. The Energy Tensor And The 4+1 Forces In The 9-Dimensional World

By extending the Schwartzschild equation ( 21 ) we obtain :
$\gamma=\mathbf{c}^{\mathbf{2}}(\mathbf{1}+\mathbf{i}+\mathbf{j}+\mathbf{k})-\left(2 \mathbf{m}_{\mathrm{G}} \mathbf{G}\right) / \mathbf{r}-\left(\mathbf{i} \mathbf{a}_{2} / \mathbf{r}\right)-\left(\mathbf{j} \mathbf{a}_{3} / \mathbf{r}\right)-\left(\mathbf{k} \mathbf{a}_{4} / \mathbf{r}\right)$
where G is the gravitation constant and the a-factors arecoefficient representing physical parameters at the separation of the ( $2 \mathrm{~m} / \mathrm{r}$ ) -term of equation ( 21 ).The $\mathrm{i}, \mathrm{j}$ and k are Quaternion indices.
Equation (22) is a solution of the extended $t$-parameter in the modified Einstein energy tensor, Einstein (4): $E_{i j}{ }^{\mathrm{t}}=\mathrm{h}_{\mathrm{ij}}\left(\mathbf{d r} \mathrm{r}_{\mathrm{i}} / \mathbf{d t}\right)\left(\mathbf{d r} \mathrm{r}_{\mathrm{j}} / \mathbf{d t}\right)$
which gives the following derivative :
$\mathrm{dE}_{\mathrm{ij}}{ }^{\mathrm{s}} / \mathrm{ds}=\mathbf{0}$
from which conservation of energy is seen and that the sum of all forces $\sum \mathrm{F}_{\mathrm{k}}=0$ by using geodetic lines and derivations with respect of the s-parameter.

## IV. Our System Of Forces

A. By letting
$\mathbf{G}_{\mu \mathrm{v}}=\mathbf{0}$
theSchwartzschild solution can be expanded as :
$\mathbf{v}^{2}=\gamma=\left(\mathbf{c}^{2}-(\mathbf{2 m / r})\right)(\mathbf{1}+\mathbf{i}+\mathbf{j}+\mathbf{k})$
and is a solution of the gravitation equation by Einstein. We can express this equation as:
$\gamma=(\mathbf{1}+\mathbf{i}+\mathbf{j}+\mathbf{k})-\left(2 \mathrm{~m}_{\mathrm{G}}(\mathbf{G}) / \mathbf{r} \mathbf{c}^{2}\right)-\left(\mathbf{i} 2 \mathrm{~m}_{\mathrm{E}}(\mathrm{EM}) / \mathbf{r} \mathbf{c}^{2}\right)-\left(\mathbf{j} 2 \mathrm{~m}_{\mathrm{W}}(\mathrm{WI}) / \mathbf{r} \mathbf{c}^{2}\right)-\left(\mathbf{k} \mathbf{2} \mathrm{m}_{\mathrm{S}}(\mathrm{SI}) / \mathbf{r} \mathbf{c}^{2}\right)$ ( 27 )
In this equation $G$ means gravitation, $\mathrm{E} / \mathrm{E} \mathrm{M}$ means electromagnetic force, W / WI means weak interaction and S / SI means strong interaction. The symbol $\gamma$ is dark energy, which has the symbol D in the coming formulas. By using ordinary physics nomenclature equation ( 27 ) has the following appearance :
$\left.\left.\left.\gamma=(1+i+j+k)-(2 m G) / r c^{2}\right)-\left(i 2 Q / 4 \pi \epsilon_{0} r c^{2}\right)-\left(j q^{2} 2 m_{\pi} k_{\pi}\right) / r c^{2}\right)-\left(k d \Gamma M^{2}\right) / r c^{2}\right)$ ( 28 )
$\mathrm{k}_{\pi}$ is here a constant.

