

Internal Differentiation, Comparative Variability, Structural Morphology, Normative Aspect Of Prognostication Of Ipse Dixit Np Hard Problems-A Totalistic Paradigmatic Statement

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ABSTRACT: We develop a progressive comparable to Bank's General Ledger, and call it a General Theory of all the problems under the head of NP hard problems. Problems have their variables. For instance "Travelling Sales man problem" can have various different cities. Based upon parameters classification is done and stability analysis, asymptotic stability and Solutional behaviour of the equations are investigated We eschew from stating any primary predications, coextensive representations, predicational anteriority of the Problems attributed to space constraints. In consideration to the parametric based classification and there is both ontological consonance, primordial exactitude, and phenomenological testimony as one finds in Bank's ledgers. General Ledger is in fact the statement of all inflows and outflows, such a one as that occurs in problems, and theories in some conditions, like for example the conservation of energy breaking down in Hawking's radiation. Emphasis is laid on the fact that for instance a travelling salesman makes some move and then retracts to redress his move or starts another move to further his final destination. And this destination is General Ledger.-The General Theory Of all the NP (hard) problems. .It is a journey, a journey to find the final balance which probably never ends like an account never closes. So we are on to the journey

KEY WORDS: Boolean satisfiability Problem, N Puzzle, Knapsack Problem, Hamiltonian Path problem, Travelling Salesman Problem

I. INTRODUCTION:

As stated in the abstract we will not give any introduction, inconsideration to the leviathans' material and humungous literature on each subject matter for fear of missing woods for trees. On the other hand, for the interested reader the literature provides a rich receptacle, repository and treasure-trove of knowledge, And also because of space constraints. We note that the NP (HARD) problems are classified as follows:

- 1) Boolean satisfiability Problem
- 2) N Puzzle
- 3) Knapsack Problem
- 4) Hamiltonian Path problem
- 5) Travelling Salesman Problem
- 6) Sub graph Isomorphism Problem
- 7) Subset Sum problem
- 8) Clique Problem
- 9) Vertex Cover Problem
- 10) Independent Set problem
- 11) Dominating set problem.
- 12) Graph Coloring Problem

As in a Bank, various parameters are there for an account like balance standing, rate of interest,

implications of inflation, money depression, depreciation of the currency, implications of Policies, philosophies and programs of the Government, each problem has certain parameters. That Gravity is constant does not mean it does not depend upon the masses of individual particles and there is no total gravity. Stratification is done based on the parameters of each problem and then consummated with the other to form a monolithic Diaspora for building the Model, which essentially as said is a progressive, nay a General Theory Of all the ways and means in which the problem can be solved be it by invocation or by abnegation and revocation of the action. Everything is recorded in the Computer and we draw up a Final General Ledger-nay The General Theory Of all NP (HARD) Problems. Essentially a prediction model, it as said analyses various other facets too.

GLOSSARY OF THE SYSTEM BOOLEAN SATISFIABILITY PROBLEM AND N PUZZLE

NOTATION :

- G_{13} : Category One Of Boolean Satisfiability Problem
- G_{14} : Category Two Of Boolean Satisfiability Problem
- G_{15} : Category Three Of Boolean Satisfiability Problem
- T_{13} : Category One Of N Puzzle
- T_{14} :Category Two Of N Puzzle
- T_{15} :Category Three Of N Puzzle

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GLOSSARY OF THE SYSTEM KNAPSACK PROBLEM AND HAMILTONIAN PATH PROBLEM:

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- G_{16} : Category One Of Knapsack Problem
 - G_{17} : Category Two Of Knapsack Problem
 - G_{18} : Category Three Of Knapsack Problem
 - T_{16} : Category One Of Hamiltonian Path Problem
 - T_{17} : Category Two Of Hamiltonian Path Problem
 - T_{18} : Category Three Of Hamiltonian Path Problem

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GLOSSARY OF THE SYSTEM:TRAVELLING SALESMAN PROBLEM AND SUBGRAPH ISOMERISM PROBLEM

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- G_{20} : Category One Of Travelling Salesman Problem
 - G_{21} : Category Two Of Travelling Salesman Problem
 - G_{22} :Category Three Of Travelling Salesman Problem
 - T_{20} : Category One Of Sub graph Isomerism Problem

T_{21} : Category Two Of Sub graph Isomerism Problem

T_{22} : Category Three Of Sub graph Isomerism Problem

GLOSSARY FOR THE SYSTEM: SUBSET SUM PROBLEM AND CLIQUE PROBLEM

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 G_{24} : Category One Of Subset Sum Problem

G_{25} : Category Two Of Subset Sum Problem

G_{26} : Category Three Of Subset Sum Problem

T_{24} : Category One Of Clique Problem

T_{25} : Category Two Of Clique Problem

T_{26} : Category Three Of Clique Problem

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GLOSSARY FOR THE SYSTEM: VERTEX COVER PROBLEM AND INDEPENDENT SET PROBLEM

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 G_{28} : Category One Of Vertex Cover Problem

G_{29} : Category Two Of Vertex Cover Problem

G_{30} : Category Three Of Vertex Cover Problem

T_{28} : Category One Of Independent Set Problem

T_{29} : Category Two Of Independent Set Problem

T_{30} : Category Three Of Independent Set Problem

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GLOSSARY OF THE SYSTEM: DOMINATING SET PROBLEM AND GRAPH COLORING PROBLEM

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 G_{32} : Category One Of Dominating Set Problem

G_{33} : Category Two Of Dominating Set Problem

G_{34} : Category Three Of Dominating Set Problem

T_{32} : Category One Of Graph Coloring Problem

T_{33} : Category Two Of Graph Coloring Problem

T_{34} : Category Three Of Graph Coloring Problem

ACCENTUATION COEFFICIENTS

$$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}, (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}, (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}; (a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}, (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}, (a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}, (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$$

DISSIPATION COEFFICIENTS

$$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}, (b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}, (a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}, (a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}, (a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}, (a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$$

GOVERNING EQUATIONS OF THE SYSTEM BOOLEAN SATISFIABILITY PROBLEM AND N PUZZLE

The differential system of this model is now

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)]G_{13} \tag{1}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t)]G_{14} \tag{2}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t)]G_{15} \tag{3}$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t)]T_{13} \tag{4}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t)]T_{14} \tag{5}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t)]T_{15} \tag{6}$$

$$+(a''_{13})^{(1)}(T_{14}, t) = \text{First augmentation factor} \tag{7}$$

$$-(b''_{13})^{(1)}(G, t) = \text{First detritions factor} \tag{8}$$

GOVERNING EQUATIONS:OF THE SYSTEM KANPSACK PROBLEM AND HAMILTONIAN PATH PROBLEM

The differential system of this model is now

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t)]G_{16} \tag{9}$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t)]G_{17} \tag{10}$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t)]G_{18} \tag{11}$$

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}((G_{19}), t)]T_{16} \tag{12}$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}((G_{19}), t)]T_{17} \quad 13$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}((G_{19}), t)]T_{18} \quad 14$$

$$+(a''_{16})^{(2)}(T_{17}, t) = \text{First augmentation factor} \quad 15$$

$$-(b''_{16})^{(2)}((G_{19}), t) = \text{First detritions factor} \quad 16$$

GOVERNING EQUATIONS: OF THE SYSTEM TRAVELLING SALESMAN PROBLEM AND SUBGRAPH ISOMERISM PROBLEM:

The differential system of this model is now

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)]G_{20} \quad 17$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)]G_{21} \quad 18$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)]G_{22} \quad 19$$

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)]T_{20} \quad 20$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)]T_{21} \quad 21$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)]T_{22} \quad 22$$

$$+(a''_{20})^{(3)}(T_{21}, t) = \text{First augmentation factor} \quad 23$$

$$-(b''_{20})^{(3)}(G_{23}, t) = \text{First detritions factor} \quad 24$$

GOVERNING EQUATIONS:OF THE SYSTEM SUBSET SUM PROBLEM AND CLIQUE PROBLEM

The differential system of this model is now

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)]G_{24} \quad 25$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)]G_{25} \quad 26$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)]G_{26} \quad 27$$

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}((G_{27}), t)]T_{24} \quad 28$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}((G_{27}), t)]T_{25} \quad 29$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}((G_{27}), t)]T_{26} \quad 30$$

$$+(a''_{24})^{(4)}(T_{25}, t) = \text{First augmentation factor} \quad 31$$

$$-(b''_{24})^{(4)}((G_{27}), t) = \text{First detritions factor} \quad 32$$

GOVERNING EQUATIONS:OF THE SYSTEM VERTEX COVER PROBLEM AND INDEPENDENT SET PROBLEM

The differential system of this model is now

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)]G_{28} \quad 33$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t)]G_{29} \quad 34$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t)]G_{30} \quad 35$$

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}((G_{31}), t)]T_{28} \quad 36$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}((G_{31}), t)]T_{29} \quad 37$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}((G_{31}), t)]T_{30} \quad 38$$

$$+(a''_{28})^{(5)}(T_{29}, t) = \text{First augmentation factor} \quad 39$$

$$-(b''_{28})^{(5)}((G_{31}), t) = \text{First detritions factor} \quad 40$$

GOVERNING EQUATIONS:OF THE DOMINATING SET PROBLEM AND GRAPH COLORING PROBLEM:

The differential system of this model is now

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)]G_{32} \quad 42$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t)]G_{33} \quad 43$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t)]G_{34} \quad 44$$

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}((G_{35}), t)]T_{32} \quad 45$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}((G_{35}), t)]T_{33} \quad 46$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}((G_{35}), t)]T_{34} \quad 47$$

$$+(a''_{32})^{(6)}(T_{33}, t) = \text{First augmentation factor} \quad 48$$

$$-(b''_{32})^{(6)}((G_{35}), t) = \text{First detritions factor} \quad 49$$

FINAL CONCATENATED GOVERNING EQUATIONS OF THE SYSTEM:

1. BOOLEAN SATISFIABILITY PROBLEM
2. N PUZZLE
3. KNAPSACK PROBLEM
4. HAMILTONIAN PATH PROBLEM
5. TRAVELLING SALESMAN PROBLEM
6. SUB GRAPH ISOMERISM PROBLEM

7. SUBSET SUM PROBLEM
8. CLIQUE PROBLEM
9. VERTEX COVER PROBLEM
10. INDEPENDENT SET PROBLEM
11. DOMINATING SET PROBLEM
12. GRAPH COLORING PROBLEM

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{|c|} \hline (a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) + (a''_{16})^{(2,2)}(T_{17}, t) + (a''_{20})^{(3,3)}(T_{21}, t) \\ \hline + (a''_{24})^{(4,4,4,4)}(T_{25}, t) + (a''_{28})^{(5,5,5,5)}(T_{29}, t) + (a''_{32})^{(6,6,6,6)}(T_{33}, t) \\ \hline \end{array} \right] G_{13} \quad 50$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\begin{array}{|c|} \hline (a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t) + (a''_{17})^{(2,2)}(T_{17}, t) + (a''_{21})^{(3,3)}(T_{21}, t) \\ \hline + (a''_{25})^{(4,4,4,4)}(T_{25}, t) + (a''_{29})^{(5,5,5,5)}(T_{29}, t) + (a''_{33})^{(6,6,6,6)}(T_{33}, t) \\ \hline \end{array} \right] G_{14} \quad 51$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\begin{array}{|c|} \hline (a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t) + (a''_{18})^{(2,2)}(T_{17}, t) + (a''_{22})^{(3,3)}(T_{21}, t) \\ \hline + (a''_{26})^{(4,4,4,4)}(T_{25}, t) + (a''_{30})^{(5,5,5,5)}(T_{29}, t) + (a''_{34})^{(6,6,6,6)}(T_{33}, t) \\ \hline \end{array} \right] G_{15} \quad 52$$

Where $(a''_{13})^{(1)}(T_{14}, t)$, $(a''_{14})^{(1)}(T_{14}, t)$, $(a''_{15})^{(1)}(T_{14}, t)$ are first augmentation coefficients for category 1, 2 and 3 54

$(a''_{16})^{(2,2)}(T_{17}, t)$, $(a''_{17})^{(2,2)}(T_{17}, t)$, $(a''_{18})^{(2,2)}(T_{17}, t)$ are second augmentation coefficient for category 1, 2 and 3 55

$(a''_{20})^{(3,3)}(T_{21}, t)$, $(a''_{21})^{(3,3)}(T_{21}, t)$, $(a''_{22})^{(3,3)}(T_{21}, t)$ are third augmentation coefficient for category 1, 2 and 3 56

$(a''_{24})^{(4,4,4,4)}(T_{25}, t)$, $(a''_{25})^{(4,4,4,4)}(T_{25}, t)$, $(a''_{26})^{(4,4,4,4)}(T_{25}, t)$ are fourth augmentation coefficient for category 1, 2 and 3 57

$(a''_{28})^{(5,5,5,5)}(T_{29}, t)$, $(a''_{29})^{(5,5,5,5)}(T_{29}, t)$, $(a''_{30})^{(5,5,5,5)}(T_{29}, t)$ are fifth augmentation coefficient for category 1, 2 and 3 58

$(a''_{32})^{(6,6,6,6)}(T_{33}, t)$, $(a''_{33})^{(6,6,6,6)}(T_{33}, t)$, $(a''_{34})^{(6,6,6,6)}(T_{33}, t)$ are sixth augmentation coefficient for category 1, 2 and 3 59

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[\begin{array}{|c|} \hline (b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{16})^{(2,2)}(G_{19}, t) - (b''_{20})^{(3,3)}(G_{23}, t) \\ \hline - (b''_{24})^{(4,4,4,4)}(G_{27}, t) - (b''_{28})^{(5,5,5,5)}(G_{31}, t) - (b''_{32})^{(6,6,6,6)}(G_{35}, t) \\ \hline \end{array} \right] T_{13} \quad 60$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[\begin{array}{|c|} \hline (b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t) - (b''_{17})^{(2,2)}(G_{19}, t) - (b''_{21})^{(3,3)}(G_{23}, t) \\ \hline - (b''_{25})^{(4,4,4,4)}(G_{27}, t) - (b''_{29})^{(5,5,5,5)}(G_{31}, t) - (b''_{33})^{(6,6,6,6)}(G_{35}, t) \\ \hline \end{array} \right] T_{14} \quad 61$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\begin{array}{|c|} \hline (b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t) - (b''_{18})^{(2,2)}(G_{19}, t) - (b''_{22})^{(3,3)}(G_{23}, t) \\ \hline - (b''_{26})^{(4,4,4,4)}(G_{27}, t) - (b''_{30})^{(5,5,5,5)}(G_{31}, t) - (b''_{34})^{(6,6,6,6)}(G_{35}, t) \\ \hline \end{array} \right] T_{15} \quad 62$$

Where $-(b''_{13})^{(1)}(G, t)$, $-(b''_{14})^{(1)}(G, t)$, $-(b''_{15})^{(1)}(G, t)$ are first detrition coefficients for category 1, 2 and 3 63

$-(b''_{16})^{(2,2)}(G_{19}, t)$, $-(b''_{17})^{(2,2)}(G_{19}, t)$, $-(b''_{18})^{(2,2)}(G_{19}, t)$ are second detrition coefficients for category 1, 2 and 3 64

$-(b''_{20})^{(3,3)}(G_{23}, t)$, $-(b''_{21})^{(3,3)}(G_{23}, t)$, $-(b''_{22})^{(3,3)}(G_{23}, t)$ are third detrition coefficients for category 1, 2 and 3 65

$-(b''_{24})^{(4,4,4,4)}(G_{27}, t)$, $-(b''_{25})^{(4,4,4,4)}(G_{27}, t)$, $-(b''_{26})^{(4,4,4,4)}(G_{27}, t)$ are fourth detrition coefficients for category 1, 2 and 3 66

$-(b''_{28})^{(5,5,5,5)}(G_{31}, t)$, $-(b''_{29})^{(5,5,5,5)}(G_{31}, t)$, $-(b''_{30})^{(5,5,5,5)}(G_{31}, t)$ are fifth detrition coefficients for category 1, 2 and 3 67

$-(b''_{32})^{(6,6,6,6)}(G_{35}, t)$, $-(b''_{33})^{(6,6,6,6)}(G_{35}, t)$, $-(b''_{34})^{(6,6,6,6)}(G_{35}, t)$ are sixth detrition coefficients for category 1, 2 and 3 68

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t) & + (a''_{13})^{(1,1)}(T_{14}, t) & + (a''_{20})^{(3,3,3)}(T_{21}, t) \\ + (a''_{24})^{(4,4,4,4)}(T_{25}, t) & + (a''_{28})^{(5,5,5,5)}(T_{29}, t) & + (a''_{32})^{(6,6,6,6)}(T_{33}, t) \end{array} \right] G_{16} \quad 69$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{array}{ccc} (a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t) & + (a''_{14})^{(1,1)}(T_{14}, t) & + (a''_{21})^{(3,3,3)}(T_{21}, t) \\ + (a''_{25})^{(4,4,4,4)}(T_{25}, t) & + (a''_{29})^{(5,5,5,5)}(T_{29}, t) & + (a''_{33})^{(6,6,6,6)}(T_{33}, t) \end{array} \right] G_{17} \quad 70$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{array}{ccc} (a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t) & + (a''_{15})^{(1,1)}(T_{14}, t) & + (a''_{22})^{(3,3,3)}(T_{21}, t) \\ + (a''_{26})^{(4,4,4,4)}(T_{25}, t) & + (a''_{30})^{(5,5,5,5)}(T_{29}, t) & + (a''_{34})^{(6,6,6,6)}(T_{33}, t) \end{array} \right] G_{18} \quad 71$$

Where $+(a''_{16})^{(2)}(T_{17}, t)$, $+(a''_{17})^{(2)}(T_{17}, t)$, $+(a''_{18})^{(2)}(T_{17}, t)$ are first augmentation coefficients for category 1, 2 and 3 72

$+(a''_{13})^{(1,1)}(T_{14}, t)$, $+(a''_{14})^{(1,1)}(T_{14}, t)$, $+(a''_{15})^{(1,1)}(T_{14}, t)$ are second augmentation coefficient for category 1, 2 and 3 73

$+(a''_{20})^{(3,3,3)}(T_{21}, t)$, $+(a''_{21})^{(3,3,3)}(T_{21}, t)$, $+(a''_{22})^{(3,3,3)}(T_{21}, t)$ are third augmentation coefficient for category 1, 2 and 3 74

$+(a''_{24})^{(4,4,4,4)}(T_{25}, t)$, $+(a''_{25})^{(4,4,4,4)}(T_{25}, t)$, $+(a''_{26})^{(4,4,4,4)}(T_{25}, t)$ are fourth augmentation coefficient for category 1, 2 and 3 75

$+(a''_{28})^{(5,5,5,5)}(T_{29}, t)$, $+(a''_{29})^{(5,5,5,5)}(T_{29}, t)$, $+(a''_{30})^{(5,5,5,5)}(T_{29}, t)$ are fifth augmentation coefficient for category 1, 2 and 3 76

$+(a''_{32})^{(6,6,6,6)}(T_{33}, t)$, $+(a''_{33})^{(6,6,6,6)}(T_{33}, t)$, $+(a''_{34})^{(6,6,6,6)}(T_{33}, t)$ are sixth augmentation coefficient for category 1, 2 and 3 77

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}, t) & - (b''_{13})^{(1,1)}(G, t) & - (b''_{20})^{(3,3,3)}(G_{23}, t) \\ - (b''_{24})^{(4,4,4,4)}(G_{27}, t) & - (b''_{28})^{(5,5,5,5)}(G_{31}, t) & - (b''_{32})^{(6,6,6,6)}(G_{35}, t) \end{array} \right] T_{16} \quad 78$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{array}{ccc} (b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}, t) & - (b''_{14})^{(1,1)}(G, t) & - (b''_{21})^{(3,3,3)}(G_{23}, t) \\ - (b''_{25})^{(4,4,4,4)}(G_{27}, t) & - (b''_{29})^{(5,5,5,5)}(G_{31}, t) & - (b''_{33})^{(6,6,6,6)}(G_{35}, t) \end{array} \right] T_{17} \quad 79$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[\begin{array}{ccc} (b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}, t) & - (b''_{15})^{(1,1)}(G, t) & - (b''_{22})^{(3,3,3)}(G_{23}, t) \\ - (b''_{26})^{(4,4,4,4)}(G_{27}, t) & - (b''_{30})^{(5,5,5,5)}(G_{31}, t) & - (b''_{34})^{(6,6,6,6)}(G_{35}, t) \end{array} \right] T_{18} \quad 80$$

where $-(b''_{16})^{(2)}(G_{19}, t)$, $-(b''_{17})^{(2)}(G_{19}, t)$, $-(b''_{18})^{(2)}(G_{19}, t)$ are first detrition coefficients for category 1, 2 and 3 81

$-(b''_{13})^{(1,1)}(G, t)$, $-(b''_{14})^{(1,1)}(G, t)$, $-(b''_{15})^{(1,1)}(G, t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{20})^{(3,3,3)}(G_{23}, t)$, $-(b''_{21})^{(3,3,3)}(G_{23}, t)$, $-(b''_{22})^{(3,3,3)}(G_{23}, t)$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27}, t)}$ are fourth detrition coefficients for category 1,2 and 3 82
 $\boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31}, t)}$ are fifth detrition coefficients for category 1,2 and 3
 $\boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35}, t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35}, t)}$ are sixth detrition coefficients for category 1,2 and 3

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{array}{ccc} (a'_{20})^{(3)} \boxed{+(a''_{20})^{(3)}(T_{21}, t)} & \boxed{+(a''_{16})^{(2,2,2)}(T_{17}, t)} & \boxed{+(a''_{13})^{(1,1,1)}(T_{14}, t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25}, t)} & \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29}, t)} & \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33}, t)} \end{array} \right] G_{20} \quad 83$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{array}{ccc} (a'_{21})^{(3)} \boxed{+(a''_{21})^{(3)}(T_{21}, t)} & \boxed{+(a''_{17})^{(2,2,2)}(T_{17}, t)} & \boxed{+(a''_{14})^{(1,1,1)}(T_{14}, t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25}, t)} & \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29}, t)} & \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33}, t)} \end{array} \right] G_{21} \quad 84$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{array}{ccc} (a'_{22})^{(3)} \boxed{+(a''_{22})^{(3)}(T_{21}, t)} & \boxed{+(a''_{18})^{(2,2,2)}(T_{17}, t)} & \boxed{+(a''_{15})^{(1,1,1)}(T_{14}, t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25}, t)} & \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29}, t)} & \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33}, t)} \end{array} \right] G_{22} \quad 85$$

$\boxed{+(a''_{20})^{(3)}(T_{21}, t)}$, $\boxed{+(a''_{21})^{(3)}(T_{21}, t)}$, $\boxed{+(a''_{22})^{(3)}(T_{21}, t)}$ are first augmentation coefficients for category 1, 2 and 3 86

$\boxed{+(a''_{16})^{(2,2,2)}(T_{17}, t)}$, $\boxed{+(a''_{17})^{(2,2,2)}(T_{17}, t)}$, $\boxed{+(a''_{18})^{(2,2,2)}(T_{17}, t)}$ are second augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{14})^{(1,1,1)}(T_{14}, t)}$, $\boxed{+(a''_{15})^{(1,1,1)}(T_{14}, t)}$ are third augmentation coefficients for category 1, 2 and 3 87

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25}, t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25}, t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25}, t)}$ are fourth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29}, t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29}, t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29}, t)}$ are fifth augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33}, t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33}, t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33}, t)}$ are sixth augmentation coefficients for category 1, 2 and 3

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{array}{ccc} (b'_{20})^{(3)} \boxed{-(b''_{20})^{(3)}(G_{23}, t)} & \boxed{-(b''_{16})^{(2,2,2)}(G_{19}, t)} & \boxed{-(b''_{13})^{(1,1,1)}(G, t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,4)}(G_{27}, t)} & \boxed{-(b''_{28})^{(5,5,5,5,5)}(G_{31}, t)} & \boxed{-(b''_{32})^{(6,6,6,6,6)}(G_{35}, t)} \end{array} \right] T_{20} \quad 88$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[\begin{array}{ccc} (b'_{21})^{(3)} \boxed{-(b''_{21})^{(3)}(G_{23}, t)} & \boxed{-(b''_{17})^{(2,2,2)}(G_{19}, t)} & \boxed{-(b''_{14})^{(1,1,1)}(G, t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,4)}(G_{27}, t)} & \boxed{-(b''_{29})^{(5,5,5,5,5)}(G_{31}, t)} & \boxed{-(b''_{33})^{(6,6,6,6,6)}(G_{35}, t)} \end{array} \right] T_{21} \quad 89$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[\begin{array}{ccc} (b'_{22})^{(3)} \boxed{-(b''_{22})^{(3)}(G_{23}, t)} & \boxed{-(b''_{18})^{(2,2,2)}(G_{19}, t)} & \boxed{-(b''_{15})^{(1,1,1)}(G, t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,4)}(G_{27}, t)} & \boxed{-(b''_{30})^{(5,5,5,5,5)}(G_{31}, t)} & \boxed{-(b''_{34})^{(6,6,6,6,6)}(G_{35}, t)} \end{array} \right] T_{22} \quad 90$$

$\boxed{-(b''_{20})^{(3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3)}(G_{23}, t)}$ are first detrition coefficients for category 1, 2 and 3 91

$\boxed{-(b''_{16})^{(2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2,2,2)}(G_{19}, t)}$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,1)}(G, t)$, $-(b''_{14})^{(1,1,1)}(G, t)$, $-(b''_{15})^{(1,1,1)}(G, t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27}, t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27}, t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27}, t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31}, t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31}, t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31}, t)$ are fifth detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35}, t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35}, t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35}, t)$ are sixth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{array}{cccc} (a'_{24})^{(4)} & +(a''_{24})^{(4)}(T_{25}, t) & +(a''_{28})^{(5,5)}(T_{29}, t) & +(a''_{32})^{(6,6)}(T_{33}, t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14}, t) & +(a''_{16})^{(2,2,2,2)}(T_{17}, t) & +(a''_{20})^{(3,3,3,3)}(T_{21}, t) & \end{array} \right] G_{24} \tag{92}$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{array}{cccc} (a'_{25})^{(4)} & +(a''_{25})^{(4)}(T_{25}, t) & +(a''_{29})^{(5,5)}(T_{29}, t) & +(a''_{33})^{(6,6)}(T_{33}, t) \\ +(a''_{14})^{(1,1,1,1)}(T_{14}, t) & +(a''_{17})^{(2,2,2,2)}(T_{17}, t) & +(a''_{21})^{(3,3,3,3)}(T_{21}, t) & \end{array} \right] G_{25} \tag{94}$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{array}{cccc} (a'_{26})^{(4)} & +(a''_{26})^{(4)}(T_{25}, t) & +(a''_{30})^{(5,5)}(T_{29}, t) & +(a''_{34})^{(6,6)}(T_{33}, t) \\ +(a''_{15})^{(1,1,1,1)}(T_{14}, t) & +(a''_{18})^{(2,2,2,2)}(T_{17}, t) & +(a''_{22})^{(3,3,3,3)}(T_{21}, t) & \end{array} \right] G_{26} \tag{95}$$

Where $(a''_{24})^{(4)}(T_{25}, t)$, $(a''_{25})^{(4)}(T_{25}, t)$, $(a''_{26})^{(4)}(T_{25}, t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5)}(T_{29}, t)$, $+(a''_{29})^{(5,5)}(T_{29}, t)$, $+(a''_{30})^{(5,5)}(T_{29}, t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6)}(T_{33}, t)$, $+(a''_{33})^{(6,6)}(T_{33}, t)$, $+(a''_{34})^{(6,6)}(T_{33}, t)$ are third augmentation coefficient for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1)}(T_{14}, t)$, $+(a''_{14})^{(1,1,1,1)}(T_{14}, t)$, $+(a''_{15})^{(1,1,1,1)}(T_{14}, t)$ are fourth augmentation coefficients for category 1, 2, and 3

$+(a''_{16})^{(2,2,2,2)}(T_{17}, t)$, $+(a''_{17})^{(2,2,2,2)}(T_{17}, t)$, $+(a''_{18})^{(2,2,2,2)}(T_{17}, t)$ are fifth augmentation coefficients for category 1, 2, and 3

$+(a''_{20})^{(3,3,3,3)}(T_{21}, t)$, $+(a''_{21})^{(3,3,3,3)}(T_{21}, t)$, $+(a''_{22})^{(3,3,3,3)}(T_{21}, t)$ are sixth augmentation coefficients for category 1, 2, and 3

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[\begin{array}{cccc} (b'_{24})^{(4)} & -(b''_{24})^{(4)}(G_{27}, t) & -(b''_{28})^{(5,5)}(G_{31}, t) & -(b''_{32})^{(6,6)}(G_{35}, t) \\ -(b''_{13})^{(1,1,1,1)}(G, t) & -(b''_{16})^{(2,2,2,2)}(G_{19}, t) & -(b''_{20})^{(3,3,3,3)}(G_{23}, t) & \end{array} \right] T_{24} \tag{97}$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[\begin{array}{cccc} (b'_{25})^{(4)} & -(b''_{25})^{(4)}(G_{27}, t) & -(b''_{29})^{(5,5)}(G_{31}, t) & -(b''_{33})^{(6,6)}(G_{35}, t) \\ -(b''_{14})^{(1,1,1,1)}(G, t) & -(b''_{17})^{(2,2,2,2)}(G_{19}, t) & -(b''_{21})^{(3,3,3,3)}(G_{23}, t) & \end{array} \right] T_{25} \tag{98}$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[\begin{array}{cccc} (b'_{26})^{(4)} & -(b''_{26})^{(4)}(G_{27}, t) & -(b''_{30})^{(5,5)}(G_{31}, t) & -(b''_{34})^{(6,6)}(G_{35}, t) \\ -(b''_{15})^{(1,1,1,1)}(G, t) & -(b''_{18})^{(2,2,2,2)}(G_{19}, t) & -(b''_{22})^{(3,3,3,3)}(G_{23}, t) & \end{array} \right] T_{26} \tag{99}$$

Where $-(b''_{24})^{(4)}(G_{27}, t)$, $-(b''_{25})^{(4)}(G_{27}, t)$, $-(b''_{26})^{(4)}(G_{27}, t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{28})^{(5,5)}(G_{31}, t)$, $-(b''_{29})^{(5,5)}(G_{31}, t)$, $-(b''_{30})^{(5,5)}(G_{31}, t)$ are second detrition coefficients for category 1, 2 and 3

$-(b''_{32})^{(6,6)}(G_{35}, t)$, $-(b''_{33})^{(6,6)}(G_{35}, t)$, $-(b''_{34})^{(6,6)}(G_{35}, t)$ are third detrition coefficients for category 1, 2 and 3 101

$-(b''_{13})^{(1,1,1,1)}(G, t)$, $-(b''_{14})^{(1,1,1,1)}(G, t)$, $-(b''_{15})^{(1,1,1,1)}(G, t)$ are fourth detrition coefficients for category 1, 2 and 3

$-(b''_{16})^{(2,2,2,2)}(G_{19}, t)$, $-(b''_{17})^{(2,2,2,2)}(G_{19}, t)$, $-(b''_{18})^{(2,2,2,2)}(G_{19}, t)$ are fifth detrition coefficients for category 1, 2 and 3 102

$-(b''_{20})^{(3,3,3,3)}(G_{23}, t)$, $-(b''_{21})^{(3,3,3,3)}(G_{23}, t)$, $-(b''_{22})^{(3,3,3,3)}(G_{23}, t)$ are sixth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t) & + (a'_{24})^{(4,4)}(T_{25}, t) & + (a'_{32})^{(6,6,6)}(T_{33}, t) \\ + (a'_{13})^{(1,1,1,1,1)}(T_{14}, t) & + (a'_{16})^{(2,2,2,2,2)}(T_{17}, t) & + (a'_{20})^{(3,3,3,3,3)}(T_{21}, t) \end{array} \right] G_{28} \quad 103$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[\begin{array}{ccc} (a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t) & + (a'_{25})^{(4,4)}(T_{25}, t) & + (a'_{33})^{(6,6,6)}(T_{33}, t) \\ + (a'_{14})^{(1,1,1,1,1)}(T_{14}, t) & + (a'_{17})^{(2,2,2,2,2)}(T_{17}, t) & + (a'_{21})^{(3,3,3,3,3)}(T_{21}, t) \end{array} \right] G_{29} \quad 104$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[\begin{array}{ccc} (a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t) & + (a'_{26})^{(4,4)}(T_{25}, t) & + (a'_{34})^{(6,6,6)}(T_{33}, t) \\ + (a'_{15})^{(1,1,1,1,1)}(T_{14}, t) & + (a'_{18})^{(2,2,2,2,2)}(T_{17}, t) & + (a'_{22})^{(3,3,3,3,3)}(T_{21}, t) \end{array} \right] G_{30} \quad 105$$

Where $+(a''_{28})^{(5)}(T_{29}, t)$, $+(a''_{29})^{(5)}(T_{29}, t)$, $+(a''_{30})^{(5)}(T_{29}, t)$ are first augmentation coefficients for category 1, 2 and 3 106

And $+(a''_{24})^{(4,4)}(T_{25}, t)$, $+(a''_{25})^{(4,4)}(T_{25}, t)$, $+(a''_{26})^{(4,4)}(T_{25}, t)$ are second augmentation coefficient for category 1, 2 and 3

$+(a''_{32})^{(6,6,6)}(T_{33}, t)$, $+(a''_{33})^{(6,6,6)}(T_{33}, t)$, $+(a''_{34})^{(6,6,6)}(T_{33}, t)$ are third augmentation coefficient for category 1, 2 and 3 107

$+(a''_{13})^{(1,1,1,1,1)}(T_{14}, t)$, $+(a''_{14})^{(1,1,1,1,1)}(T_{14}, t)$, $+(a''_{15})^{(1,1,1,1,1)}(T_{14}, t)$ are fourth augmentation coefficients for category 1, 2, and 3

$+(a''_{16})^{(2,2,2,2,2)}(T_{17}, t)$, $+(a''_{17})^{(2,2,2,2,2)}(T_{17}, t)$, $+(a''_{18})^{(2,2,2,2,2)}(T_{17}, t)$ are fifth augmentation coefficients for category 1, 2, and 3

$+(a''_{20})^{(3,3,3,3,3)}(T_{21}, t)$, $+(a''_{21})^{(3,3,3,3,3)}(T_{21}, t)$, $+(a''_{22})^{(3,3,3,3,3)}(T_{21}, t)$ are sixth augmentation coefficients for category 1, 2, 3

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[\begin{array}{ccc} (b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31}, t) & - (b'_{24})^{(4,4)}(G_{27}, t) & - (b'_{32})^{(6,6,6)}(G_{35}, t) \\ - (b'_{13})^{(1,1,1,1,1)}(G, t) & - (b'_{16})^{(2,2,2,2,2)}(G_{19}, t) & - (b'_{20})^{(3,3,3,3,3)}(G_{23}, t) \end{array} \right] T_{28} \quad 108$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[\begin{array}{ccc} (b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31}, t) & - (b'_{25})^{(4,4)}(G_{27}, t) & - (b'_{33})^{(6,6,6)}(G_{35}, t) \\ - (b'_{14})^{(1,1,1,1,1)}(G, t) & - (b'_{17})^{(2,2,2,2,2)}(G_{19}, t) & - (b'_{21})^{(3,3,3,3,3)}(G_{23}, t) \end{array} \right] T_{29} \quad 109$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[\begin{array}{ccc} (b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31}, t) & - (b'_{26})^{(4,4)}(G_{27}, t) & - (b'_{34})^{(6,6,6)}(G_{35}, t) \\ - (b'_{15})^{(1,1,1,1,1)}(G, t) & - (b'_{18})^{(2,2,2,2,2)}(G_{19}, t) & - (b'_{22})^{(3,3,3,3,3)}(G_{23}, t) \end{array} \right] T_{30} \quad 110$$

where $-(b''_{28})^{(5)}(G_{31}, t)$, $-(b''_{29})^{(5)}(G_{31}, t)$, $-(b''_{30})^{(5)}(G_{31}, t)$ are first detrition coefficients for category 1, 2 and 3 111

$-(b''_{24})^{(4,4,4)}(G_{27}, t)$, $-(b''_{25})^{(4,4,4)}(G_{27}, t)$, $-(b''_{26})^{(4,4,4)}(G_{27}, t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6)}(G_{35}, t)$, $-(b''_{33})^{(6,6,6)}(G_{35}, t)$, $-(b''_{34})^{(6,6,6)}(G_{35}, t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{13})^{(1,1,1,1,1,1)}(G, t)$, $-(b''_{14})^{(1,1,1,1,1,1)}(G, t)$, $-(b''_{15})^{(1,1,1,1,1,1)}(G, t)$ are fourth detrition coefficients 112

$-(b''_{16})^{(2,2,2,2,2,2)}(G_{19}, t)$, $-(b''_{17})^{(2,2,2,2,2,2)}(G_{19}, t)$, $-(b''_{18})^{(2,2,2,2,2,2)}(G_{19}, t)$ are fifth detrition coefficients for category 1,2, and 3

$-(b''_{20})^{(3,3,3,3,3,3)}(G_{23}, t)$, $-(b''_{21})^{(3,3,3,3,3,3)}(G_{23}, t)$, $-(b''_{22})^{(3,3,3,3,3,3)}(G_{23}, t)$ are sixth detrition coefficients for category 1,2, and 3