## B. Coupling constants

From the extended Schwartzschildequation :
$\gamma=\mathbf{c}^{2}(\mathbf{1}+\mathbf{i}+\mathbf{j}+\mathbf{k})-\left(2 \mathbf{m}_{\mathrm{G}} \mathbf{G} / \mathbf{r}\right)-\left(\mathbf{i} \mathbf{a}_{2} / \mathbf{r}\right)-\left(\mathbf{j} \mathbf{a}_{3} / \mathbf{r}\right)-\left(\mathbf{k} \mathbf{a}_{4} / \mathbf{r}\right)$
$\gamma=\mathbf{c}^{2}(\mathbf{1}+\mathrm{i}+\mathrm{j}+\mathrm{k})-\left(2 \mathrm{~m}_{\mathrm{G}}(\mathrm{G}) / \mathbf{r}\right)-\left(\mathrm{i} 2 \mathrm{~m}_{\mathrm{E}}\left(\mathrm{G}_{\mathrm{E}}\right) / \mathbf{r}\right)-\left(\mathrm{j} 2 \mathrm{~m}_{\mathrm{W}}\left(\mathrm{G}_{\mathrm{W}}\right) / \mathbf{r}\right)-$
( $\mathbf{k} 2 \mathrm{~m}_{\mathrm{S}} / \mathbf{r}$ ))
We can calculate the coupling constants by using an empirical equation between the different coupling constants and their values from the literature by using the following empirical formula :
$\mathbf{G}_{\mathrm{N}+1}=\mathbf{G}\left(\mathbf{G}_{\mathbf{E}} / \mathbf{G}\right)^{\mathbf{N} / 2}$
Table 1.

1. $\mathrm{G}=\mathrm{G}_{1}=6,67310^{-11} \mathrm{Nm}^{2} \mathrm{~kg}^{-2} \quad$ for gravitational force from tables
2. $\mathrm{G}_{\mathrm{E}}=\mathrm{G}_{3}=8.9910^{9} \mathrm{Nm}^{2} /(\mathrm{As})^{2}$
3. $\mathrm{G}_{\mathrm{W}}=\mathrm{G}_{4}=1.04310^{20}$ for electromagnetic force from tables
for weak interaction from deductive estimations
4. $\mathrm{G}_{\mathrm{S}}=\mathrm{G}_{5}=1.2110^{29}$ for strong interaction
5. $\mathrm{G}_{\mathrm{D}}=\mathrm{G}_{2}=0.775$ for dark energy

## V. Dynamic Relativistic Invariant Laplace Equation ( DRLE )

We define the dynamic relativistic invariant Laplace equation DRLE, Barrera and Thelin( 5 ) as the equation :
$\mathbf{f}(\mathbf{t})^{2} \nabla^{2} \boldsymbol{\psi}-\left(\mathbf{f}(\mathbf{r})^{2} / \mathbf{c}^{2}\right) \psi_{t t}=\mathbf{0}$
where the two dynamic functions are $f(t)=v(t) / t$ and $f(r)=v(r) / r$, where $v(t)$ is rotation velocity at a certain time.Introducing these dynamic functions into equation ( 32 ) gives:
Inserting these functions in the equation ( 32 ) will lead to equation 33 ):
$|(\mathbf{v}(\mathbf{t}) /(\mathbf{t}))|^{2} \nabla^{2} \psi-\left(|(\mathbf{v}(\mathbf{r}) / \mathbf{r})|^{2} / \mathbf{c}^{2}\right) \psi_{\mathrm{tt}}=\mathbf{0}$
This formula can be rewritten as :
$\nabla^{2} \psi-\left(\mathbf{1} / \mathbf{c}^{2}\right)\left(|(\mathbf{v}(\mathbf{r}) / \mathbf{r})|^{2} / \mid\left(\mathbf{v}(\mathbf{t}) /\left.(\mathbf{t})\right|^{2}\right) \psi_{\mathrm{tt}}=\mathbf{0}\right.$
with the solutions :
$\left.\psi_{1}=(R / r) \exp \pm i\left(\left(2 \pi v(r) / \lambda_{1} r\right)(r \pm c t)\right)+n \varphi / 2\right)$ and
$\psi_{2}=\exp \pm\left(\left(2 \pi v(r) / \lambda_{2} r\right)(r \pm c t)\right)$
where $\varphi$ is the phase angle and $\lambda_{1}$ and $\lambda_{2}$ are the bandwidths
where the combined solution will be :
$\psi=\psi_{1} \psi_{2}$
which are the combined solutions of equation( 32 )
Letting $\mathrm{r}=\mathrm{ct}$ the differential equation (32) is reduced to :
$\nabla^{2} \psi-\left(\mathbf{1} / \mathbf{c}^{2}\right) \psi_{\mathrm{tt}}=\mathbf{0}$
which is the relativistic invariant Laplace equation RLE, Barrera and Thelin ( 5 ) and ( 6 ).