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{32})^{(6)} & + (a''_{32})^{(6)}(T_{33}, t) & + (a''_{28})^{(5,5,5)}(T_{29}, t) & + (a''_{24})^{(4,4,4)}(T_{25}, t) \\ + (a''_{13})^{(1,1,1,1,1,1)}(T_{14}, t) & + (a''_{16})^{(2,2,2,2,2,2)}(T_{17}, t) & + (a''_{20})^{(3,3,3,3,3,3)}(T_{21}, t) & \end{array} \right] G_{32} \quad 113$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{ccc} (a'_{33})^{(6)} & + (a''_{33})^{(6)}(T_{33}, t) & + (a''_{29})^{(5,5,5)}(T_{29}, t) & + (a''_{25})^{(4,4,4)}(T_{25}, t) \\ + (a''_{14})^{(1,1,1,1,1,1)}(T_{14}, t) & + (a''_{17})^{(2,2,2,2,2,2)}(T_{17}, t) & + (a''_{21})^{(3,3,3,3,3,3)}(T_{21}, t) & \end{array} \right] G_{33} \quad 114$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{ccc} (a'_{34})^{(6)} & + (a''_{34})^{(6)}(T_{33}, t) & + (a''_{30})^{(5,5,5)}(T_{29}, t) & + (a''_{26})^{(4,4,4)}(T_{25}, t) \\ + (a''_{15})^{(1,1,1,1,1,1)}(T_{14}, t) & + (a''_{18})^{(2,2,2,2,2,2)}(T_{17}, t) & + (a''_{22})^{(3,3,3,3,3,3)}(T_{21}, t) & \end{array} \right] G_{34} \quad 115$$

$+(a''_{32})^{(6)}(T_{33}, t)$, $+(a''_{33})^{(6)}(T_{33}, t)$, $+(a''_{34})^{(6)}(T_{33}, t)$ are first augmentation coefficients for category 1, 2 and 3 116

$+(a''_{28})^{(5,5,5)}(T_{29}, t)$, $+(a''_{29})^{(5,5,5)}(T_{29}, t)$, $+(a''_{30})^{(5,5,5)}(T_{29}, t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4)}(T_{25}, t)$, $+(a''_{25})^{(4,4,4)}(T_{25}, t)$, $+(a''_{26})^{(4,4,4)}(T_{25}, t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,1,1,1)}(T_{14}, t)$, $+(a''_{14})^{(1,1,1,1,1,1)}(T_{14}, t)$, $+(a''_{15})^{(1,1,1,1,1,1)}(T_{14}, t)$ - are fourth augmentation coefficients

$+(a''_{16})^{(2,2,2,2,2,2)}(T_{17}, t)$, $+(a''_{17})^{(2,2,2,2,2,2)}(T_{17}, t)$, $+(a''_{18})^{(2,2,2,2,2,2)}(T_{17}, t)$ - fifth augmentation coefficients

$+(a''_{20})^{(3,3,3,3,3,3)}(T_{21}, t)$, $+(a''_{21})^{(3,3,3,3,3,3)}(T_{21}, t)$, $+(a''_{22})^{(3,3,3,3,3,3)}(T_{21}, t)$ sixth augmentation coefficients

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{32})^{(6)} & -(b''_{32})^{(6)}(G_{35}, t) & -(b''_{28})^{(5,5,5)}(G_{31}, t) & -(b''_{24})^{(4,4,4)}(G_{27}, t) \\ -(b''_{13})^{(1,1,1,1,1,1)}(G, t) & -(b''_{16})^{(2,2,2,2,2,2)}(G_{19}, t) & -(b''_{20})^{(3,3,3,3,3,3)}(G_{23}, t) & \end{array} \right] T_{32} \quad 117$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[\begin{array}{ccc} (b'_{33})^{(6)} & -(b''_{33})^{(6)}(G_{35}, t) & -(b''_{29})^{(5,5,5)}(G_{31}, t) & -(b''_{25})^{(4,4,4)}(G_{27}, t) \\ -(b''_{14})^{(1,1,1,1,1,1)}(G, t) & -(b''_{17})^{(2,2,2,2,2,2)}(G_{19}, t) & -(b''_{21})^{(3,3,3,3,3,3)}(G_{23}, t) & \end{array} \right] T_{33} \quad 118$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[\begin{array}{ccc} (b'_{34})^{(6)} & -(b''_{34})^{(6)}(G_{35}, t) & -(b''_{30})^{(5,5,5)}(G_{31}, t) & -(b''_{26})^{(4,4,4)}(G_{27}, t) \\ -(b''_{15})^{(1,1,1,1,1,1)}(G, t) & -(b''_{18})^{(2,2,2,2,2,2)}(G_{19}, t) & -(b''_{22})^{(3,3,3,3,3,3)}(G_{23}, t) & \end{array} \right] T_{34} \quad 119$$

$-(b''_{32})^{(6)}(G_{35}, t)$, $-(b''_{33})^{(6)}(G_{35}, t)$, $-(b''_{34})^{(6)}(G_{35}, t)$ are first detrition coefficients for category 1, 2 and 3 120

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31}, t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31}, t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{25})^{(4,4,4)}(G_{27}, t)}$, $\boxed{-(b''_{26})^{(4,4,4)}(G_{27}, t)}$ are third detrition coefficients for category 1,2 and 3 121

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G, t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G, t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G, t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19}, t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19}, t)}$ are fifth detrition coefficients for category 1, 2, and 3 122

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23}, t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23}, t)}$ are sixth detrition coefficients for category 1, 2, and 3

Where we suppose

(A) $(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0$, 123
 $i, j = 13, 14, 15$

(B) The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}, (r_i)^{(1)}$:

$$(a''_i)^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)} \quad 124$$

$$(b''_i)^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b'_i)^{(1)} \leq (\hat{B}_{13})^{(1)}$$

(C) $\lim_{T_2 \rightarrow \infty} (a''_i)^{(1)}(T_{14}, t) = (p_i)^{(1)}$ 125
 $\lim_{G \rightarrow \infty} (b''_i)^{(1)}(G, t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13, 14, 15}$

They satisfy Lipschitz condition: 126

$$|(a''_i)^{(1)}(T'_{14}, t) - (a''_i)^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)} |T'_{14} - T_{14}| e^{-(\hat{M}_{13})^{(1)}t} \quad 127$$

$$|(b''_i)^{(1)}(G', t) - (b''_i)^{(1)}(G, T)| < (\hat{k}_{13})^{(1)} \|G - G'\| e^{-(\hat{M}_{13})^{(1)}t} \quad 128$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(1)}(T'_{14}, t)$ and $(a''_i)^{(1)}(T_{14}, t)$. (T'_{14}, t) and (T_{14}, t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a''_i)^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a''_i)^{(1)}(T_{14}, t)$, the first augmentation coefficient would be absolutely continuous. 129

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$: 130

(D) $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$:

- (E) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a'_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$ 131

satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(a_i)^{(1)} + (a'_i)^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(b_i)^{(1)} + (b'_i)^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Where we suppose

- (F) $(a_i)^{(2)}, (a'_i)^{(2)}, (a''_i)^{(2)}, (b_i)^{(2)}, (b'_i)^{(2)}, (b''_i)^{(2)} > 0, \quad i, j = 16,17,18$ 132

- (G) The functions $(a''_i)^{(2)}, (b''_i)^{(2)}$ are positive continuous increasing and bounded. 133

Definition of $(p_i)^{(2)}, (r_i)^{(2)}$: 134

$$(a''_i)^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)} \quad 135$$

$$(b''_i)^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (b'_i)^{(2)} \leq (\hat{B}_{16})^{(2)} \quad 136$$

- (H) $\lim_{T_2 \rightarrow \infty} (a''_i)^{(2)}(T_{17}, t) = (p_i)^{(2)}$ 137

$$\lim_{G \rightarrow \infty} (b''_i)^{(2)}(G_{19}, t) = (r_i)^{(2)} \quad 138$$

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 139

Where $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}$ are positive constants and $i = 16,17,18$

They satisfy Lipschitz condition:

$$|(a''_i)^{(2)}(T'_{17}, t) - (a''_i)^{(2)}(T_{17}, t)| \leq (\hat{k}_{16})^{(2)} |T'_{17} - T_{17}| e^{-(\hat{M}_{16})^{(2)}t} \quad 140$$

$$|(b''_i)^{(2)}((G_{19})', t) - (b''_i)^{(2)}((G_{19}), T_{19})| < (\hat{k}_{16})^{(2)} \|(G_{19}) - (G_{19})'\| e^{-(\hat{M}_{16})^{(2)}t} \quad 141$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(2)}(T'_{17}, t)$ and $(a''_i)^{(2)}(T_{17}, t)$. (T'_{17}, t) and (T_{17}, t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a''_i)^{(2)}(T_{17}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a''_i)^{(2)}(T_{17}, t)$, the SECOND augmentation coefficient would be absolutely continuous.

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$: 142

- (I) $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 143

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$: 144

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)}$ and $(\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a'_i)^{(2)}, (b_i)^{(2)}, (b'_i)^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(a_i)^{(2)} + (a'_i)^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 145$$

$$\frac{1}{(\hat{M}_{16})^{(2)}} [(b_i)^{(2)} + (b'_i)^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)} (\hat{k}_{16})^{(2)}] < 1 \quad 146$$

Where we suppose

$$(J) \quad (a_i)^{(3)}, (a'_i)^{(3)}, (a''_i)^{(3)}, (b_i)^{(3)}, (b'_i)^{(3)}, (b''_i)^{(3)} > 0, \quad i, j = 20, 21, 22 \quad 147$$

The functions $(a''_i)^{(3)}, (b''_i)^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, (r_i)^{(3)}$:

$$(a''_i)^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b''_i)^{(3)}(G, t) \leq (r_i)^{(3)} \leq (b'_i)^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$\lim_{T_2 \rightarrow \infty} (a''_i)^{(3)}(T_{21}, t) = (p_i)^{(3)} \quad 148$$

$$\lim_{G \rightarrow \infty} (b''_i)^{(3)}(G, t) = (r_i)^{(3)} \quad 149$$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$: 150

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20, 21, 22}$

They satisfy Lipschitz condition:

$$|(a''_i)^{(3)}(T'_{21}, t) - (a''_i)^{(3)}(T_{21}, t)| \leq (\hat{k}_{20})^{(3)} |T'_{21} - T_{21}| e^{-(\hat{M}_{20})^{(3)}t} \quad 151$$

$$|(b''_i)^{(3)}(G', t) - (b''_i)^{(3)}(G, T)| < (\hat{k}_{20})^{(3)} \|G - G'\| e^{-(\hat{M}_{20})^{(3)}t} \quad 152$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(3)}(T'_{21}, t)$ and $(a''_i)^{(3)}(T_{21}, t) \cdot (T'_{21}, t)$ and (T_{21}, t) are points belonging to the interval $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$. It is to be noted that $(a''_i)^{(3)}(T_{21}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a''_i)^{(3)}(T_{21}, t)$, the third augmentation coefficient would be absolutely continuous.

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 153

(K) $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$$

There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)}$ and $(\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a'_i)^{(3)}, (b_i)^{(3)}, (b'_i)^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20, 21, 22$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{20})^{(3)}} [(a_i)^{(3)} + (a'_i)^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1 \quad 154$$

$$\frac{1}{(\hat{M}_{20})^{(3)}} [(b_i)^{(3)} + (b'_i)^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}] < 1 \quad 155$$

Where we suppose

$$(L) \quad (a_i)^{(4)}, (a'_i)^{(4)}, (a''_i)^{(4)}, (b_i)^{(4)}, (b'_i)^{(4)}, (b''_i)^{(4)} > 0, \quad i, j = 24, 25, 26 \quad 156$$

(M) The functions $(a''_i)^{(4)}, (b''_i)^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, (r_i)^{(4)}$:

$$(a''_i)^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$$

$$(b''_i)^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b'_i)^{(4)} \leq (\hat{B}_{24})^{(4)}$$

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$$(N) \quad \lim_{T_2 \rightarrow \infty} (a''_i)^{(4)}(T_{25}, t) = (p_i)^{(4)}$$

$$\lim_{G \rightarrow \infty} (b''_i)^{(4)}((G_{27}), t) = (r_i)^{(4)}$$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}$ are positive constants and $i = 24, 25, 26$

They satisfy Lipschitz condition:

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$$|(a''_i)^{(4)}(T'_{25}, t) - (a''_i)^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)} |T'_{25} - T_{25}| e^{-(\hat{M}_{24})^{(4)}t}$$

$$|(b''_i)^{(4)}((G'_{27}), t) - (b''_i)^{(4)}((G_{27}), T)| < (\hat{k}_{24})^{(4)} |(G'_{27}) - (G_{27})| e^{-(\hat{M}_{24})^{(4)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(4)}(T'_{25}, t)$ and $(a''_i)^{(4)}(T_{25}, t)$. (T'_{25}, t) and (T_{25}, t) are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a''_i)^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 4$ then the function $(a''_i)^{(4)}(T_{25}, t)$, the fourth **augmentation coefficient** would be absolutely continuous.

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$:

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$$(O) \quad (\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, \text{ are positive constants}$$

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$:

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(P) There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)}$ and $(\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a'_i)^{(4)}, (b_i)^{(4)}, (b'_i)^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24, 25, 26$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{24})^{(4)}} [(a_i)^{(4)} + (a'_i)^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

$$\frac{1}{(\hat{M}_{24})^{(4)}} [(b_i)^{(4)} + (b'_i)^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$$

Where we suppose

- (Q) $(a_i)^{(5)}, (a'_i)^{(5)}, (a''_i)^{(5)}, (b_i)^{(5)}, (b'_i)^{(5)}, (b''_i)^{(5)} > 0, \quad i, j = 28, 29, 30$ 162
 (R) The functions $(a''_i)^{(5)}, (b''_i)^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, (r_i)^{(5)}$:

$$(a''_i)^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b''_i)^{(5)}((G_{31}), t) \leq (r_i)^{(5)} \leq (b'_i)^{(5)} \leq (\hat{B}_{28})^{(5)}$$

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- (S) $\lim_{T_2 \rightarrow \infty} (a''_i)^{(5)}(T_{29}, t) = (p_i)^{(5)}$
 $\lim_{G \rightarrow \infty} (b''_i)^{(5)}(G_{31}, t) = (r_i)^{(5)}$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}$ are positive constants and $i = 28, 29, 30$

They satisfy Lipschitz condition:

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$$|(a''_i)^{(5)}(T'_{29}, t) - (a''_i)^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)} |T'_{29} - T_{29}| e^{-(M_{28})^{(5)}t}$$

$$|(b''_i)^{(5)}((G_{31})', t) - (b''_i)^{(5)}((G_{31}), (T_{31}))| < (\hat{k}_{28})^{(5)} \|(G_{31}) - (G_{31})'\| e^{-(M_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(5)}(T'_{29}, t)$ and $(a''_i)^{(5)}(T_{29}, t)$. (T'_{29}, t) and (T_{29}, t) are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a''_i)^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 5$ then the function $(a''_i)^{(5)}(T_{29}, t)$, the fifth **augmentation coefficient** would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$:

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(T) $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$:

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- (U) There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)}$ and $(\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a'_i)^{(5)}, (b_i)^{(5)}, (b'_i)^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}, i = 28, 29, 30$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}} [(a_i)^{(5)} + (a'_i)^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}} [(b_i)^{(5)} + (b'_i)^{(5)} + (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)} (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose

- $(a_i)^{(6)}, (a'_i)^{(6)}, (a''_i)^{(6)}, (b_i)^{(6)}, (b'_i)^{(6)}, (b''_i)^{(6)} > 0, \quad i, j = 32, 33, 34$ 168
 (V) The functions $(a''_i)^{(6)}, (b''_i)^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, (r_i)^{(6)}$:

$$(a''_i)^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i'')^{(6)}((G_{35}), t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$

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$$(W) \quad \lim_{T_2 \rightarrow \infty} (a_i'')^{(6)}(T_{33}, t) = (p_i)^{(6)}$$

$$\lim_{G \rightarrow \infty} (b_i'')^{(6)}((G_{35}), t) = (r_i)^{(6)}$$

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32, 33, 34}$

They satisfy Lipschitz condition:

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$$|(a_i'')^{(6)}(T'_{33}, t) - (a_i'')^{(6)}(T_{33}, t)| \leq (\hat{k}_{32})^{(6)} |T_{33} - T'_{33}| e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})', t) - (b_i'')^{(6)}((G_{35}), (T_{35}))| < (\hat{k}_{32})^{(6)} \|(G_{35}) - (G_{35})'\| e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T'_{33}, t)$ and $(a_i'')^{(6)}(T_{33}, t)$. (T'_{33}, t) and (T_{33}, t) are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 6$ then the function $(a_i'')^{(6)}(T_{33}, t)$, the sixth **augmentation coefficient** would be absolutely continuous.