## VI. Second Large Solution Of The Universal Theory Of Relativity. Null Geodetic Curvature

Let the distribution $\psi^{i}=x^{i}$ energy so that the energy density $\psi^{i}$ of a wave is represented as the spatial position $x^{i}$
, Kay ( 7 ). Set General Geodetic curvature :
$\left(d^{2} \psi_{i} / d t^{2}\right)+\Gamma_{k l}^{i}\left(d \psi_{k} / d t\right)\left(d \psi_{1} / d t\right)=0$
Equal to the energy tensor equation

Then by integrating :
$\iint\left(\mathbf{d}^{2} \psi_{\mathrm{i}} / \mathbf{d} \mathbf{t}^{2}\right)$
we have $\mathrm{g}_{11}=\mathrm{g}_{22}=\mathrm{g}_{33}=-\mathrm{g}_{44}$ and the general solution
$\psi^{i}=\mathbf{x}_{0}{ }^{i}+\gamma^{i} \mathbf{t}$
with $\mathrm{g}_{\mathrm{ij}}{ }^{\mathrm{i}} \gamma^{i} \gamma^{\mathrm{j}}=\mathbf{0}$
so that $\quad \mathbf{g}_{\mathrm{ij}}\left(\mathbf{x}^{\mathbf{i}}-\mathbf{x}_{\mathbf{0}}{ }^{\mathbf{i}}\right)\left(\mathbf{x}^{\mathbf{j}}-\mathbf{x}_{\mathbf{0}}{ }^{\mathbf{j}}\right)=\mathbf{0}$
this is in Cartesian coordinates and gives the special relativity solution:
$\left(\mathrm{x}^{1}-\mathrm{x}_{0}{ }^{1}\right)^{2}+\left(\mathrm{x}^{2}-\mathrm{x}_{0}{ }^{2}\right)^{2}+\left(\mathrm{x}^{3}-\mathrm{x}_{0}{ }^{3}\right)^{2}=\left(\mathrm{x}^{4}-\mathrm{x}_{0}{ }^{4}\right)^{2}$
or simply
$r^{2}=c^{2} t^{2}(46)$
i.e.
$\mathbf{r}-\mathbf{c t}=\mathbf{0}=\boldsymbol{\beta}$
the propagation of a light pulse. Using a phase $\mathrm{n} \varphi / 2$ then this pulse can be inserted into the Ultrasphere in equation ( 48 ), expressed in term of velocities using the theorem by De Moivre :
$\left(v_{D}{ }^{2} / 2 c^{2}\right)+\left(v_{G}{ }^{2} / 2 c^{2}\right)+\left(v_{E}{ }^{2} / 2 c^{2}\right)+\left(v_{W}{ }^{2} / 2 c^{2}\right)+\left(v_{S}{ }^{2} / 2 c^{2}\right)=1(48)$
where $u_{i}=\beta=(r-c t) / \lambda+n \varphi / 2$ and we could choose a solution such as the light wave
$\psi=C \sin 2 \pi((\mathbf{r}-\mathbf{c t}) / \lambda)+\mathbf{n} \boldsymbol{\varphi} / 2)$
or generally
$\boldsymbol{\psi}=\mathbf{C} \operatorname{expi} 2 \pi((\mathbf{r}-\mathbf{c t}) / \lambda)+\mathbf{n} \boldsymbol{\varphi} / 2+m \theta)$
then transforming these equations into spherical coordinates we have the gravitation wave(a galaxy).

$$
\psi=\mathbf{C} \exp i 2 \pi((\mathbf{r}-\mathbf{c t}) / \lambda)+\mathbf{n} \varphi / 2) / \mathbf{r}
$$

( 51 )

## VII. Egregia Demonstrandum, Proof Nr 5, Flat Space Time

Consider the general geodetic line ( $\mathrm{t}=\mathrm{s}=\operatorname{arc}$ length $)$
$\left(d^{2} \psi_{i} / d r^{2}\right)+\Gamma_{k l}{ }^{i}\left(d \psi_{k} / d t\right)\left(d \psi_{\mathrm{l}} / d t\right)=0$
where we assume that:
$\left(d^{2} \psi_{i} / d r^{2}\right)-\left(1 / c^{2}\right)\left(d^{2} \psi_{i} / d t^{2}\right)=0$
according to Barrera and Thelin ( 5 ) and ( 6 ), then set the positive definite :
$|\psi|=\mathbf{v}^{2}$
where v denotes velocity, the a subset of the geodetic equation can be written :
$\mathbf{d v} / \mathrm{dt}=\mathrm{v}^{2}$
then there exists a solution in rectangular coordinates :
$\psi=C(\sin 2 \pi(r-c t) / \lambda)+i \cos (2 \pi(r-c t) / \lambda))$
C is a complex constant and as usual c denotes velocity of light, also
$\left.\psi_{t}=\mathbf{C c}(\cos 2 \pi(r-c t) / \lambda)-i \sin (2 \pi(r-c t) / \lambda)\right)$
in spherical coordinates this means that we have the Bessel solution :
$\psi=(1 / r) C(\sin 2 \pi(r-c t) / \lambda)+i \cos (2 \pi(r-c t) / \lambda))=$
$\psi=(1 / r) A(\sin 2 \pi(r-c t) / \lambda)+i B \cos (2 \pi(r-c t) / \lambda))$
with the rotation velocity of papers ( 5 ),( 6 ), ( 8 ) and ( 9 ).
$\mathbf{v}_{\text {rot }}=A \sin \alpha+i B \cos \alpha$ ( 59 )
where $\alpha=\arcsin (\mathrm{mG} / \mathrm{r})^{1 / 2}$ in spherical coordinates. Formula (59) is a solution to the Schwarzschild formula :
$v_{\text {rot }}=c^{2}-2 \mathrm{mG} / \mathbf{r}$
VIII. Egregia Demonstrandum Proof Nr 6. Linear radial distribution. Flat space time

Consider the FriedmannRobertsson Walker Metric :
$\mathbf{d s}^{2}=\mathbf{a}^{2} \mathbf{d r}^{2}-\mathbf{c}^{2} \mathbf{d t}^{2}(61)$
Now set it equal to zero then
$\mathbf{d s}^{2}=\mathbf{a}^{2} \mathbf{d r}^{2}-\mathbf{c}^{2} \mathbf{d t}^{2}=\mathbf{0}$
This is the so called "scaled Milne Metric" Milne (10) which means:
$\mathbf{a}^{2} \mathrm{dr}^{2}=\mathrm{c}^{2} \mathrm{dt}^{2}(63)$
which implies $\mathrm{ds}^{2}=0$ and $\int \mathrm{ds}=0$, which means in flat space time :
$\mathbf{d}^{2} \mathbf{X}_{\mathrm{i}} / \mathbf{d t}^{2}+\Gamma_{\mathrm{rs}}{ }^{\mathbf{i}}\left(\mathbf{d} \mathbf{X}_{\mathrm{i}}{ }^{\mathbf{r}} / \mathbf{d t}\right)\left(\mathbf{d} \mathbf{X}_{\mathrm{i}}^{\mathrm{s}} / \mathbf{d t}\right)=\mathbf{0}(\mathbf{6 4})$
implies that
$\mathbf{d}^{2} \mathbf{X}_{\mathrm{i}} / \mathbf{d t}^{2}=\mathbf{0}$
but the scaled Milne Metric, Milne ( 10 ) can be written
$a^{2} \mathrm{dr}^{2}=\mathrm{c}^{2} \mathrm{dt}^{2}(66)$
or
$\mathbf{a d r}=\mathbf{c d t}$
then by integrating
$\mathbf{a r}=\mathbf{c t + p}$
where $p$ is a constant of integration.
This implies that there exists a wave propagation phase $2 \pi(\mathrm{kr}-\omega \mathrm{t}) / \lambda=\mathrm{p}$ which also satisfies the wave equation (RLE) in equation ( 38 ).
$\mathbf{p}=|\psi|=($ const $)=\mathbf{v}^{2}$
where
$d^{2} \psi / d r^{2}-\left(\mathbf{1} / \mathbf{c}^{2}\right)\left(d^{2} \psi /{d t^{2}}^{2}\right)=0$
and
$\psi=\cos p+i \sin p=\cos (2 \pi(k r-\omega t) / \lambda)+i \sin (2 \pi(k r-\omega t) / \lambda)$
Our required wave velocity is from the Schwarzschild equation :
$\mathbf{v}^{2}=\gamma=\mathbf{c}^{2}-2 \mathbf{m G} / \mathbf{r}=\boldsymbol{\varepsilon}^{2} \mathbf{c}^{2} \geq 0$
which implies $\mathrm{v}=\varepsilon \mathrm{c} \geq 0, \varepsilon \leq 1$
This means that we have a linear radial distribution of the variable $r$ and a light wave.
In spherical coordinates this represents atoms, stars, galaxies.

This means that the Milne Model Ref (10) accounts when we are on geodetic lines, but if we are not, the so called Friedmann- Robertsson-Walker metric is valid.