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Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$:

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$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$:1

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There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)}$ and $(\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32, 33, 34$, satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}} [(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}} [(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)} (\hat{k}_{32})^{(6)}] < 1$$

Theorem 1: if the conditions (A)-(E) (first five conditions related to the system Boolean satisfiability problem) above are fulfilled, there exists a solution satisfying the conditions

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Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$

If the conditions of second module pertaining to Knapsack problem and Hamiltonian Path Problem above are fulfilled, there exists a solution satisfying the conditions

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Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)}t}, \quad T_i(0) = T_i^0 > 0$$

If the conditions pertaining to the third module Sub graph Isomorphism problem and Subset sum problem above are fulfilled, there exists a solution satisfying the conditions 176

$$G_i(t) \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad G_i(0) = G_i^0 > 0$$

$$T_i(t) \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}, \quad T_i(0) = T_i^0 > 0$$

If the conditions of the fourth module Subset Sum Problem and Clique problem above are fulfilled, there exists a solution satisfying the conditions 177

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

If the conditions pertaining to the module five namely Vertex Cover Problem and Independent Set problem are fulfilled, there exists a solution satisfying the conditions 178

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

If the conditions pertaining to Dominating set problem and Graph Coloring Problem above are fulfilled, there exists a solution satisfying the conditions 179

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof: 180

Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{13})^{(1)}, T_i^0 \leq (\hat{Q}_{13})^{(1)}, \quad 181$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 182$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}t} \quad 183$$

By 184

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a'_{13})^{(1)} + a''_{13}(1)(T_{14}(s_{(13)}), s_{(13)}) \right) G_{13}(s_{(13)}) \right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}(s_{(13)}), s_{(13)}) \right) G_{14}(s_{(13)}) \right] ds_{(13)} \quad 185$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}(s_{(13)}), s_{(13)}) \right) G_{15}(s_{(13)}) \right] ds_{(13)} \quad 186$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b'_{13})^{(1)} - (b''_{13})^{(1)}(G(s_{(13)}), s_{(13)}) \right) T_{13}(s_{(13)}) \right] ds_{(13)} \quad 187$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b'_{14})^{(1)} - (b''_{14})^{(1)}(G(s_{(13)}), s_{(13)}) \right) T_{14}(s_{(13)}) \right] ds_{(13)} \quad 188$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b'_{15})^{(1)} - (b''_{15})^{(1)}(G(s_{(13)}), s_{(13)}) \right) T_{15}(s_{(13)}) \right] ds_{(13)} \quad 189$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{16})^{(2)}, T_i^0 \leq (\hat{Q}_{16})^{(2)}, \quad 190$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(M_{16})^{(2)}t} \quad 191$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(M_{16})^{(2)}t} \quad 192$$

By 193

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a'_{16})^{(2)} + a''_{16}(2)(T_{17}(s_{(16)}), s_{(16)}) \right) G_{16}(s_{(16)}) \right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}(s_{(16)}), s_{(17)}) \right) G_{17}(s_{(16)}) \right] ds_{(16)} \quad 194$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}(s_{(16)}), s_{(16)}) \right) G_{18}(s_{(16)}) \right] ds_{(16)} \quad 195$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b'_{16})^{(2)} - (b''_{16})^{(2)}(G(s_{(16)}), s_{(16)}) \right) T_{16}(s_{(16)}) \right] ds_{(16)} \quad 196$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b'_{17})^{(2)} - (b''_{17})^{(2)}(G(s_{(16)}), s_{(16)}) \right) T_{17}(s_{(16)}) \right] ds_{(16)} \quad 197$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b'_{18})^{(2)} - (b''_{18})^{(2)}(G(s_{(16)}), s_{(16)}) \right) T_{18}(s_{(16)}) \right] ds_{(16)} \quad 198$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy 199

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{20})^{(3)}, T_i^0 \leq (\hat{Q}_{20})^{(3)}, \quad 200$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(M_{20})^{(3)}t} \quad 201$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(M_{20})^{(3)}t} \quad 202$$

By 203

$$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a'_{20})^{(3)} + a''_{20}(3)(T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$$

$$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a'_{21})^{(3)} + (a''_{21})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)} \quad 204$$

$$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a'_{22})^{(3)} + (a''_{22})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)} \quad 205$$

$$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b'_{20})^{(3)} - (b''_{20})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)} \quad 206$$

$$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b'_{21})^{(3)} - (b''_{21})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)} \quad 207$$

$$\bar{T}_{22}(t) = T_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b'_{22})^{(3)} - (b''_{22})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)} \quad 208$$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Proof: Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{24})^{(4)}, T_i^0 \leq (\hat{Q}_{24})^{(4)}, \quad 209$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\mathcal{M}_{24})^{(4)}t} \quad 210$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\mathcal{M}_{24})^{(4)}t} \quad 211$$

By 212

$$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a'_{24})^{(4)} + (a''_{24})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)} \quad 213$$

$$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a'_{25})^{(4)} + (a''_{25})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)} \quad 214$$

$$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a'_{26})^{(4)} + (a''_{26})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)} \quad 215$$

$$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b'_{24})^{(4)} - (b''_{24})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)} \quad 216$$

$$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b'_{25})^{(4)} - (b''_{25})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)} \quad 217$$

$$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b'_{26})^{(4)} - (b''_{26})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)} \quad 218$$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{28})^{(5)}, T_i^0 \leq (\hat{Q}_{28})^{(5)}, \quad 219$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\mathcal{M}_{28})^{(5)}t} \quad 220$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\mathcal{M}_{28})^{(5)}t} \quad 221$$

By 221

$$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a'_{28})^{(5)} + a''_{28}{}^{(5)}(T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)} \quad 222$$

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)} \quad 223$$

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b'_{28})^{(5)} - (b''_{28})^{(5)}(G(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)} \quad 224$$

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b'_{29})^{(5)} - (b''_{29})^{(5)}(G(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)} \quad 225$$

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b'_{30})^{(5)} - (b''_{30})^{(5)}(G(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)} \quad 226$$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{32})^{(6)}, T_i^0 \leq (\hat{Q}_{32})^{(6)}, \quad 227$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} \quad 228$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} \quad 229$$

By 230

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32}{}^{(6)}(T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)} \quad 231$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)} \quad 232$$

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)}(G(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)} \quad 233$$

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)}(G(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)} \quad 234$$

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)}(G(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)} \quad 235$$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

(a) The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying into itself. Indeed it is obvious that 236

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)}s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)}t \right) G_{14}^0 + \frac{(a_{13})^{(1)}(\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)}t} - 1 \right)$$

From which it follows that 237

$$(G_{13}(t) - G_{13}^0)e^{-(\hat{M}_{13})^{(1)}t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[((\hat{P}_{13})^{(1)} + G_{14}^0)e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{\hat{G}_{14}^0}\right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{14}, G_{15}, T_{13}, T_{14}, T_{15}$

(b) The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying into itself .Indeed it is obvious that

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)}s_{(16)}} \right) \right] ds_{(16)} = (1 + (a_{16})^{(2)}t)G_{17}^0 + \frac{(a_{16})^{(2)}(\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)}t} - 1 \right) \tag{238}$$

From which it follows that 239

$$(G_{16}(t) - G_{16}^0)e^{-(\hat{M}_{16})^{(2)}t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[((\hat{P}_{16})^{(2)} + G_{17}^0)e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{\hat{G}_{17}^0}\right)} + (\hat{P}_{16})^{(2)} \right]$$

Analogous inequalities hold also for $G_{17}, G_{18}, T_{16}, T_{17}, T_{18}$

(a) The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 240

$$G_{20}(t) \leq G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}s_{(20)}} \right) \right] ds_{(20)} = (1 + (a_{20})^{(3)}t)G_{21}^0 + \frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left(e^{(\hat{M}_{20})^{(3)}t} - 1 \right)$$

From which it follows that 241

$$(G_{20}(t) - G_{20}^0)e^{-(\hat{M}_{20})^{(3)}t} \leq \frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}} \left[((\hat{P}_{20})^{(3)} + G_{21}^0)e^{\left(-\frac{(\hat{P}_{20})^{(3)} + G_{21}^0}{\hat{G}_{21}^0}\right)} + (\hat{P}_{20})^{(3)} \right] \tag{277}$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$ 278

(b) The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying into itself .Indeed it is obvious that 242

$$G_{24}(t) \leq G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)}s_{(24)}} \right) \right] ds_{(24)} = (1 + (a_{24})^{(4)}t)G_{25}^0 + \frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)}t} - 1 \right)$$

From which it follows that 243

$$(G_{24}(t) - G_{24}^0)e^{-(\hat{M}_{24})^{(4)}t} \leq \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[((\hat{P}_{24})^{(4)} + G_{25}^0)e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{\hat{G}_{25}^0}\right)} + (\hat{P}_{24})^{(4)} \right]$$

(G_i^0) is as defined in the statement of theorem 4

(c) The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying 35,35,36 into itself .Indeed it is obvious that 244

$$G_{28}(t) \leq G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}s_{(28)}} \right) \right] ds_{(28)} = (1 + (a_{28})^{(5)}t)G_{29}^0 + \frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)}t} - 1 \right)$$

From which it follows that

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$$(G_{28}(t) - G_{28}^0)e^{-(\bar{M}_{28})^{(5)}t} \leq \frac{(a_{28})^{(5)}}{(\bar{M}_{28})^{(5)}} \left[((\hat{P}_{28})^{(5)} + G_{29}^0)e^{-\left(\frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0}\right)} + (\hat{P}_{28})^{(5)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

(d) The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

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$$G_{32}(t) \leq G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)}s_{(32)}} \right) \right] ds_{(32)} = \\ (1 + (a_{32})^{(6)}t)G_{33}^0 + \frac{(a_{32})^{(6)}(\bar{P}_{32})^{(6)}}{(\bar{M}_{32})^{(6)}} \left(e^{(\bar{M}_{32})^{(6)}t} - 1 \right)$$

From which it follows that

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$$(G_{32}(t) - G_{32}^0)e^{-(\bar{M}_{32})^{(6)}t} \leq \frac{(a_{32})^{(6)}}{(\bar{M}_{32})^{(6)}} \left[((\hat{P}_{32})^{(6)} + G_{33}^0)e^{-\left(\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0}\right)} + (\hat{P}_{32})^{(6)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\bar{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\bar{M}_{13})^{(1)}} < 1$ and to choose

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$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\bar{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + ((\hat{P}_{13})^{(1)} + G_j^0)e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)} \right] \leq (\hat{P}_{13})^{(1)}$$

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$$\frac{(b_i)^{(1)}}{(\bar{M}_{13})^{(1)}} \left[((\hat{Q}_{13})^{(1)} + T_j^0)e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)} \right] \leq (\hat{Q}_{13})^{(1)}$$

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In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i into itself

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric

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$$d((G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)})) =$$

$$\sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{13})^{(1)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{13})^{(1)}t} \}$$

Indeed if we denote

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Definition of $\tilde{G}, \tilde{T} : (\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)$

It results

$$\begin{aligned}
 |\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{13})^{(1)} |G_{14}^{(1)} - G_{14}^{(2)}| e^{-(\bar{M}_{13})^{(1)}s_{(13)}} e^{(\bar{M}_{13})^{(1)}s_{(13)}} ds_{(13)} + \\
 &\int_0^t \{(a'_{13})^{(1)} |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\bar{M}_{13})^{(1)}s_{(13)}} e^{-(\bar{M}_{13})^{(1)}s_{(13)}} + \\
 &(a''_{13})^{(1)} (T_{14}^{(1)}, s_{(13)}) |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\bar{M}_{13})^{(1)}s_{(13)}} e^{(\bar{M}_{13})^{(1)}s_{(13)}} + \\
 &G_{13}^{(2)} |(a''_{13})^{(1)} (T_{14}^{(1)}, s_{(13)}) - (a''_{13})^{(1)} (T_{14}^{(2)}, s_{(13)})| e^{-(\bar{M}_{13})^{(1)}s_{(13)}} e^{(\bar{M}_{13})^{(1)}s_{(13)}}\} ds_{(13)}
 \end{aligned}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\begin{aligned}
 |G^{(1)} - G^{(2)}| e^{-(\bar{M}_{13})^{(1)}t} &\leq & 253 \\
 \frac{1}{(\bar{M}_{13})^{(1)}} ((a_{13})^{(1)} + (a'_{13})^{(1)} + (\bar{A}_{13})^{(1)} + (\bar{P}_{13})^{(1)} (\bar{k}_{13})^{(1)}) d((G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}))
 \end{aligned}$$

And analogous inequalities for G_i and T_i . Taking into account the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\bar{P}_{13})^{(1)} e^{(\bar{M}_{13})^{(1)}t}$ and $(\bar{Q}_{13})^{(1)} e^{(\bar{M}_{13})^{(1)}t}$ respectively of \mathbb{R}_+ . 254

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}$, $i = 13, 14, 15$ depend only on T_{14} and respectively on G (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 255

From the governing equations of the holistic system it results

$$\begin{aligned}
 G_i(t) &\geq G_i^0 e^{-\int_0^t ((a'_i)^{(1)} - (a''_i)^{(1)})(T_{14}(s_{(13)}), s_{(13)}) ds_{(13)}} \geq 0 \\
 T_i(t) &\geq T_i^0 e^{-(b'_i)^{(1)}t} > 0 \quad \text{for } t > 0
 \end{aligned}$$

Definition of $((\bar{M}_{13})^{(1)})_1, ((\bar{M}_{13})^{(1)})_2$ and $((\bar{M}_{13})^{(1)})_3$: 256

Remark 3: if G_{13} is bounded, the same property have also G_{14} and G_{15} . indeed if

$$G_{13} < ((\bar{M}_{13})^{(1)})_1 \text{ it follows } \frac{dG_{14}}{dt} \leq ((\bar{M}_{13})^{(1)})_1 - (a'_{14})^{(1)} G_{14} \text{ and by integrating}$$

$$G_{14} \leq ((\bar{M}_{13})^{(1)})_2 = G_{14}^0 + 2(a_{14})^{(1)} ((\bar{M}_{13})^{(1)})_1 / (a'_{14})^{(1)}$$

In the same way, one can obtain

$$G_{15} \leq ((\bar{M}_{13})^{(1)})_3 = G_{15}^0 + 2(a_{15})^{(1)} ((\bar{M}_{13})^{(1)})_2 / (a'_{15})^{(1)}$$

If G_{14} or G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} is bounded, from below, the same property holds for G_{14} and G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 257

Remark 5: If T_{13} is bounded from below and $\lim_{t \rightarrow \infty} ((b''_i)^{(1)}(G(t), t)) = (b'_{14})^{(1)}$ then $T_{14} \rightarrow \infty$. 258

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to 259

$$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1} \right) (1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_1 t} = \frac{1}{2} \text{ it results}$$

$$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_1} \text{ By taking now } \varepsilon_1 \text{ sufficiently small one sees that } T_{14} \text{ is unbounded.}$$

The same property holds for T_{15} if $\lim_{t \rightarrow \infty} (b_{15}'')^{(1)}(G(t), t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations solution to the governing equations of the global system

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\bar{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\bar{M}_{16})^{(2)}} < 1$ and to choose 260

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\bar{M}_{16})^{(2)}} \left[(\hat{P}_{16})^{(2)} + ((\hat{P}_{16})^{(2)} + G_j^0) e^{-\left(\frac{(\hat{P}_{16})^{(2)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{16})^{(2)} \quad 261$$

$$\frac{(b_i)^{(2)}}{(\bar{M}_{16})^{(2)}} \left[((\hat{Q}_{16})^{(2)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{16})^{(2)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{16})^{(2)} \right] \leq (\hat{Q}_{16})^{(2)} \quad 262$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i into itself 263

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 264

$$d\left(((G_{19})^{(1)}, (T_{19})^{(1)}), ((G_{19})^{(2)}, (T_{19})^{(2)}) \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{16})^{(2)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{16})^{(2)}t} \right\}$$

Indeed if we denote 265

$$\widetilde{G}_{19}, \widetilde{T}_{19} : (\widetilde{G}_{19}, \widetilde{T}_{19}) = \mathcal{A}^{(2)}(G_{19}, T_{19})$$

It results 266

$$|\widetilde{G}_{16}^{(1)} - \widetilde{G}_{16}^{(2)}| \leq \int_0^t (a_{16})^{(2)} |G_{17}^{(1)} - G_{17}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} ds_{(16)} +$$

$$\int_0^t \{ (a'_{16})^{(2)} |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{-(\bar{M}_{16})^{(2)}s_{(16)}} +$$

$$(a''_{16})^{(2)}(T_{17}^{(1)}, s_{(16)}) |G_{16}^{(1)} - G_{16}^{(2)}| e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} +$$

$$G_{16}^{(2)} | (a''_{16})^{(2)}(T_{17}^{(1)}, s_{(16)}) - (a''_{16})^{(2)}(T_{17}^{(2)}, s_{(16)}) | e^{-(\bar{M}_{16})^{(2)}s_{(16)}} e^{(\bar{M}_{16})^{(2)}s_{(16)}} \} ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 267

From the hypotheses it follows

$$\left| (G_{19})^{(1)} - (G_{19})^{(2)} \right| e^{-(\widehat{M}_{16})^{(2)}t} \leq \frac{1}{(\widehat{M}_{16})^{(2)}} \left((a_{16})^{(2)} + (a'_{16})^{(2)} + (\widehat{A}_{16})^{(2)} + (\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)} \right) d \left(((G_{19})^{(1)}, (T_{19})^{(1)}); (G_{19})^{(2)}, (T_{19})^{(2)} \right) \quad 268$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis (34,35,36) the result follows 269

Remark 1: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of \mathbb{R}_+ . 270

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(2)}$ and $(b''_i)^{(2)}$, $i = 16, 17, 18$ depend only on T_{17} and respectively on (G_{19}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 271

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{ (a'_i)^{(2)} - (a''_i)^{(2)} \} (T_{17}(s_{(16)}), s_{(16)}) \} ds_{(16)} \right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(2)}t} > 0 \quad \text{for } t > 0$$

Definition of $((\widehat{M}_{16})^{(2)})_1$, $((\widehat{M}_{16})^{(2)})_2$ and $((\widehat{M}_{16})^{(2)})_3$: 272

Remark 3: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$$G_{16} < ((\widehat{M}_{16})^{(2)})_1 \text{ it follows } \frac{dG_{17}}{dt} \leq ((\widehat{M}_{16})^{(2)})_1 - (a'_{17})^{(2)}G_{17} \text{ and by integrating}$$

$$G_{17} \leq ((\widehat{M}_{16})^{(2)})_2 = G_{17}^0 + 2(a_{17})^{(2)}((\widehat{M}_{16})^{(2)})_1 / (a'_{17})^{(2)}$$

In the same way, one can obtain

$$G_{18} \leq ((\widehat{M}_{16})^{(2)})_3 = G_{18}^0 + 2(a_{18})^{(2)}((\widehat{M}_{16})^{(2)})_2 / (a'_{18})^{(2)}$$

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 4: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 273

Remark 5: If T_{16} is bounded from below and $\lim_{t \rightarrow \infty} ((b'_i)^{(2)}((G_{19})(t), t)) = (b'_{17})^{(2)}$ then $T_{17} \rightarrow \infty$. 274

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$$(b_{17})^{(2)} - (b'_i)^{(2)}((G_{19})(t), t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 275

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right) (1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right)$, $t = \log \frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. 276

The same property holds for T_{18} if $\lim_{t \rightarrow \infty} (b''_{18})^{(2)}((G_{19})(t), t) = (b'_{18})^{(2)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

It is now sufficient to take $\frac{(a_i)^{(3)}}{(M_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(M_{20})^{(3)}} < 1$ and to choose 277

$(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(M_{20})^{(3)}} \left[(\hat{P}_{20})^{(3)} + ((\hat{P}_{20})^{(3)} + G_j^0) e^{-\left(\frac{(P_{20})^{(3)} + G_j^0}{G_j^0}\right)} \right] \leq (\hat{P}_{20})^{(3)} \quad 278$$

$$\frac{(b_i)^{(3)}}{(M_{20})^{(3)}} \left[((\hat{Q}_{20})^{(3)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{20})^{(3)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{20})^{(3)} \right] \leq (\hat{Q}_{20})^{(3)} \quad 279$$

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i into itself

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 280

$$d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)}\right), \left((G_{23})^{(2)}, (T_{23})^{(2)}\right)\right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(M_{20})^{(3)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(M_{20})^{(3)}t} \right\}$$

Indeed if we denote 281

Definition of $\widetilde{G}_{23}, \widetilde{T}_{23} : (\widetilde{G}_{23}, \widetilde{T}_{23}) = \mathcal{A}^{(3)}((G_{23}), (T_{23}))$

It results 282

$$\begin{aligned} |\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{20})^{(3)} |G_{21}^{(1)} - G_{21}^{(2)}| e^{-(M_{20})^{(3)}s_{(20)}} e^{(M_{20})^{(3)}s_{(20)}} ds_{(20)} + \\ &\int_0^t \{ (a'_{20})^{(3)} |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(M_{20})^{(3)}s_{(20)}} e^{-(M_{20})^{(3)}s_{(20)}} + \\ &(a''_{20})^{(3)} (T_{21}^{(1)}, s_{(20)}) |G_{20}^{(1)} - G_{20}^{(2)}| e^{-(M_{20})^{(3)}s_{(20)}} e^{(M_{20})^{(3)}s_{(20)}} + \\ &G_{20}^{(2)} | (a''_{20})^{(3)} (T_{21}^{(1)}, s_{(20)}) - (a''_{20})^{(3)} (T_{21}^{(2)}, s_{(20)}) | e^{-(M_{20})^{(3)}s_{(20)}} e^{(M_{20})^{(3)}s_{(20)}} \} ds_{(20)} \end{aligned}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\begin{aligned} |G^{(1)} - G^{(2)}| e^{-(M_{20})^{(3)}t} &\leq \quad 283 \\ \frac{1}{(M_{20})^{(3)}} &\left((a_{20})^{(3)} + (a'_{20})^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{k}_{20})^{(3)} \right) d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)}\right); (G_{23})^{(2)}, (T_{23})^{(2)}\right) \end{aligned}$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}$ and $(\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)}t}$ respectively of \mathbb{R}_+ . 284

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(3)}$ and $(b''_i)^{(3)}$, $i = 20, 21, 22$ depend only on T_{21} and respectively on (G_{23}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 285

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{-\int_0^t ((a'_i)^{(3)} - (a''_i)^{(3)})(T_{21}(s_{(20)}), s_{(20)}) ds_{(20)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b'_i)^{(3)}t} > 0 \quad \text{for } t > 0$$

Definition of $((\hat{M}_{20})^{(3)})_1$, $((\hat{M}_{20})^{(3)})_2$ and $((\hat{M}_{20})^{(3)})_3$: 286

Remark 3: if G_{20} is bounded, the same property have also G_{21} and G_{22} . indeed if

$$G_{20} < ((\hat{M}_{20})^{(3)})_1 \text{ it follows } \frac{dG_{21}}{dt} \leq ((\hat{M}_{20})^{(3)})_1 - (a'_{21})^{(3)}G_{21} \text{ and by integrating}$$

$$G_{21} \leq ((\hat{M}_{20})^{(3)})_2 = G_{21}^0 + 2(a_{21})^{(3)}((\hat{M}_{20})^{(3)})_1 / (a'_{21})^{(3)}$$

In the same way, one can obtain 287

$$G_{22} \leq ((\hat{M}_{20})^{(3)})_3 = G_{22}^0 + 2(a_{22})^{(3)}((\hat{M}_{20})^{(3)})_2 / (a'_{22})^{(3)}$$

If G_{21} or G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 4: If G_{20} is bounded, from below, the same property holds for G_{21} and G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 288

Remark 5: If T_{20} is bounded from below and $\lim_{t \rightarrow \infty} ((b''_i)^{(3)}((G_{23})(t), t)) = (b'_{21})^{(3)}$ then $T_{21} \rightarrow \infty$. 289

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$

$$(b_{21})^{(3)} - (b''_i)^{(3)}((G_{23})(t), t) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 290

$$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3} \right) (1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_3 t} = \frac{1}{2} \text{ it results}$$

$$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_3} \text{ By taking now } \varepsilon_3 \text{ sufficiently small one sees that } T_{21} \text{ is unbounded.}$$

The same property holds for T_{22} if $\lim_{t \rightarrow \infty} (b''_{22})^{(3)}((G_{23})(t), t) = (b'_{22})^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions:

It is now sufficient to take $\frac{(a_i)^{(4)}}{(M_{24})^{(4)}} , \frac{(b_i)^{(4)}}{(M_{24})^{(4)}} < 1$ and to choose 291

$(\widehat{P}_{24})^{(4)}$ and $(\widehat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\overline{M}_{24})^{(4)}} \left[(\widehat{P}_{24})^{(4)} + ((\widehat{P}_{24})^{(4)} + G_j^0) e^{-\left(\frac{(\widehat{P}_{24})^{(4)} + G_j^0}{G_j^0}\right)} \right] \leq (\widehat{P}_{24})^{(4)} \tag{292}$$

$$\frac{(b_i)^{(4)}}{(\overline{M}_{24})^{(4)}} \left[((\widehat{Q}_{24})^{(4)} + T_j^0) e^{-\left(\frac{(\widehat{Q}_{24})^{(4)} + T_j^0}{T_j^0}\right)} + (\widehat{Q}_{24})^{(4)} \right] \leq (\widehat{Q}_{24})^{(4)} \tag{293}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i into itself 294

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 295

$$d\left((G_{27})^{(1)}, (T_{27})^{(1)}, (G_{27})^{(2)}, (T_{27})^{(2)} \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\overline{M}_{24})^{(4)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\overline{M}_{24})^{(4)}t} \right\}$$

Indeed if we denote 296

Definition of $(\widehat{G}_{27}), (\widehat{T}_{27}) : (\widehat{G}_{27}), (\widehat{T}_{27}) = \mathcal{A}^{(4)}((G_{27}), (T_{27}))$

It results

$$\begin{aligned} |\tilde{G}_{24}^{(1)} - \tilde{G}_{24}^{(2)}| &\leq \int_0^t (a_{24})^{(4)} |G_{25}^{(1)} - G_{25}^{(2)}| e^{-(\overline{M}_{24})^{(4)}s_{(24)}} e^{(\overline{M}_{24})^{(4)}s_{(24)}} ds_{(24)} + \\ &\int_0^t \{ (a'_{24})^{(4)} |G_{24}^{(1)} - G_{24}^{(2)}| e^{-(\overline{M}_{24})^{(4)}s_{(24)}} e^{-(\overline{M}_{24})^{(4)}s_{(24)}} + \\ &(a''_{24})^{(4)} (T_{25}^{(1)}, s_{(24)}) |G_{24}^{(1)} - G_{24}^{(2)}| e^{-(\overline{M}_{24})^{(4)}s_{(24)}} e^{(\overline{M}_{24})^{(4)}s_{(24)}} + \\ &G_{24}^{(2)} | (a_{24})^{(4)} (T_{25}^{(1)}, s_{(24)}) - (a_{24})^{(4)} (T_{25}^{(2)}, s_{(24)}) | e^{-(\overline{M}_{24})^{(4)}s_{(24)}} e^{(\overline{M}_{24})^{(4)}s_{(24)}} \} ds_{(24)} \end{aligned}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\begin{aligned} |(G_{27})^{(1)} - (G_{27})^{(2)}| e^{-(\overline{M}_{24})^{(4)}t} &\leq \\ \frac{1}{(\overline{M}_{24})^{(4)}} &\left((a_{24})^{(4)} + (a'_{24})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)} \right) d\left((G_{27})^{(1)}, (T_{27})^{(1)}; (G_{27})^{(2)}, (T_{27})^{(2)} \right) \end{aligned} \tag{297}$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{24})^{(4)}$ and $(b''_{24})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\overline{M}_{24})^{(4)}t}$ and $(\widehat{Q}_{24})^{(4)} e^{(\overline{M}_{24})^{(4)}t}$ respectively of \mathbb{R}_+ . 298

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(4)}$ and $(b''_i)^{(4)}$, $i = 24, 25, 26$ depend only on T_{25} and respectively on

(G_{27}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 299

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(4)} - (a_i'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i')^{(4)}t} > 0 \quad \text{for } t > 0$$

Definition of $((\widehat{M}_{24})^{(4)})_1$, $((\widehat{M}_{24})^{(4)})_2$ and $((\widehat{M}_{24})^{(4)})_3$: 300

Remark 3: if G_{24} is bounded, the same property have also G_{25} and G_{26} . indeed if

$G_{24} < ((\widehat{M}_{24})^{(4)})_1$ it follows $\frac{dG_{25}}{dt} \leq ((\widehat{M}_{24})^{(4)})_1 - (a_{25}')^{(4)}G_{25}$ and by integrating

$$G_{25} \leq ((\widehat{M}_{24})^{(4)})_2 = G_{25}^0 + 2(a_{25}')^{(4)}((\widehat{M}_{24})^{(4)})_1 / (a_{25}')^{(4)}$$

In the same way, one can obtain

$$G_{26} \leq ((\widehat{M}_{24})^{(4)})_3 = G_{26}^0 + 2(a_{26}')^{(4)}((\widehat{M}_{24})^{(4)})_2 / (a_{26}')^{(4)}$$

If G_{25} or G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 4: If G_{24} is bounded, from below, the same property holds for G_{25} and G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 301

Remark 5: If T_{24} is bounded from below and $\lim_{t \rightarrow \infty} ((b_i'')^{(4)}((G_{27})(t), t)) = (b_{25}')^{(4)}$ then $T_{25} \rightarrow \infty$. 302

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$$(b_{25}')^{(4)} - (b_i'')^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$$

Then $\frac{dT_{25}}{dt} \geq (a_{25}')^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 303

$$T_{25} \geq \left(\frac{(a_{25}')^{(4)}(m)^{(4)}}{\varepsilon_4}\right) (1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t} \quad \text{If we take } t \text{ such that } e^{-\varepsilon_4 t} = \frac{1}{2} \text{ it results}$$

$$T_{25} \geq \left(\frac{(a_{25}')^{(4)}(m)^{(4)}}{2}\right), \quad t = \log \frac{2}{\varepsilon_4} \quad \text{By taking now } \varepsilon_4 \text{ sufficiently small one sees that } T_{25} \text{ is unbounded.}$$

The same property holds for T_{26} if $\lim_{t \rightarrow \infty} (b_{26}'')^{(4)}((G_{27})(t), t) = (b_{26}')^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions ;

Analogous inequalities hold also for G_{29} , G_{30} , T_{28} , T_{29} , T_{30}

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose 304

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\bar{M}_{28})^{(5)}} \left[(\hat{P}_{28})^{(5)} + ((\hat{P}_{28})^{(5)} + G_j^0) e^{-\left(\frac{(P_{28})^{(5)} + G_j^0}{G_j^0}\right)} \right] \leq (\hat{P}_{28})^{(5)} \quad 305$$

$$\frac{(b_i)^{(5)}}{(\bar{M}_{28})^{(5)}} \left[((\hat{Q}_{28})^{(5)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{28})^{(5)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{28})^{(5)} \right] \leq (\hat{Q}_{28})^{(5)} \quad 306$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i into itself

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 307

$$d\left((G_{31})^{(1)}, (T_{31})^{(1)}, (G_{31})^{(2)}, (T_{31})^{(2)} \right) =$$

$$\sup_i \left\{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{28})^{(5)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{28})^{(5)}t} \right\}$$

Indeed if we denote

Definition of $(\widehat{G}_{31}), (\widehat{T}_{31}) : ((\widehat{G}_{31}), (\widehat{T}_{31})) = \mathcal{A}^{(5)}((G_{31}), (T_{31}))$ 308

It results

$$\begin{aligned} |\tilde{G}_{28}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{28})^{(5)} |G_{29}^{(1)} - G_{29}^{(2)}| e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{(\bar{M}_{28})^{(5)}s_{(28)}} ds_{(28)} + \\ &\int_0^t \{ (a'_{28})^{(5)} |G_{28}^{(1)} - G_{28}^{(2)}| e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{-(\bar{M}_{28})^{(5)}s_{(28)}} + \\ &(a''_{28})^{(5)} (T_{29}^{(1)}, s_{(28)}) |G_{28}^{(1)} - G_{28}^{(2)}| e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{(\bar{M}_{28})^{(5)}s_{(28)}} + \\ &G_{28}^{(2)} | (a''_{28})^{(5)} (T_{29}^{(1)}, s_{(28)}) - (a''_{28})^{(5)} (T_{29}^{(2)}, s_{(28)}) | e^{-(\bar{M}_{28})^{(5)}s_{(28)}} e^{(\bar{M}_{28})^{(5)}s_{(28)}} \} ds_{(28)} \end{aligned}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\begin{aligned} |(G_{31})^{(1)} - (G_{31})^{(2)}| e^{-(\bar{M}_{28})^{(5)}t} &\leq \quad 309 \\ \frac{1}{(\bar{M}_{28})^{(5)}} &\left((a_{28})^{(5)} + (a'_{28})^{(5)} + (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)} (\hat{k}_{28})^{(5)} \right) d\left(((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}) \right) \end{aligned}$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis (35,35,36) the result follows

Remark 1: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as 310
not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)} e^{(\bar{M}_{28})^{(5)}t}$ and $(\hat{Q}_{28})^{(5)} e^{(\bar{M}_{28})^{(5)}t}$ respectively of \mathbb{R}_+ .

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(5)}$ and $(b''_i)^{(5)}$, $i = 28,29,30$ depend only on T_{29} and respectively on (G_{31}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 311

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(5)} - (a_i'')^{(5)}\} (T_{29}(s_{(28)}), s_{(28)}) ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i')^{(5)}t} > 0 \quad \text{for } t > 0$$

Definition of $((\widehat{M}_{28})^{(5)})_1, ((\widehat{M}_{28})^{(5)})_2$ and $((\widehat{M}_{28})^{(5)})_3$: 312

Remark 3: if G_{28} is bounded, the same property have also G_{29} and G_{30} . indeed if

$$G_{28} < ((\widehat{M}_{28})^{(5)})_1 \text{ it follows } \frac{dG_{29}}{dt} \leq ((\widehat{M}_{28})^{(5)})_1 - (a_{29}')^{(5)}G_{29} \text{ and by integrating}$$

$$G_{29} \leq ((\widehat{M}_{28})^{(5)})_2 = G_{29}^0 + 2(a_{29}')^{(5)}((\widehat{M}_{28})^{(5)})_1 / (a_{29}')^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq ((\widehat{M}_{28})^{(5)})_3 = G_{30}^0 + 2(a_{30}')^{(5)}((\widehat{M}_{28})^{(5)})_2 / (a_{30}')^{(5)}$$

If G_{29} or G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 4: If G_{28} is bounded, from below, the same property holds for G_{29} and G_{30} . The proof is 313
analogous with the preceding one. An analogous property is true if G_{29} is bounded from below.

Remark 5: If T_{28} is bounded from below and $\lim_{t \rightarrow \infty} ((b_i'')^{(5)}((G_{31})(t), t)) = (b_{29}')^{(5)}$ then $T_{29} \rightarrow \infty$. 314

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29}')^{(5)} - (b_i'')^{(5)}((G_{31})(t), t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

Then $\frac{dT_{29}}{dt} \geq (a_{29}')^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 315

$$T_{29} \geq \left(\frac{(a_{29}')^{(5)}(m)^{(5)}}{\varepsilon_5} \right) (1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_5 t} = \frac{1}{2} \text{ it results}$$

$$T_{29} \geq \left(\frac{(a_{29}')^{(5)}(m)^{(5)}}{2} \right), \quad t = \log \frac{2}{\varepsilon_5} \text{ By taking now } \varepsilon_5 \text{ sufficiently small one sees that } T_{29} \text{ is unbounded.}$$

The same property holds for T_{30} if $\lim_{t \rightarrow \infty} (b_{30}'')^{(5)}((G_{31})(t), t) = (b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions;

Analogous inequalities hold also for $G_{33}, G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(M_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(M_{32})^{(6)}} < 1$ and to choose 316

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(M_{32})^{(6)}} \left[(\widehat{P}_{32})^{(6)} + ((\widehat{P}_{32})^{(6)} + G_j^0) e^{-\left(\frac{(\widehat{P}_{32})^{(6)} + G_j^0}{G_j^0} \right)} \right] \leq (\widehat{P}_{32})^{(6)} \quad \text{317}$$

$$\frac{(b_i)^{(6)}}{(\bar{M}_{32})^{(6)}} \left[((\hat{Q}_{32})^{(6)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{32})^{(6)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{32})^{(6)} \right] \leq (\hat{Q}_{32})^{(6)} \quad 318$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i into itself

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 319

$$d\left(((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)}) \right) = \sup_i \{ \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\bar{M}_{32})^{(6)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\bar{M}_{32})^{(6)}t} \}$$

Indeed if we denote

Definition of $(\widetilde{G}_{35}), (\widetilde{T}_{35}) : ((\widetilde{G}_{35}), (\widetilde{T}_{35})) = \mathcal{A}^{(6)}((G_{35}), (T_{35}))$

It results 320

$$\begin{aligned} |\tilde{G}_{32}^{(1)} - \tilde{G}_i^{(2)}| &\leq \int_0^t (a_{32})^{(6)} |G_{33}^{(1)} - G_{33}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} ds_{(32)} + \\ &\int_0^t \{ (a'_{32})^{(6)} |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{-(\bar{M}_{32})^{(6)}s_{(32)}} + \\ &(a''_{32})^{(6)} (T_{33}^{(1)}, s_{(32)}) |G_{32}^{(1)} - G_{32}^{(2)}| e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} + \\ &G_{32}^{(2)} | (a''_{32})^{(6)} (T_{33}^{(1)}, s_{(32)}) - (a''_{32})^{(6)} (T_{33}^{(2)}, s_{(32)}) | e^{-(\bar{M}_{32})^{(6)}s_{(32)}} e^{(\bar{M}_{32})^{(6)}s_{(32)}} \} ds_{(32)} \end{aligned}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows

$$\begin{aligned} |(G_{35})^{(1)} - (G_{35})^{(2)}| e^{-(\bar{M}_{32})^{(6)}t} &\leq \\ \frac{1}{(\bar{M}_{32})^{(6)}} &((a_{32})^{(6)} + (a'_{32})^{(6)} + (\bar{A}_{32})^{(6)} + (\bar{P}_{32})^{(6)} (\hat{k}_{32})^{(6)}) d\left(((G_{35})^{(1)}, (T_{35})^{(1)}); (G_{35})^{(2)}, (T_{35})^{(2)} \right) \end{aligned} \quad 321$$

And analogous inequalities for G_i and T_i . Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\bar{P}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)}t}$ and $(\hat{Q}_{32})^{(6)} e^{(\bar{M}_{32})^{(6)}t}$ respectively of \mathbb{R}_+ . 322

If instead of proving the existence of the solution on \mathbb{R}_+ , we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i = 32, 33, 34$ depend only on T_{33} and respectively on (G_{35}) (and not on t) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 323

From governing equations it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(6)} - (a_i'')^{(6)}\} (T_{33}(s_{(32)}), s_{(32)}) ds_{(32)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i')^{(6)}t} > 0 \quad \text{for } t > 0$$

Definition of $((\widehat{M}_{32})^{(6)})_1, ((\widehat{M}_{32})^{(6)})_2$ and $((\widehat{M}_{32})^{(6)})_3$:

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Remark 3: if G_{32} is bounded, the same property have also G_{33} and G_{34} . indeed if

$$G_{32} < ((\widehat{M}_{32})^{(6)}) \text{ it follows } \frac{dG_{33}}{dt} \leq ((\widehat{M}_{32})^{(6)})_1 - (a_{33}')^{(6)}G_{33} \text{ and by integrating}$$

$$G_{33} \leq ((\widehat{M}_{32})^{(6)})_2 = G_{33}^0 + 2(a_{33}')^{(6)}((\widehat{M}_{32})^{(6)})_1 / (a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34} \leq ((\widehat{M}_{32})^{(6)})_3 = G_{34}^0 + 2(a_{34}')^{(6)}((\widehat{M}_{32})^{(6)})_2 / (a_{34}')^{(6)}$$

If G_{33} or G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 4: If G_{32} is bounded, from below, the same property holds for G_{33} and G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below.

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Remark 5: If T_{32} is bounded from below and $\lim_{t \rightarrow \infty} ((b_i'')^{(6)}((G_{35})(t), t)) = (b_{33}')^{(6)}$ then $T_{33} \rightarrow \infty$.