## IX. Extension Of The Schwarzschild Solution Proof Nr 7 Using Quaternions

The Schwarzschild solution can be extended as the Quaternion vector $\gamma^{\wedge}$ with the Quaternion velocity vector $\left(v^{\wedge}\right)^{2}{ }_{D}=v^{2}(-1+i+j+k)$ where $D$ denotes dark energy:

$1+\mathbf{i}+\mathbf{j}+\mathbf{k})-(2 \mathrm{~m} / \mathbf{r})-\left(\mathbf{i} 2 \mathrm{~m}_{\mathrm{EM}} / \mathbf{r}\right)-\left(\mathbf{j} 2 \mathrm{~m}_{\mathrm{W}} / \mathbf{r}\right)-\left(\mathbf{k} 2 \mathrm{~m}_{\mathrm{S}} / \mathbf{r}\right)$
or
$\mathbf{v}_{\mathbf{D}}^{2}(-1+\mathbf{i}+\mathbf{j}+\mathbf{k})=\mathbf{c}^{2}(\mathbf{1}+\mathbf{i}+\mathbf{j}+\mathbf{k})-(2 \mathrm{~m} / \mathbf{r})-\left(\mathbf{i} 2 \mathrm{~m}_{\mathrm{EM}} / \mathbf{r}\right)-\left(\mathbf{j} 2 \mathrm{~m}_{\mathrm{W}} / \mathbf{r}\right)-\left(\mathbf{k} 2 \mathrm{~m}_{\mathrm{S} /} \mathbf{r}\right)=$ $\mathbf{c}^{2}(1+i+j+k)-2 v_{G}^{2}-2 i v_{\text {EM }}^{2}-2 j v_{w}^{2}-2 k v_{S}^{2} \quad$ (74)
$\underset{v_{D}^{2}}{\text { or }}(-1+i+j+k)=\left(c^{2}-(2 m / r)\right)+i\left(c^{2}-\left(i 2 m_{E M} / r\right)\right)+j\left(c^{2}-\left(j 2 m_{W} / r\right)\right)+k\left(c^{2}-\left(k 2 m_{S} /\right.\right.$ r))
if we have a defined dot product s,t :
$(\mathbf{1}+\mathbf{i}+\mathbf{j}+\mathbf{k})(\mathbf{1}+\mathbf{i}+\mathbf{j}+\mathbf{k})=(\mathbf{1}+\mathbf{i} \mathbf{i}+j \mathbf{j}+\mathrm{k} k)=(\mathbf{1}-\mathbf{1 - 1}-\mathbf{1})=-\mathbf{2}$
( 76 )
then we multiply the equation by ( $\mathbf{1}-\mathbf{i}-\mathbf{j}-\mathbf{k})$, which reduces the equation to :
$2 \mathrm{v}_{\mathrm{D}}^{2}=4 \mathrm{c}^{2}-2 \mathrm{v}_{\mathrm{G}}^{2}-2 \mathrm{v}_{\mathrm{EM}}^{2}-2 \mathrm{v}_{\mathrm{w}}^{2}-2 \mathrm{v}_{\mathrm{S}}^{2}$
By dividing this equation with 2 and rearranging the terms yields :
$\mathbf{v}^{2}{ }_{\mathrm{D}}+\mathbf{v}_{\mathbf{G}}^{2}+\mathbf{v}_{\mathrm{EM}}^{2}+\mathbf{v}^{2}{ }_{\mathrm{w}}+\mathbf{v}_{\mathrm{S}}{ }^{2}=2 \mathbf{c}^{2}$
By multiplying by M and dividing by 2 gives:

(79)
or
$\mathbf{E}=\mathbf{E}_{\mathrm{D}}+\mathbf{E}_{\mathrm{G}}+\mathbf{E}_{\mathrm{EM}}+\mathbf{E}_{\mathrm{W}}+\mathbf{E}_{\mathrm{S}}=\mathbf{M} \mathbf{c}^{2}$
The total rest energy equals the sum of the kinetic energies of all forces.

## X. Extension Of The Schwarzschild Solution, Proof Nr 8 Using Real Numbers.

The Schwarzschild solution can be extended for the five forces in the following way:

$$
\begin{align*}
& \gamma=\left(\mathbf{c}^{2}-(\mathbf{2} \mathbf{m} / \mathbf{r})\right)  \tag{81}\\
& \text { let } \mathrm{m}=\left(\mathrm{m}_{1}+\mathrm{m}_{2}+\mathrm{m}_{3}+\mathrm{m}_{4}\right) / 4 \text { then } \\
& \boldsymbol{\gamma}=\left(\mathrm{c}^{2}-\left(\mathbf{2}\left(\mathbf{m}_{\mathbf{1}}+\mathbf{m}_{\mathbf{2}}+\mathbf{m}_{3}+\mathbf{m}_{4}\right) / \mathbf{4 r}\right)\right.  \tag{82}\\
& \text { expanding } \\
& \boldsymbol{\gamma = \mathbf { c } ^ { 2 } - ( \mathbf { m } _ { \mathbf { 1 } } / \mathbf { 2 r } ) - ( \mathbf { m } _ { \mathbf { 2 } } / \mathbf { 2 r } ) - ( \mathbf { m } _ { \mathbf { 3 } } / \mathbf { 2 r } ) - ( \mathbf { m } _ { 4 } / \mathbf { 2 r } )} \tag{83}
\end{align*}
$$

now substituting in the forces units $\mathrm{m}_{2}=\mathrm{Q}, \mathrm{m}_{3}=\mathrm{q}, \mathrm{m}_{4}=\Gamma$ and reusing the variable m for $\mathrm{m}_{1}$ :
$\gamma=\mathbf{c}^{2}-(\mathbf{m} / 2 \mathbf{r})-(\mathbf{Q} / 2 \mathbf{r})-(\mathbf{q} / 2 \mathbf{r})-(\Gamma / 2 \mathbf{r})$
multiplying by M then
$\mathbf{M} \boldsymbol{\gamma}=\mathbf{M c} \mathbf{c}^{\mathbf{2}}$ - (Mm/2r)-(MQ/2r)-(Mq/2r)-(M「/2r)
or expressed as Keplarian velocities and kinetic energy

rearranging the terms

( 87 )
or
$E=E_{G}+E_{E M}+E_{W}+E_{S}+E_{D}=M c^{2}$
As in chapter IX, the total rest energy equals the sum of the kinetic energies of all forces.

## XI. Experimental Results and Pictures

In this section "experimental" results and pictures from computer simulations made are presented.All the computer simulation pictures are obtained by using the new DRLE equation ( 32 ) on atoms, molecules, planetary systems, stars and galaxies. This is shown in Figs (1-12) and are tabled in Table 2., which show atoms, molecules, a planetary system, a stars (Gemini) and 2 galaxies NGC( 1417 ) and NGC 3200.We have earlier presented many galaxy pictures in papers Barrera and Thelin ( $5,6,8$ and 9 ) with a very realistic appearance and are the results of further developments of the RLE and DRLE formulas and new computer programs.
Table 2

1. Ne- atom
2. U 235 nucleus
3. Hydrogen molecule, $\mathrm{H}_{2}$
4. Benzene molecule, $\mathrm{C}_{6} \mathrm{H}_{6}$
5. Phenol molecule, $\mathrm{C}_{6} \mathrm{H}_{5} \mathrm{OH}$
6. Naftalen molecule, $\mathrm{C}_{10} \mathrm{H}_{8}$
7. Velocity distribution in a planet system
8. Velocity distribution in a planet system versus distance of the largest planets
9. The star Gemini with protuberances.
10. The galaxy NGC 1417, simulation and photography
11. The galaxy NGC 3200, simulation and photography
12. Rotation velocity distribution of galaxy NGC 3200

## XII. Discussion

One new idea in this paper about the Grand Unified Theory GUT, is applying the Schwarzschild solution of the Einstein gravitational equations. This is done by using a Riemann-Christoffel tensor together with Christoffel symbols of general relativity. In this way a useful potential function is obtained. By combining the Schwarzschild solution with quaternions, a solution is obtained of the extended $t$-parameter energy tensor by Einstein. By using geodetic lines here will lead to information of conservation of energy and that the sum of all forces in a system is zero. This means that it is possible to expand the Schwarzschild solution to a sum of terms including gravitation( G ), electromagnetic force( E/E M ), weak interaction( W / WI ), strong interaction( S / SI ) and dark energy( D ) (equation 27 ). This can also expressed in ordinary physics nomenclature like equation ( 28 ) with the individual coupling constants of these different forces, without using any common merged constant. The coupling constants can be calculated using an empirical equation ( 31 ) between the different coupling constants and their values from the literature and are shown in Table 1. It is interesting to note the order of magnitude ratio around $10^{9}$ between the coupling constants nr $1,5,2,3,4$, in Table 1.

Chapter V.The dynamic relativistic Laplacian equation DRLE has been earlier presented by the authors Barrera and Thelin ( 5 ) and ( 6 ) used for galaxies and can be seen as a combination formula between quantum mechanics and relativity. In this very paper this formula has shown to be a very flexible formula used for different systems like nuclei, atoms, molecules, planets and galaxies and can be studied in Figs ( 1-12) and Table 2. This means that DRLE is a more flexible formula than the Schrödinger equation, which is only used for atoms and molecules. Similar equations to DRLE have earlier been used by us for these different systems in Barrera and Thelin( 8 ) and ( 9 ).

The advantages to use DRLE compared to the Schrödinger equation are the following main advantages : 1. DRLE is Lorenze invariant(generally), 2. Rotation Lorenze invariant, 3. Invariant during translation, 4.Invariant during scaling.