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Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$$(b_{33})^{(6)} - (b_i'')^{(6)}((G_{35})(t), t) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$$

Then $\frac{dT_{33}}{dt} \geq (a_{33}')^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to

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$$T_{33} \geq \left(\frac{(a_{33}')^{(6)}(m)^{(6)}}{\varepsilon_6}\right) (1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_6 t} = \frac{1}{2} \text{ it results}$$

$$T_{33} \geq \left(\frac{(a_{33}')^{(6)}(m)^{(6)}}{2}\right), \quad t = \log \frac{2}{\varepsilon_6} \text{ By taking now } \varepsilon_6 \text{ sufficiently small one sees that } T_{33} \text{ is unbounded.}$$

The same property holds for T_{34} if $\lim_{t \rightarrow \infty} (b_{34}'')^{(6)}((G_{35})(t), t(t), t) = (b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

Behavior of the solutions

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Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$:

(a) $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \leq -(a_{13}')^{(1)} + (a_{14}')^{(1)} - (a_{13}'')^{(1)}(T_{14}, t) + (a_{14}'')^{(1)}(T_{14}, t) \leq -(\sigma_1)^{(1)}$$

$$-(\tau_2)^{(1)} \leq -(b_{13}')^{(1)} + (b_{14}')^{(1)} - (b_{13}'')^{(1)}(G, t) - (b_{14}'')^{(1)}(G, t) \leq -(\tau_1)^{(1)}$$

Definition of $(v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}$:

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(b) By $(v_1)^{(1)} > 0, (v_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0, (u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(\bar{v}_1)^{(1)}, (\bar{v}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$: 330

By $(\bar{v}_1)^{(1)} > 0, (\bar{v}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0, (\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}(u^{(1)})^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}$:- 331

(c) If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$$(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}, \text{ if } (v_0)^{(1)} < (v_1)^{(1)}$$

$$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, \text{ if } (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},$$

$$\text{and } (v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}$$

$$(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}, \text{ if } (\bar{v}_1)^{(1)} < (v_0)^{(1)}$$

and analogously 332

$$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \text{ if } (u_0)^{(1)} < (u_1)^{(1)}$$

$$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, \text{ if } (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},$$

$$\text{and } (u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}$$

$$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, \text{ if } (\bar{u}_1)^{(1)} < (u_0)^{(1)} \text{ where } (u_1)^{(1)}, (\bar{u}_1)^{(1)}$$

are defined above

Then the solution satisfies the inequalities 333

$$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{13}(t) \leq G_{13}^0 e^{(S_1)^{(1)}t}$$

where $(p_i)^{(1)}$ is defined above

$$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{14}(t) \leq \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$$

$$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)} ((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})} \left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t} \right] + G_{15}^0 e^{-(S_2)^{(1)}t} \right) \leq G_{15}(t) \leq \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)} ((S_1)^{(1)} - (a'_{15})^{(1)})} \left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t} \right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}$$
 334

$$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}}$$
 335

$$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}$$
 336

$$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)} ((R_1)^{(1)} - (b'_{15})^{(1)})} \left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t} \right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \leq T_{15}(t) \leq$$
 337

$$\frac{(a_{15})^{(1)}T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)}+(r_{13})^{(1)}+(R_2)^{(1)})} \left[e^{((R_1)^{(1)}+(r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t} \right] + T_{15}^0 e^{-(R_2)^{(1)}t}$$

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:- 338

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$$

$$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$$

$$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$$

Behavior of the solutions 339

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$: 340

(d) $\sigma_1^{(2)}, \sigma_2^{(2)}, \tau_1^{(2)}, \tau_2^{(2)}$ four constants satisfying

$$-(\sigma_2)^{(2)} \leq -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \leq -(\sigma_1)^{(2)} \quad 341$$

$$-(\tau_2)^{(2)} \leq -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}((G_{19}), t) - (b''_{17})^{(2)}((G_{19}), t) \leq -(\tau_1)^{(2)} \quad 342$$

Definition of $(v_1)^{(2)}, (v_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$: 343

By $(v_1)^{(2)} > 0, (v_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0, (u_2)^{(2)} < 0$ the roots 344

(e) of the equations $(a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$ 345

and $(b_{14})^{(2)}(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and 346

Definition of $(\bar{v}_1)^{(2)}, (\bar{v}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$: 347

By $(\bar{v}_1)^{(2)} > 0, (\bar{v}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0, (\bar{u}_2)^{(2)} < 0$ the 348

roots of the equations $(a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} = 0$ 349

and $(b_{17})^{(2)}(u^{(2)})^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ 350

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:- 351

(f) If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by 352

$$(m_2)^{(2)} = (v_0)^{(2)}, (m_1)^{(2)} = (v_1)^{(2)}, \text{ if } (v_0)^{(2)} < (v_1)^{(2)} \quad 353$$

$$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (\bar{v}_1)^{(2)}, \text{ if } (v_1)^{(2)} < (v_0)^{(2)} < (\bar{v}_1)^{(2)}, \quad 354$$

and
$$(v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$$

$$(m_2)^{(2)} = (v_1)^{(2)}, (m_1)^{(2)} = (v_0)^{(2)}, \text{ if } (\bar{v}_1)^{(2)} < (v_0)^{(2)} \quad 355$$

and analogously 356

$$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}, \text{ if } (u_0)^{(2)} < (u_1)^{(2)}$$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \text{ if } (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$$

$$\text{and } \boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$$

$$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}, \text{ if } (\bar{u}_1)^{(2)} < (u_0)^{(2)} \tag{357}$$

Then the solution satisfies the inequalities 358

$$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{16}(t) \leq G_{16}^0 e^{(S_1)^{(2)}t}$$

$(p_i)^{(2)}$ is defined by equation above 359

$$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t} \tag{360}$$

$$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)} ((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})} \left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t} \right] + G_{18}^0 e^{-(S_2)^{(2)}t} \right) \leq G_{18}(t) \leq \tag{361}$$

$$\frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)} ((S_1)^{(2)} - (a_{18})^{(2)})} \left[e^{(S_1)^{(2)}t} - e^{-(a'_{18})^{(2)}t} \right] + G_{18}^0 e^{-(a'_{18})^{(2)}t}$$

$$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T_{16}^0 e^{(R_1)^{(2)} + (r_{16})^{(2)}t} \tag{362}$$

$$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{(R_1)^{(2)} + (r_{16})^{(2)}t} \tag{363}$$

$$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)} ((R_1)^{(2)} - (b'_{18})^{(2)})} \left[e^{(R_1)^{(2)}t} - e^{-(b'_{18})^{(2)}t} \right] + T_{18}^0 e^{-(b'_{18})^{(2)}t} \leq T_{18}(t) \leq \tag{364}$$

$$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)} ((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})} \left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t} \right] + T_{18}^0 e^{-(R_2)^{(2)}t}$$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 365

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a'_{16})^{(2)}$ 366

$$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$$

$$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b'_{16})^{(2)} \tag{367}$$

$$(R_2)^{(2)} = (b'_{18})^{(2)} - (r_{18})^{(2)}$$

Behavior of the solutions 368

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

(a) $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$$-(\sigma_2)^{(3)} \leq -(a'_{20})^{(3)} + (a'_{21})^{(3)} - (a''_{20})^{(3)}(T_{21}, t) + (a''_{21})^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)}$$

$$-(\tau_2)^{(3)} \leq -(b'_{20})^{(3)} + (b'_{21})^{(3)} - (b''_{20})^{(3)}(G, t) - (b''_{21})^{(3)}((G_{23}), t) \leq -(\tau_1)^{(3)}$$

Definition of $(v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 369

(b) By $(v_1)^{(3)} > 0, (v_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0, (u_2)^{(3)} < 0$ the roots of the equations

$$(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$$

$$\text{and } (b_{21})^{(3)}(u^{(3)})^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0 \text{ and}$$

By $(\bar{v}_1)^{(3)} > 0, (\bar{v}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0, (\bar{u}_2)^{(3)} < 0$ the

$$\text{roots of the equations } (a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} = 0$$

$$\text{and } (b_{21})^{(3)}(u^{(3)})^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:-

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(c) If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$$(m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}, \text{ if } (v_0)^{(3)} < (v_1)^{(3)}$$

$$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}, \text{ if } (v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)},$$

$$\text{and } \boxed{(v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$$

$$(m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}, \text{ if } (\bar{v}_1)^{(3)} < (v_0)^{(3)}$$

and analogously

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$$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}, \text{ if } (u_0)^{(3)} < (u_1)^{(3)}$$

$$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \text{ if } (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}, \text{ and } \boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$$

$$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}, \text{ if } (\bar{u}_1)^{(3)} < (u_0)^{(3)}$$

Then the solution satisfies the inequalities

$$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{20}(t) \leq G_{20}^0 e^{(S_1)^{(3)}t}$$

$(p_i)^{(3)}$ is defined by equation above

$$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{21}(t) \leq \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t} \quad 372$$

$$\left(\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)} - (p_{20})^{(3)} - (S_2)^{(3)})} \left[e^{((S_1)^{(3)} - (p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t} \right] + G_{22}^0 e^{-(S_2)^{(3)}t} \right) \leq G_{22}(t) \leq \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)} - (a'_{22})^{(3)})} \left[e^{(S_1)^{(3)}t} - e^{-(a'_{22})^{(3)}t} \right] + G_{22}^0 e^{-(a'_{22})^{(3)}t} \quad 373$$

$$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t}} \quad 374$$

$$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)} + (r_{20})^{(3)})t} \quad 375$$

$$\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)} - (b_{22})^{(3)})} \left[e^{(R_1)^{(3)}t} - e^{-(b_{22})^{(3)}t} \right] + T_{22}^0 e^{-(b_{22})^{(3)}t} \leq T_{22}(t) \leq \quad 376$$

$$\frac{(a_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)} + (r_{20})^{(3)} + (R_2)^{(3)})} \left[e^{((R_1)^{(3)} + (r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t} \right] + T_{22}^0 e^{-(R_2)^{(3)}t}$$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:-

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$$\text{Where } (S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a'_{20})^{(3)}$$

$$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$$

$$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b'_{20})^{(3)}$$

$$(R_2)^{(3)} = (b'_{22})^{(3)} - (r_{22})^{(3)}$$

Behavior of the solutions

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If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

(d) $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$$-(\sigma_2)^{(4)} \leq -(a'_{24})^{(4)} + (a'_{25})^{(4)} - (a''_{24})^{(4)}(T_{25}, t) + (a''_{25})^{(4)}(T_{25}, t) \leq -(\sigma_1)^{(4)}$$

$$-(\tau_2)^{(4)} \leq -(b'_{24})^{(4)} + (b'_{25})^{(4)} - (b''_{24})^{(4)}((G_{27}), t) - (b''_{25})^{(4)}((G_{27}), t) \leq -(\tau_1)^{(4)}$$

Definition of $(v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, v^{(4)}, u^{(4)}$:

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(e) By $(v_1)^{(4)} > 0, (v_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0, (u_2)^{(4)} < 0$ the roots of the equations

$$(a_{25})^{(4)}(v^{(4)})^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$$

$$\text{and } (b_{25})^{(4)}(u^{(4)})^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0 \text{ and}$$

Definition of $(\bar{v}_1)^{(4)}, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$:

380

By $(\bar{v}_1)^{(4)} > 0, (\bar{v}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0, (\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}(v^{(4)})^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0$

$$\text{and } (b_{25})^{(4)}(u^{(4)})^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}$:-

(f) If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$$(m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}, \text{ if } (v_0)^{(4)} < (v_1)^{(4)}$$

$$(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}, \text{ if } (v_4)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)},$$

$$\text{and } (v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}$$

$$(m_2)^{(4)} = (v_4)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}, \text{ if } (\bar{v}_4)^{(4)} < (v_0)^{(4)}$$

and analogously

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$$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}, \text{ if } (u_0)^{(4)} < (u_1)^{(4)}$$

$$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \text{ if } (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$$

$$\text{and } (u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}$$

$$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \text{ if } (\bar{u}_1)^{(4)} < (u_0)^{(4)} \text{ where } (u_1)^{(4)}, (\bar{u}_1)^{(4)}$$

are defined

Then the solution satisfies the inequalities 382

$$G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \leq G_{24}(t) \leq G_{24}^0 e^{(S_1)^{(4)}t}$$

where $(p_i)^{(4)}$ is defined by equation above

$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \leq G_{25}(t) \leq \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t} \quad 383$$

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)})} \left[e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t} \right] + G_{26}^0 e^{-(S_2)^{(4)}t} \right) \leq G_{26}(t) \leq (a_{26})^{(4)} G_{24}^0 (m_2)^{(4)} (S_1)^{(4)} - (a_{26}')^{(4)} e^{(S_1)^{(4)}t} - e^{-(a_{26}')^{(4)}t} + G_{26}^0 e^{-(a_{26}')^{(4)}t} \quad 384$$

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t}} \quad 385$$

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)} + (r_{24})^{(4)})t} \quad 386$$

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)} - (b_{26}')^{(4)})} \left[e^{(R_1)^{(4)}t} - e^{-(b_{26}')^{(4)}t} \right] + T_{26}^0 e^{-(b_{26}')^{(4)}t} \leq T_{26}(t) \leq \quad 387$$

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)})} \left[e^{((R_1)^{(4)} + (r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t} \right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 388

$$\text{Where } (S_1)^{(4)} = (a_{24})^{(4)} (m_2)^{(4)} - (a_{24}')^{(4)}$$

$$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$$

$$(R_1)^{(4)} = (b_{24})^{(4)} (\mu_2)^{(4)} - (b_{24}')^{(4)}$$

$$(R_2)^{(4)} = (b_{26}')^{(4)} - (r_{26})^{(4)}$$

Behavior of the solutions 389

If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

(g) $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$$-(\sigma_2)^{(5)} \leq -(a_{28}')^{(5)} + (a_{29}')^{(5)} - (a_{28}'')^{(5)}(T_{29}, t) + (a_{29}'')^{(5)}(T_{29}, t) \leq -(\sigma_1)^{(5)}$$

$$-(\tau_2)^{(5)} \leq -(b_{28}')^{(5)} + (b_{29}')^{(5)} - (b_{28}'')^{(5)}((G_{31}), t) - (b_{29}'')^{(5)}((G_{31}), t) \leq -(\tau_1)^{(5)}$$

Definition of $(v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}$: 390

(h) By $(v_1)^{(5)} > 0, (v_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0, (u_2)^{(5)} < 0$ the roots of the equations

$$(a_{29})^{(5)} (v^{(5)})^2 + (\sigma_1)^{(5)} v^{(5)} - (a_{28})^{(5)} = 0$$

$$\text{and } (b_{29})^{(5)} (u^{(5)})^2 + (\tau_1)^{(5)} u^{(5)} - (b_{28})^{(5)} = 0 \text{ and}$$

Definition of $(\bar{v}_1)^{(5)}, (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$:

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By $(\bar{v}_1)^{(5)} > 0, (\bar{v}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0, (\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} = 0$

and $(b_{29})^{(5)}(u^{(5)})^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (v_0)^{(5)}$:-

(i) If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$$(m_2)^{(5)} = (v_0)^{(5)}, (m_1)^{(5)} = (v_1)^{(5)}, \text{ if } (v_0)^{(5)} < (v_1)^{(5)}$$

$$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (\bar{v}_1)^{(5)}, \text{ if } (v_1)^{(5)} < (v_0)^{(5)} < (\bar{v}_1)^{(5)},$$

$$\text{and } (v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}$$

$$(m_2)^{(5)} = (v_1)^{(5)}, (m_1)^{(5)} = (v_0)^{(5)}, \text{ if } (\bar{v}_1)^{(5)} < (v_0)^{(5)}$$

and analogously

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$$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \text{ if } (u_0)^{(5)} < (u_1)^{(5)}$$

$$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \text{ if } (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$$

$$\text{and } (u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}$$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \text{ if } (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$ are defined respectively

Then the solution satisfies the inequalities

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$$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)}t}$$

where $(p_i)^{(5)}$ is defined by equation above

$$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t}$$

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$$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)} ((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})} \left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t} \right] + G_{30}^0 e^{-(S_2)^{(5)}t} \right) \leq G_{30}(t) \leq (a_{30})^{(5)} G_{28}^0 (m_2)^{(5)} (S_1)^{(5)} - (a_{30}')^{(5)} e^{(S_1)^{(5)}t} - e^{-(a_{30}')^{(5)}t} + G_{30}^0 e^{-(a_{30}')^{(5)}t}$$

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$$T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$$

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$$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$$

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$$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)} ((R_1)^{(5)} - (b_{30})^{(5)})} \left[e^{(R_1)^{(5)}t} - e^{-(b_{30}')^{(5)}t} \right] + T_{30}^0 e^{-(b_{30}')^{(5)}t} \leq T_{30}(t) \leq$$

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$$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)} ((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})} \left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t} \right] + T_{30}^0 e^{-(R_2)^{(5)}t}$$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:-

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Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a'_{28})^{(5)}$

$$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$$

$$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b'_{28})^{(5)}$$

$$(R_2)^{(5)} = (b'_{30})^{(5)} - (r_{30})^{(5)}$$

Behavior of the solutions

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If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

(j) $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33}, t) + (a''_{33})^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$$

$$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}((G_{35}), t) - (b''_{33})^{(6)}((G_{35}), t) \leq -(\tau_1)^{(6)}$$

Definition of $(v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}$:

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(k) By $(v_1)^{(6)} > 0, (v_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0, (u_2)^{(6)} < 0$ the roots of the equations

$$(a_{33})^{(6)}(v^{(6)})^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$$

$$\text{and } (b_{33})^{(6)}(u^{(6)})^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0 \text{ and}$$

Definition of $(\bar{v}_1)^{(6)}, (\bar{v}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$:

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By $(\bar{v}_1)^{(6)} > 0, (\bar{v}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0, (\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}(v^{(6)})^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} = 0$

$$\text{and } (b_{33})^{(6)}(u^{(6)})^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (v_0)^{(6)}$:-

(l) If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$$(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}, \text{ if } (v_0)^{(6)} < (v_1)^{(6)}$$

$$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_6)^{(6)}, \text{ if } (v_1)^{(6)} < (v_0)^{(6)} < (\bar{v}_1)^{(6)},$$

$$\text{and } (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}$$

$$(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}, \text{ if } (\bar{v}_1)^{(6)} < (v_0)^{(6)}$$

and analogously

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$$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \text{ if } (u_0)^{(6)} < (u_1)^{(6)}$$

$$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}, \text{ if } (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$$

$$\text{and } (u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}$$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \text{ if } (\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$ are defined respectively

Then the solution satisfies the inequalities

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$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation above

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t} \quad 405$$

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})} \left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t} \right] + G_{34}^0 e^{-(S_2)^{(6)}t} \right) \leq G_{34}(t) \leq (a_{34})^{(6)} G_{32}^0 (m_2)^{(6)} (S_1)^{(6)} - (a_{34}')^{(6)} e^{(S_1)^{(6)}t} - e^{-(a_{34}')^{(6)}t} + G_{34}^0 e^{-(a_{34}')^{(6)}t} \quad 406$$

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}} \quad 407$$

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t} \quad 408$$

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b_{34}')^{(6)})} \left[e^{(R_1)^{(6)}t} - e^{-(b_{34}')^{(6)}t} \right] + T_{34}^0 e^{-(b_{34}')^{(6)}t} \leq T_{34}(t) \leq \quad 409$$

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})} \left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t} \right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 410

$$\text{Where } (S_1)^{(6)} = (a_{32})^{(6)} (m_2)^{(6)} - (a_{32}')^{(6)}$$

$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)} (\mu_2)^{(6)} - (b_{32}')^{(6)}$$

$$(R_2)^{(6)} = (b_{34}')^{(6)} - (r_{34})^{(6)}$$

Proof : From Governing equations we obtain 411

$$\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left((a_{13}')^{(1)} - (a_{14}')^{(1)} + (a_{13}'')^{(1)}(T_{14}, t) \right) - (a_{14}'')^{(1)}(T_{14}, t)v^{(1)} - (a_{14})^{(1)}v^{(1)}$$

Definition of $v^{(1)}$:- $\boxed{v^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$$- \left((a_{14})^{(1)} (v^{(1)})^2 + (\sigma_2)^{(1)} v^{(1)} - (a_{13})^{(1)} \right) \leq \frac{dv^{(1)}}{dt} \leq - \left((a_{14})^{(1)} (v^{(1)})^2 + (\sigma_1)^{(1)} v^{(1)} - (a_{13})^{(1)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(1)}, (v_0)^{(1)}$:-

(a) For $0 < \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)}$

$$v^{(1)}(t) \geq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{[-(a_{14})^{(1)}(v_1)^{(1)} - (v_0)^{(1)}]t}}{1 + (C)^{(1)} e^{[-(a_{14})^{(1)}(v_1)^{(1)} - (v_0)^{(1)}]t}}, \quad \boxed{(C)^{(1)} = \frac{(v_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (v_2)^{(1)}}$$

it follows $(v_0)^{(1)} \leq v^{(1)}(t) \leq (v_1)^{(1)}$

In the same manner , we get

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$$v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}}, \quad \boxed{(\bar{C})^{(1)} = \frac{(\bar{v}_1)^{(1)} - (v_0)^{(1)}}{(v_0)^{(1)} - (\bar{v}_2)^{(1)}}$$

From which we deduce $(v_0)^{(1)} \leq v^{(1)}(t) \leq (\bar{v}_1)^{(1)}$

(b) If $0 < (v_1)^{(1)} < (v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{v}_1)^{(1)}$ we find like in the previous case,

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$$(v_1)^{(1)} \leq \frac{(v_1)^{(1)} + (C)^{(1)}(v_2)^{(1)} e^{[-(a_{14})^{(1)}(v_1)^{(1)} - (v_2)^{(1)}]t}}{1 + (C)^{(1)} e^{[-(a_{14})^{(1)}(v_1)^{(1)} - (v_2)^{(1)}]t}} \leq v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}} \leq (\bar{v}_1)^{(1)}$$

(c) If $0 < (v_1)^{(1)} \leq (\bar{v}_1)^{(1)} \leq \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain

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$$(v_1)^{(1)} \leq v^{(1)}(t) \leq \frac{(\bar{v}_1)^{(1)} + (\bar{C})^{(1)}(\bar{v}_2)^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}(\bar{v}_1)^{(1)} - (\bar{v}_2)^{(1)}]t}} \leq (v_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$$

Now, using this result and replacing it in concatenated equations of global system we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}$, then $(\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(v_1)^{(1)} = (\bar{v}_1)^{(1)}$ if in addition $(v_0)^{(1)} = (v_1)^{(1)}$ then $v^{(1)}(t) = (v_0)^{(1)}$ and as a consequence $G_{13}(t) = (v_0)^{(1)}G_{14}(t)$ this also defines $(v_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}$, then $(\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then

$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$ if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(v_1)^{(1)}$ and $(\bar{v}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

Proof : From the concatenated set of global governing equations we obtain

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$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17}, t) \right) - (a''_{17})^{(2)}(T_{17}, t)v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

Definition of $v^{(2)}$:- $v^{(2)} = \frac{G_{16}}{G_{17}}$ 416

It follows 417

$$- \left((a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} \right) \leq \frac{dv^{(2)}}{dt} \leq - \left((a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} \right)$$

From which one obtains 418

Definition of $(\bar{v}_1)^{(2)}, (v_0)^{(2)}$:-

(d) For $0 < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (v_1)^{(2)} < (\bar{v}_1)^{(2)}$

$$v^{(2)}(t) \geq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)} e^{[-(a_{17})^{(2)}(v_1)^{(2)} - (v_0)^{(2)}]t}}{1 + (C)^{(2)} e^{[-(a_{17})^{(2)}(v_1)^{(2)} - (v_0)^{(2)}]t}}, \quad \left(C \right)^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}$$

it follows $(v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}$

In the same manner , we get 419

$$v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)}(\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}]t}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}(\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}]t}}, \quad \left(\bar{C} \right)^{(2)} = \frac{(\bar{v}_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (\bar{v}_2)^{(2)}}$$

From which we deduce $(v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}$ 420

(e) If $0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{v}_1)^{(2)}$ we find like in the previous case, 421

$$(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)} e^{[-(a_{17})^{(2)}(v_1)^{(2)} - (v_2)^{(2)}]t}}{1 + (C)^{(2)} e^{[-(a_{17})^{(2)}(v_1)^{(2)} - (v_2)^{(2)}]t}} \leq v^{(2)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)}(\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}]t}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}(\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}]t}} \leq (\bar{v}_1)^{(2)}$$

(f) If $0 < (v_1)^{(2)} \leq (\bar{v}_1)^{(2)} \leq (v_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain 422

$$(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (\bar{C})^{(2)}(\bar{v}_2)^{(2)} e^{[-(a_{17})^{(2)}(\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}]t}}{1 + (\bar{C})^{(2)} e^{[-(a_{17})^{(2)}(\bar{v}_1)^{(2)} - (\bar{v}_2)^{(2)}]t}} \leq (v_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(2)}(t)$:- 423

$$(m_2)^{(2)} \leq v^{(2)}(t) \leq (m_1)^{(2)}, \quad \left(v^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)} \right)$$

In a completely analogous way, we obtain 424

Definition of $u^{(2)}(t)$:-

$$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}, \quad \boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

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If $(a''_{16})^{(2)} = (a''_{17})^{(2)}$, then $(\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(v_1)^{(2)} = (\bar{v}_1)^{(2)}$ if in addition $(v_0)^{(2)} = (v_1)^{(2)}$ then $v^{(2)}(t) = (v_0)^{(2)}$ and as a consequence $G_{16}(t) = (v_0)^{(2)}G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}$, then $(\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$ if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)}T_{17}(t)$ This is an important consequence of the relation between $(v_1)^{(2)}$ and $(\bar{v}_1)^{(2)}$

Proof : From Global equations we obtain

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$$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t) \right) - (a''_{21})^{(3)}(T_{21}, t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$$

Definition of $v^{(3)}$:-

$$\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$$

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It follows

$$- \left((a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} \right) \leq \frac{dv^{(3)}}{dt} \leq - \left((a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)} \right)$$

From which one obtains

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(a) For $0 < (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$$v^{(3)}(t) \geq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)} e^{[-(a_{21})^{(3)}(v_1)^{(3)} - (v_0)^{(3)}]t}}{1 + (C)^{(3)} e^{[-(a_{21})^{(3)}(v_1)^{(3)} - (v_0)^{(3)}]t}}, \quad \boxed{(C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}$$

it follows $(v_0)^{(3)} \leq v^{(3)}(t) \leq (v_1)^{(3)}$

In the same manner , we get

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$$v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)}(\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}]t}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)}(\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}]t}}, \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (\bar{v}_2)^{(3)}}$$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \leq v^{(3)}(t) \leq (\bar{v}_1)^{(3)}$

(b) If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{v}_1)^{(3)}$ we find like in the previous case,

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$$(v_1)^{(3)} \leq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)} e^{[-(a_{21})^{(3)}(v_1)^{(3)} - (v_2)^{(3)}]t}}{1 + (C)^{(3)} e^{[-(a_{21})^{(3)}(v_1)^{(3)} - (v_2)^{(3)}]t}} \leq v^{(3)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(3)} + (\bar{C})^{(3)}(\bar{v}_2)^{(3)} e^{[-(a_{21})^{(3)}(\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}]t}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)}(\bar{v}_1)^{(3)} - (\bar{v}_2)^{(3)}]t}} \leq (\bar{v}_1)^{(3)}$$

(c) If $0 < (v_1)^{(3)} \leq (\bar{v}_1)^{(3)} \leq (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain

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$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)} (\bar{\nu}_2)^{(3)} e^{[-(a_{21})^{(3)} (\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}] t}}{1 + (\bar{C})^{(3)} e^{[-(a_{21})^{(3)} (\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}] t}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq \nu^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}, \quad \boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$$

Now, using this result and replacing it in Global Equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}$, then $(\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}$, then $(\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$ if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

Proof : From Global equations we obtain

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$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)} (T_{25}, t) \right) - (a''_{25})^{(4)} (T_{25}, t) \nu^{(4)} - (a_{25})^{(4)} \nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$- \left((a_{25})^{(4)} (\nu^{(4)})^2 + (\sigma_2)^{(4)} \nu^{(4)} - (a_{24})^{(4)} \right) \leq \frac{d\nu^{(4)}}{dt} \leq - \left((a_{25})^{(4)} (\nu^{(4)})^2 + (\sigma_4)^{(4)} \nu^{(4)} - (a_{24})^{(4)} \right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

(d) For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (\bar{C})^{(4)} (\nu_2)^{(4)} e^{[-(a_{25})^{(4)} (\nu_1)^{(4)} - (\nu_0)^{(4)}] t}}{4 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)} (\nu_1)^{(4)} - (\nu_0)^{(4)}] t}}, \quad \boxed{(\bar{C})^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)} (\bar{\nu}_2)^{(4)} e^{[-(a_{25})^{(4)} (\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}] t}}{4 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)} (\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}] t}}, \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}$$

From which we deduce $(v_0)^{(4)} \leq v^{(4)}(t) \leq (\bar{v}_1)^{(4)}$

(e) If $0 < (v_1)^{(4)} < (v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{v}_1)^{(4)}$ we find like in the previous case, 433

$$(v_1)^{(4)} \leq \frac{(v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)} e^{[-(a_{25})^{(4)}(v_1)^{(4)} - (v_2)^{(4)}]t}}{1 + (C)^{(4)} e^{[-(a_{25})^{(4)}(v_1)^{(4)} - (v_2)^{(4)}]t}} \leq v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)}(\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}]t}}{1 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)}(\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}]t}} \leq (\bar{v}_1)^{(4)}$$

(f) If $0 < (v_1)^{(4)} \leq (\bar{v}_1)^{(4)} \leq \boxed{(v_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain 434

$$(v_1)^{(4)} \leq v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(\bar{v}_2)^{(4)} e^{[-(a_{25})^{(4)}(\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}]t}}{1 + (\bar{C})^{(4)} e^{[-(a_{25})^{(4)}(\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}]t}} \leq (v_0)^{(4)}$$

And so with the notation of the first part of condition (c), we have

Definition of $v^{(4)}(t)$:-

$$(m_2)^{(4)} \leq v^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{v^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$$

Now, using this result and replacing it in Global equations we get easily the result stated in the theorem.

Particular case :

If $(a_{24}''^{(4)}) = (a_{25}''^{(4)})$, then $(\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(v_1)^{(4)} = (\bar{v}_1)^{(4)}$ if in addition $(v_0)^{(4)} = (v_1)^{(4)}$ then $v^{(4)}(t) = (v_0)^{(4)}$ and as a consequence $G_{24}(t) = (v_0)^{(4)}G_{25}(t)$ **this also defines** $(v_0)^{(4)}$ **for the special case** .

Analogously if $(b_{24}''^{(4)}) = (b_{25}''^{(4)})$, then $(\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$ if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(v_1)^{(4)}$ and $(\bar{v}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

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Proof : From concatenated set of equations we obtain

$$\frac{dv^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t) \right) - (a''_{29})^{(5)}(T_{29}, t)v^{(5)} - (a_{29})^{(5)}v^{(5)}$$

Definition of $v^{(5)}$:- $\boxed{v^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$- \left((a_{29})^{(5)}(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} \right) \leq \frac{dv^{(5)}}{dt} \leq - \left((a_{29})^{(5)}(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(5)}, (v_0)^{(5)}$:-

(g) For $0 < \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (v_1)^{(5)} < (\bar{v}_1)^{(5)}$

$$v^{(5)}(t) \geq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_0)^{(5)})t]}}{5 + (C)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_0)^{(5)})t]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(v_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (v_2)^{(5)}}$$

it follows $(v_0)^{(5)} \leq v^{(5)}(t) \leq (v_1)^{(5)}$

In the same manner , we get

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$$v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}}{5 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{v}_1)^{(5)} - (v_0)^{(5)}}{(v_0)^{(5)} - (\bar{v}_2)^{(5)}}$$

From which we deduce $(v_0)^{(5)} \leq v^{(5)}(t) \leq (\bar{v}_5)^{(5)}$

(h) If $0 < (v_1)^{(5)} < (v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{v}_1)^{(5)}$ we find like in the previous case,

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$$(v_1)^{(5)} \leq \frac{(v_1)^{(5)} + (C)^{(5)}(v_2)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_2)^{(5)})t]}}{1 + (C)^{(5)} e^{[-(a_{29})^{(5)}((v_1)^{(5)} - (v_2)^{(5)})t]}} \leq v^{(5)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}}{1 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}} \leq (\bar{v}_1)^{(5)}$$

(i) If $0 < (v_1)^{(5)} \leq (\bar{v}_1)^{(5)} \leq \boxed{(v_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain

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$$(v_1)^{(5)} \leq v^{(5)}(t) \leq \frac{(\bar{v}_1)^{(5)} + (\bar{C})^{(5)}(\bar{v}_2)^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}}{1 + (\bar{C})^{(5)} e^{[-(a_{29})^{(5)}((\bar{v}_1)^{(5)} - (\bar{v}_2)^{(5)})t]}} \leq (v_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(5)}(t)$:-

$$(m_2)^{(5)} \leq v^{(5)}(t) \leq (m_1)^{(5)} \quad , \quad \boxed{v^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)} \quad , \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}$, then $(\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(v_1)^{(5)} = (\bar{v}_1)^{(5)}$ if in addition $(v_0)^{(5)} = (v_5)^{(5)}$ then $v^{(5)}(t) = (v_0)^{(5)}$ and as a consequence $G_{28}(t) = (v_0)^{(5)}G_{29}(t)$ **this also defines** $(v_0)^{(5)}$ **for the special case .**

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}$, then $(\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$ if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(v_1)^{(5)}$ and $(\bar{v}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

Proof : From Global equations we obtain

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$$\frac{dv^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t) \right) - (a''_{33})^{(6)}(T_{33}, t)v^{(6)} - (a_{33})^{(6)}v^{(6)}$$

Definition of $v^{(6)}$:-
$$v^{(6)} = \frac{G_{32}}{G_{33}}$$

It follows

$$- \left((a_{33})^{(6)}(v^{(6)})^2 + (\sigma_2)^{(6)}v^{(6)} - (a_{32})^{(6)} \right) \leq \frac{dv^{(6)}}{dt} \leq - \left((a_{33})^{(6)}(v^{(6)})^2 + (\sigma_1)^{(6)}v^{(6)} - (a_{32})^{(6)} \right)$$

From which one obtains

Definition of $(\bar{v}_1)^{(6)}, (v_0)^{(6)}$:-

(j) For $0 < \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (v_1)^{(6)} < (\bar{v}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)} e^{[-(a_{33})^{(6)}(v_1)^{(6)} - (v_0)^{(6)}]t}}{1 + (C)^{(6)} e^{[-(a_{33})^{(6)}(v_1)^{(6)} - (v_0)^{(6)}]t}}, \quad \boxed{(C)^{(6)} = \frac{(v_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (v_2)^{(6)}}$$

it follows $(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}$

In the same manner , we get

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$$v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}}, \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{v}_1)^{(6)} - (v_0)^{(6)}}{(v_0)^{(6)} - (\bar{v}_2)^{(6)}}$$

From which we deduce $(v_0)^{(6)} \leq v^{(6)}(t) \leq (\bar{v}_1)^{(6)}$

(k) If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{v}_1)^{(6)}$ we find like in the previous case,

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$$(v_1)^{(6)} \leq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)} e^{[-(a_{33})^{(6)}(v_1)^{(6)} - (v_2)^{(6)}]t}}{1 + (C)^{(6)} e^{[-(a_{33})^{(6)}(v_1)^{(6)} - (v_2)^{(6)}]t}} \leq v^{(6)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}} \leq (\bar{v}_1)^{(6)}$$

(l) If $0 < (v_1)^{(6)} \leq (\bar{v}_1)^{(6)} \leq \boxed{(v_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain

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$$(v_1)^{(6)} \leq v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (\bar{C})^{(6)}(\bar{v}_2)^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}}{1 + (\bar{C})^{(6)} e^{[-(a_{33})^{(6)}(\bar{v}_1)^{(6)} - (\bar{v}_2)^{(6)}]t}} \leq (v_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(6)}(t)$:-

$$(m_2)^{(6)} \leq v^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{v^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in global equations we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}$, then $(\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$ if in addition $(v_0)^{(6)} = (v_1)^{(6)}$ then $v^{(6)}(t) = (v_0)^{(6)}$ and as a consequence $G_{32}(t) = (v_0)^{(6)}G_{33}(t)$ **this also defines $(v_0)^{(6)}$ for the special case .**

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}$, then $(\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(v_1)^{(6)}$ and $(\bar{v}_1)^{(6)}$, **and definition of $(u_0)^{(6)}$.**

We can prove the following

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Theorem 3: If $(a_i'')^{(1)}$ and $(b_i'')^{(1)}$ are independent on t , and the conditions

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0 ,$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

with $(p_{13})^{(1)}, (r_{14})^{(1)}$ as defined are satisfied , then the system

If $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ are independent on t , and the conditions

$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$$

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$$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0 ,$$

$$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$$

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with $(p_{16})^{(2)}, (r_{17})^{(2)}$ as defined are satisfied , then the system

: If $(a_i'')^{(3)}$ and $(b_i'')^{(3)}$ are independent on t , and the conditions