The DRLE equation is a much more flexible equation than an earlier version of a RLE equation by us Barrera and Thelin( 5 ) and ( 6 ) and a still much earlier version of RLE by Dirac ( 11 )

In chapter VI the space time is realized from null geodetic curvature giving a propagation of a light pulse. At a certain phase this light pulse can be combined with the Ultrasphere giving a solution of a lightwave. This is a proof that photons have null geodetic curvature(flat space time) and do not demand energy when they propagate and they are also a solution of the DRLE equation( 32 ). When transforming these equations into spherical coordinates a gravitation wave is realized in a galaxy equation ( 51 ). This gravitation wave equation is also a solution of the DRLE equation ( 32 ). Such gravitation waves can be seen from galaxy animations in Figs 10 and 11 of this paper and in other computer animations of Barrera and Theln (5) and ( 6 ), where the wave structure (density variations) is seen in the galaxy arms. The wave length of these gravitation waves can be estimated of around $10^{4}$ l.y. from these and other computer animations.

In chapter VII there is another proof of the creation of a light wave, when using a general geodetic null line in combination with the DRLE equation( 32 ). When using spherical coordinates, a Bessel function is obtained giving another light wave in combination with our rotation velocity formula Barrera and Thelin ( 5 ) and ( 6 ), earlier used mostly on galaxies and planetary systems. This calculation shows that every rotation system in the universe from atoms to galaxies are dependent of null geodetic lines and the DRLE equation ( 32 ).

In chapter VIII is another proof of creating a light wave from using null geodetic line calculations(flat space time) and combine these calculations with thegeneral Milne Metric ( 10 ). These calculations implies that there exist a certain wave propagation phase which is also a solution to the DRLE equation (32) and from that gives a light wave equation. These calculations together with Schwartzschild equation show that we have a linear radial distribution on the variable $r$ and is in accordance to the other results in chapters VI and VII. This means that the Milne Model accounts for when we are on geodetic lines, but if we are not, then the so called Friedmann - Robertsson - Walker Metric is valid.

According to these chapters all kind of waves are identical to geodetic lines, and are propagating linearly in a flat space time. In spherical coordinates this can be represented in all kinds of systems from atoms to galaxies. Important equations to obtain flat space time are the gravitation tensor $G_{\mu \nu}=0$ and the energy tensor $\mathrm{E}_{\mathrm{ij}}{ }^{\mathrm{t}}$ in equation (23) by Einstein together with our DRLE equation. Further arguments and measurements supporting these ideas of space time, can be seen in Table 3.

Table 3
Arguments for Space- Time - $\mathbf{G}_{\mu \mathrm{v}}=\mathbf{0}$

1. Is proven to be correct for our solar system and precession of the planet Mercury.
2. Has an acceptable solution for the bending of light around the Sun.
3. Has been verified for the background radiation of our universe (Satellite measurements).
4. Causes 100 percent pure sinusoidal undistorted waves for light, Electromagenetic forcee.t.c...
5. Has been measured to a 95 to $97 \%$ correlation of the rotation velocity of spiral galaxies.

In chapter IX an extension of the Schwartzschild solution is shown. Here this solution can be extended by using Quaternion vector $\gamma$ and Quaternion velocity vector $\mathrm{v}_{\mathrm{D}}{ }^{2}$, where D stands for dark energy.At these calculations, we obtain a sum of velocity squares(equation ( 48 ). By rearrange the terms and multiply with M (total mass of a system), we obtain equation ( 79 ). This equation tells us that the total rest energy equals the sum of the kinetic energies of all forces in a system. These facts seem to be logical and are a strong evidence, that our calculations of theSchwartzschild solution with different separated potentials, seem to be correct.

In chapter $\mathbf{X}$ a similar extension of the Schwartzschild solution is shown. Here this solution can be extended by using real numbers on vector $\gamma$.At these calculations, we obtain a sum of terms of different masses of the different forces. By rearrange the terms and multiply with M (total mass of a system), we obtain equation ( 87 ). This equation tells us that the total rest energy equals the sum of the kinetic energies of all forces in a system as in chapter IX. These facts are also a strong evidence, that our calculations of the Schwartzschild solution with different separated potentials, seem to be correct.

In chapter XI pictures from different atoms, molecules, a star, a planetary system and two galaxies are shown from computer simulation calculations with the new DRLE equation and the older RLE equation. At these calculations a scaling procedure was used to manage to calculate and show pictures of different objects with completely different sizes, using the same formula.

In Fig 9 a computer simulation of the star Gemini from the DRLE equation 32 was carried out. This star is a very big star showing a lot of protuberances giving a realistic appearance. These objects appear as a consequence of angular solutions to the DRLE equation. A star can be seen as a very big atom in these calculations with the same DRLE equation. There are similarities between protuberances and electronic orbits around atom at certain electronic states. If this is correct the dynamo of the protuberances might have relativistic background and as a consequence of this, the solar wind too.

In a previous paper by Barrera and Thelin( 5 ) the DRLE equation was used on planets. This means that planet orbits have relativistic background. The question is, if the angular solution of DRLE equation have influence on upflowing ionospheric ions from earth, in the same way as the protuberances on the sun? One of us (BT), have worked in this geophysical field earlier and have done a statistical study of data from these upflowing ionospheric ions, measured with the Swedish Viking satellite Thelin et al. ( 12 ). In this geophysical field, the acceleration mechanism of these ionospheric ions is a very big question. The general opinion believes, that the acceleration mechanism behind the so called beams and conics, has plasma physical explanation of some sort. The question is, if there are similarities between protuberances on the sun and upflowing ion beams on earth? If so, these upflowing ionospheric ions might have relativistic origin too.
XII.We have also made some calculations concerning the Yukawa potential, Yukawa ( 14 ),Martin ( 13 ),using the DRLE equation, and found that a new version of this formula fit better with the gravitation equation $G_{\mu \nu}=0$, than the old one. This new formula is :
$\mathbf{E}_{\mathrm{W}}=\left(-\mathbf{j} \mathbf{g}^{\mathbf{2}} \mathbf{2} \mathbf{m}_{\mathrm{W}} \mathbf{k}_{\pi} /\left(\mathbf{r}^{\mathrm{a}-\mathrm{m}}{ }_{\mathrm{W}} / \mathrm{m}_{\mathbf{0}}+\mathrm{b}\right) \mathbf{c}^{\mathbf{2}}\right)$
In this new formula ( 89 ) $\mathrm{m}_{0}$ is a balancing constant and a and b are weighting constants.

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Figures


Fig 1 Computer simulation of the Neon atom ( Ne ) from the DRLE equation 32, where the different electronic orbits are seen.


Fig 2 Computer reconstruction of Uranium nucleus (U235) from the DRLE equation 32.


Fig 3 Computer simulation of the Hydrogen molecule ( $\mathrm{H}_{2}$ ) from the DRLE equation 32, where the different electronic orbits at different stages are seen.


Fig 4 Computer simulation of the Benzene molecule $\left(\mathrm{C}_{6} \mathrm{H}_{6}\right)$ from the DRLE equation 32, where the different electronic orbits are seen around different atoms.


Fig 5 Computer simulation of the Phenol molecule ( $\mathrm{C}_{6} \mathrm{H}_{5} \mathrm{OH}$ ) from the DRLE equation 32, where the different electronic orbits are seen around the different atoms.


Fig 6 Computer simulation of the Naftalene molecule $\left(\mathrm{C}_{10} \mathrm{H}_{8}\right)$ from the DRLE equation 32, where the different electronic orbits are seen as a cloud of electrons.


Fig 7 Velocity distributions of some planets in 3 dimensions in a planetary system. The planet orbits in one plane are clearly seen. (Computer simulations)


Fig 8 Velocity distributions versus distance of the largest planets (Jupiter, Saturn, Uranus and Neptune ) (Computer simulations)


Fig 9 Computer simulation of the star Gemini from the DRLE equation 32, where the protuberances are clearly seen and exist as a consequence of angular solutions to the DRLE equation.


Fig 10 NGC 1417 Computer simulations of randomly distributed 25000 "stars" distributed like an ellipsoid in a galaxy (left). Equation 31 of Barrera and Thelin( 6 ) about the energy density distribution versus distance to the centre has been used. The right figure is a photograph from space (Hubble), showing very good similarities. Similar computer simulations have been made using the new DRLE equation on many galaxies. Reproduction from Barrera and Thelin( 6 )


Fig 11 NGC3200Computer simulations of randomly distributed 25000 "stars" distributed like an ellipsoid in a galaxy (left). Equation 31 in Barrera and Thelin( 6 ) about the energy density distribution versus distance to the centre has been used. The right figure is a photograph from space (Hubble).

Reproduction from Barrera and Thelin( 6 )


Fig 12 Rotation velocity distributions versus distance of the galaxy NGC 3200
These graphs do follow the observational velocity distributions.
(Computer simulations) Match $95 \%$. Equation ( 24 ) of paper ( 5 ) has been followed.
Reproduction from Barrera and Thelin( 5 )