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$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$$

$$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$$

$$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0 ,$$

$$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$$

with $(p_{20})^{(3)}, (r_{21})^{(3)}$ as defined are satisfied , then the system

We can prove the following

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If $(a_i'')^{(4)}$ and $(b_i'')^{(4)}$ are independent on t , and the conditions

$$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$$

$$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$$

$$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0 ,$$

$$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$$

with $(p_{24})^{(4)}, (r_{25})^{(4)}$ as defined are satisfied , then the system

If $(a_i'')^{(5)}$ and $(b_i'')^{(5)}$ are independent on t , and the conditions

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$$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$$

$$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$$

$$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0 ,$$

$$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$$

with $(p_{28})^{(5)}, (r_{29})^{(5)}$ as defined are satisfied , then the system

If $(a_i'')^{(6)}$ and $(b_i'')^{(6)}$ are independent on t , and the conditions

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$$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$$

$$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$$

$$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0 ,$$

$$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$$

with $(p_{32})^{(6)}, (r_{33})^{(6)}$ as defined are satisfied , then the system Boolean satisfiability problem and N puzzle

$$(a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})]G_{13} = 0$$

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$$(a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})]G_{14} = 0$$

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$$(a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})]G_{15} = 0$$

452

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)]T_{13} = 0$$

453

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)]T_{14} = 0$$

454

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)]T_{15} = 0$$

455

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{16})^{(2)}G_{17} - [(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})]G_{16} = 0$$

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$$(a_{17})^{(2)}G_{16} - [(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})]G_{17} = 0$$

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$$(a_{18})^{(2)}G_{17} - [(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})]G_{18} = 0$$

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$$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})]T_{16} = 0 \quad 459$$

$$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})]T_{17} = 0 \quad 460$$

$$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})]T_{18} = 0 \quad 461$$

has a unique positive solution , which is an equilibrium solution for the system 462

$$(a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})]G_{20} = 0 \quad 463$$

$$(a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})]G_{21} = 0 \quad 464$$

$$(a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})]G_{22} = 0 \quad 465$$

$$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})]T_{20} = 0 \quad 466$$

$$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})]T_{21} = 0 \quad 467$$

$$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})]T_{22} = 0 \quad 468$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})]G_{24} = 0 \quad 469$$

$$(a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})]G_{25} = 0 \quad 470$$

$$(a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})]G_{26} = 0 \quad 471$$

$$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27})]T_{24} = 0 \quad 472$$

$$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27})]T_{25} = 0 \quad 473$$

$$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27})]T_{26} = 0 \quad 474$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{28})^{(5)}G_{29} - [(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})]G_{28} = 0 \quad 475$$

$$(a_{29})^{(5)}G_{28} - [(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})]G_{29} = 0 \quad 476$$

$$(a_{30})^{(5)}G_{29} - [(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})]G_{30} = 0 \quad 477$$

$$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0 \quad 478$$

$$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0 \quad 479$$

$$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})]T_{30} = 0 \quad 480$$

has a unique positive solution , which is an equilibrium solution for the system

$$(a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})]G_{32} = 0 \quad 481$$

$$(a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})]G_{33} = 0 \quad 482$$

$$(a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})]G_{34} = 0 \quad 483$$

$$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0 \quad 484$$

$$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} = 0 \quad 485$$

$$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0 \quad 486$$

has a unique positive solution , which is an equilibrium solution for the system

Proof: 487

(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a'_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0$$

Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if 488

$$F(T_{19}) = (a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a''_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a''_{16})^{(2)}(T_{17}) + (a'_{16})^{(2)}(T_{17})(a''_{17})^{(2)}(T_{17}) = 0$$

(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if 489

$$F(T_{23}) = (a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a''_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a''_{20})^{(3)}(T_{21}) + (a'_{20})^{(3)}(T_{21})(a''_{21})^{(3)}(T_{21}) = 0$$

(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if 490

$$F(T_{27}) = (a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a'_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$$

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$$F(T_{31}) = (a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a'_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$$

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if 491

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a'_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

$$(a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$

Definition and uniqueness of T_{14}^* :-

492

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T_{14}^* for which $f(T_{14}^*) = 0$. With this value, we obtain from the three first equations

$$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^*)]}$$

Definition and uniqueness of T_{17}^* :-

493

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T_{17}^* for which $f(T_{17}^*) = 0$. With this value, we obtain from the three first equations

$$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^*)]}$$

494

Definition and uniqueness of T_{21}^* :-

495

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value, we obtain from the three first equations

$$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :-

496

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value, we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :-

497

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value, we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :-

498

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value, we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^*)]}$$

(e) By the same argument, the equations of global system admit solutions G_{13}, G_{14} if

499

$$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

(f) By the same argument, the equations 92,93 admit solutions G_{16}, G_{17} if 500

$$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$$[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$ 501

(g) By the same argument, the equations of the global system admit solutions G_{20}, G_{21} if 502

$$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$$

$$[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

(h) By the same argument, the equations of the global system admit solutions G_{24}, G_{25} if 503

$$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$$

$$[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{25}^* such that $\varphi((G_{27})^*) = 0$

(i) By the same argument, the global equations admit solutions G_{28}, G_{29} if 504

$$\varphi(G_{31}) = (b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$$

$$[(b'_{28})^{(5)}(b''_{29})^{(5)}(G_{31}) + (b'_{29})^{(5)}(b''_{28})^{(5)}(G_{31})] + (b''_{28})^{(5)}(G_{31})(b''_{29})^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{29}^* such that $\varphi((G_{31})^*) = 0$

(j) By the same argument, the global equations admit solutions G_{32}, G_{33} if 505

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$$

$$[(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a

unique G_{33}^* such that $\varphi(G^*) = 0$

Finally we obtain the unique solution of the global system: 506

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b'_{13})^{(1)}-(b''_{13})^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b'_{15})^{(1)}-(b''_{15})^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 507

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 508

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^*)]} \quad 509$$

$$T_{16}^* = \frac{(b_{16})^{(2)}T_{17}^*}{[(b'_{16})^{(2)}-(b''_{16})^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)}T_{17}^*}{[(b'_{18})^{(2)}-(b''_{18})^{(2)}((G_{19})^*)]} \quad 510$$

Obviously, these values represent an equilibrium solution 511

Finally we obtain the unique solution 512

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)}T_{21}^*}{[(b'_{20})^{(3)}-(b''_{20})^{(3)}(G_{23}^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)}T_{21}^*}{[(b'_{22})^{(3)}-(b''_{22})^{(3)}(G_{23}^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 513

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)}G_{25}^*}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)}G_{25}^*}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)}T_{25}^*}{[(b'_{24})^{(4)}-(b''_{24})^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)}T_{25}^*}{[(b'_{26})^{(4)}-(b''_{26})^{(4)}((G_{27})^*)]} \quad 514$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 515

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)}G_{29}^*}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)}G_{29}^*}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)}T_{29}^*}{[(b_{28}')^{(5)} - (b_{28}'')^{(5)}((G_{31})^*)]} \quad , \quad T_{30}^* = \frac{(b_{30})^{(5)}T_{29}^*}{[(b_{30}')^{(5)} - (b_{30}'')^{(5)}((G_{31})^*)]} \quad 516$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 517

G_{33}^* given by $\varphi((G_{35})^*) = 0$, T_{33}^* given by $f(T_{33}^*) = 0$ and

$$G_{32}^* = \frac{(a_{32})^{(6)}G_{33}^*}{[(a_{32}')^{(6)} + (a_{32}'')^{(6)}(T_{33}^*)]} \quad , \quad G_{34}^* = \frac{(a_{34})^{(6)}G_{33}^*}{[(a_{34}')^{(6)} + (a_{34}'')^{(6)}(T_{33}^*)]} \quad 518$$

$$T_{32}^* = \frac{(b_{32})^{(6)}T_{33}^*}{[(b_{32}')^{(6)} - (b_{32}'')^{(6)}((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)}T_{33}^*}{[(b_{34}')^{(6)} - (b_{34}'')^{(6)}((G_{35})^*)]} \quad 518$$

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS 519

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ and $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of G_i, T_i :-

$$G_i = G_i^* + \mathbb{G}_i \quad , \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \quad \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations of global system neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -((a'_{13})^{(1)} + (p_{13})^{(1)})\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \quad 520$$

$$\frac{d\mathbb{G}_{14}}{dt} = -((a'_{14})^{(1)} + (p_{14})^{(1)})\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \quad 521$$

$$\frac{d\mathbb{G}_{15}}{dt} = -((a'_{15})^{(1)} + (p_{15})^{(1)})\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \quad 522$$

$$\frac{d\mathbb{T}_{13}}{dt} = -((b'_{13})^{(1)} - (r_{13})^{(1)})\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15} (s_{(13)(j)})T_{13}^*\mathbb{G}_j \quad 523$$

$$\frac{d\mathbb{T}_{14}}{dt} = -((b'_{14})^{(1)} - (r_{14})^{(1)})\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15} (s_{(14)(j)})T_{14}^*\mathbb{G}_j \quad 524$$

$$\frac{d\mathbb{T}_{15}}{dt} = -((b'_{15})^{(1)} - (r_{15})^{(1)})\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15} (s_{(15)(j)})T_{15}^*\mathbb{G}_j \quad 525$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

_Denote 526

Definition of G_i, T_i :-

$$G_i = G_i^* + \mathbb{G}_i \quad , \quad T_i = T_i^* + \mathbb{T}_i \quad 527$$

$$\frac{\partial(a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)}, \quad \frac{\partial(b_i'')^{(2)}}{\partial G_j}(G_{19}^*) = s_{ij} \tag{528}$$

taking into account equations (global) and neglecting the terms of power 2, we obtain

$$\frac{dG_{16}}{dt} = -((a'_{16})^{(2)} + (p_{16})^{(2)})G_{16} + (a_{16})^{(2)}G_{17} - (q_{16})^{(2)}G_{16}^*T_{17} \tag{529}$$

$$\frac{dG_{17}}{dt} = -((a'_{17})^{(2)} + (p_{17})^{(2)})G_{17} + (a_{17})^{(2)}G_{16} - (q_{17})^{(2)}G_{17}^*T_{17} \tag{530}$$

$$\frac{dG_{18}}{dt} = -((a'_{18})^{(2)} + (p_{18})^{(2)})G_{18} + (a_{18})^{(2)}G_{17} - (q_{18})^{(2)}G_{18}^*T_{17} \tag{531}$$

$$\frac{dT_{16}}{dt} = -((b'_{16})^{(2)} - (r_{16})^{(2)})T_{16} + (b_{16})^{(2)}T_{17} + \sum_{j=16}^{18} (s_{(16)j})T_{16}^*G_j \tag{532}$$

$$\frac{dT_{17}}{dt} = -((b'_{17})^{(2)} - (r_{17})^{(2)})T_{17} + (b_{17})^{(2)}T_{16} + \sum_{j=16}^{18} (s_{(17)j})T_{17}^*G_j \tag{533}$$

$$\frac{dT_{18}}{dt} = -((b'_{18})^{(2)} - (r_{18})^{(2)})T_{18} + (b_{18})^{(2)}T_{17} + \sum_{j=16}^{18} (s_{(18)j})T_{18}^*G_j \tag{534}$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ and $(b_i'')^{(3)}$ belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable. 535

Denote

Definition of G_i, T_i :-

$$G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i$$

$$\frac{\partial(a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)}, \quad \frac{\partial(b_i'')^{(3)}}{\partial G_j}(G_{23}^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 536

$$\frac{dG_{20}}{dt} = -((a'_{20})^{(3)} + (p_{20})^{(3)})G_{20} + (a_{20})^{(3)}G_{21} - (q_{20})^{(3)}G_{20}^*T_{21} \tag{537}$$

$$\frac{dG_{21}}{dt} = -((a'_{21})^{(3)} + (p_{21})^{(3)})G_{21} + (a_{21})^{(3)}G_{20} - (q_{21})^{(3)}G_{21}^*T_{21} \tag{538}$$

$$\frac{dG_{22}}{dt} = -((a'_{22})^{(3)} + (p_{22})^{(3)})G_{22} + (a_{22})^{(3)}G_{21} - (q_{22})^{(3)}G_{22}^*T_{21} \tag{539}$$

$$\frac{dT_{20}}{dt} = -((b'_{20})^{(3)} - (r_{20})^{(3)})T_{20} + (b_{20})^{(3)}T_{21} + \sum_{j=20}^{22} (s_{(20)j})T_{20}^*G_j \tag{540}$$

$$\frac{dT_{21}}{dt} = -((b'_{21})^{(3)} - (r_{21})^{(3)})T_{21} + (b_{21})^{(3)}T_{20} + \sum_{j=20}^{22} (s_{(21)j})T_{21}^*G_j \tag{541}$$

$$\frac{dT_{22}}{dt} = -((b'_{22})^{(3)} - (r_{22})^{(3)})T_{22} + (b_{22})^{(3)}T_{21} + \sum_{j=20}^{22} (s_{(22)j})T_{22}^*G_j \tag{542}$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ and $(b_i'')^{(4)}$ belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable. 543

Denote

Definition of G_i, T_i :- 544

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}''^{(4)})}{\partial T_{25}} (T_{25}^*) = (q_{25})^{(4)} \quad , \quad \frac{\partial (b_i''^{(4)})}{\partial G_j} ((G_{27})^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{24}}{dt} = -((a'_{24})^{(4)} + (p_{24})^{(4)})\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25} \quad 545$$

$$\frac{d\mathbb{G}_{25}}{dt} = -((a'_{25})^{(4)} + (p_{25})^{(4)})\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25} \quad 546$$

$$\frac{d\mathbb{G}_{26}}{dt} = -((a'_{26})^{(4)} + (p_{26})^{(4)})\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25} \quad 547$$

$$\frac{d\mathbb{T}_{24}}{dt} = -((b'_{24})^{(4)} - (r_{24})^{(4)})\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26} (s_{(24)(j)})T_{24}^*\mathbb{G}_j \quad 548$$

$$\frac{d\mathbb{T}_{25}}{dt} = -((b'_{25})^{(4)} - (r_{25})^{(4)})\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26} (s_{(25)(j)})T_{25}^*\mathbb{G}_j \quad 549$$

$$\frac{d\mathbb{T}_{26}}{dt} = -((b'_{26})^{(4)} - (r_{26})^{(4)})\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26} (s_{(26)(j)})T_{26}^*\mathbb{G}_j \quad 550$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ and $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

_Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 551

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{29}''^{(5)})}{\partial T_{29}} (T_{29}^*) = (q_{29})^{(5)} \quad , \quad \frac{\partial (b_i''^{(5)})}{\partial G_j} ((G_{31})^*) = s_{ij}$$

Then taking into account equations(global) and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{28}}{dt} = -((a'_{28})^{(5)} + (p_{28})^{(5)})\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29} \quad 552$$

$$\frac{d\mathbb{G}_{29}}{dt} = -((a'_{29})^{(5)} + (p_{29})^{(5)})\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29} \quad 553$$

$$\frac{d\mathbb{G}_{30}}{dt} = -((a'_{30})^{(5)} + (p_{30})^{(5)})\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29} \quad 554$$

$$\frac{d\mathbb{T}_{28}}{dt} = -((b'_{28})^{(5)} - (r_{28})^{(5)})\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30} (s_{(28)(j)})T_{28}^*\mathbb{G}_j \quad 555$$

$$\frac{d\mathbb{T}_{29}}{dt} = -((b'_{29})^{(5)} - (r_{29})^{(5)})\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30} (s_{(29)(j)})T_{29}^*\mathbb{G}_j \quad 556$$

$$\frac{d\mathbb{T}_{30}}{dt} = -((b'_{30})^{(5)} - (r_{30})^{(5)})\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30} (s_{(30)(j)})T_{30}^*\mathbb{G}_j \quad 557$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ and $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

_Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

558

$$G_i = G_i^* + \mathbb{G}_i \quad , \quad T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial(a_{33}''^{(6)})}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \quad \frac{\partial(b_i''^{(6)})}{\partial G_j}(G_{35}^*) = s_{ij}$$

Then taking into account equations(global) and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -((a'_{32})^{(6)} + (p_{32})^{(6)})\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33} \quad 559$$

$$\frac{d\mathbb{G}_{33}}{dt} = -((a'_{33})^{(6)} + (p_{33})^{(6)})\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33} \quad 560$$

$$\frac{d\mathbb{G}_{34}}{dt} = -((a'_{34})^{(6)} + (p_{34})^{(6)})\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33} \quad 561$$

$$\frac{d\mathbb{T}_{32}}{dt} = -((b'_{32})^{(6)} - (r_{32})^{(6)})\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}(s_{(32)(j)})T_{32}^*\mathbb{G}_j \quad 562$$

$$\frac{d\mathbb{T}_{33}}{dt} = -((b'_{33})^{(6)} - (r_{33})^{(6)})\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}(s_{(33)(j)})T_{33}^*\mathbb{G}_j \quad 563$$

$$\frac{d\mathbb{T}_{34}}{dt} = -((b'_{34})^{(6)} - (r_{34})^{(6)})\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}(s_{(34)(j)})T_{34}^*\mathbb{G}_j \quad 564$$

The characteristic equation of this system is

$$\begin{aligned} & ((\lambda)^{(1)} + (b'_{15})^{(1)} - (r_{15})^{(1)})\{((\lambda)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}) \\ & \left[((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)})(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)}G_{13}^* \right] \\ & \left(((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(14)}T_{14}^* + (b_{14})^{(1)}s_{(13),(14)}T_{14}^* \right) \\ & + \left(((\lambda)^{(1)} + (a'_{14})^{(1)} + (p_{14})^{(1)})(q_{13})^{(1)}G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)}G_{14}^* \right) \\ & \left(((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(13)}T_{14}^* + (b_{14})^{(1)}s_{(13),(13)}T_{13}^* \right) \\ & \left(((\lambda)^{(1)})^2 + ((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}) (\lambda)^{(1)} \right) \\ & \left(((\lambda)^{(1)})^2 + ((b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}) (\lambda)^{(1)} \right) \\ & + \left(((\lambda)^{(1)})^2 + ((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}) (\lambda)^{(1)} \right) (q_{15})^{(1)}G_{15} \\ & + ((\lambda)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}) \left((a_{15})^{(1)}(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^* \right) \\ & \left(((\lambda)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})s_{(14),(15)}T_{14}^* + (b_{14})^{(1)}s_{(13),(15)}T_{13}^* \right) \} = 0 \\ & + \\ & ((\lambda)^{(2)} + (b'_{18})^{(2)} - (r_{18})^{(2)})\{((\lambda)^{(2)} + (a'_{18})^{(2)} + (p_{18})^{(2)}) \\ & \left[((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)})(q_{17})^{(2)}G_{17}^* + (a_{17})^{(2)}(q_{16})^{(2)}G_{16}^* \right] \end{aligned} \quad 565$$

$$\begin{aligned}
 & \left(((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(17)}T_{17}^* + (b_{17})^{(2)}s_{(16),(17)}T_{17}^* \right) \\
 & + \left(((\lambda)^{(2)} + (a'_{17})^{(2)} + (p_{17})^{(2)})(q_{16})^{(2)}G_{16}^* + (a_{16})^{(2)}(q_{17})^{(2)}G_{17}^* \right) \\
 & \left(((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(16)}T_{17}^* + (b_{17})^{(2)}s_{(16),(16)}T_{16}^* \right) \\
 & \left(((\lambda)^{(2)})^2 + ((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}) (\lambda)^{(2)} \right) \\
 & \left(((\lambda)^{(2)})^2 + ((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}) (\lambda)^{(2)} \right) \\
 & + \left(((\lambda)^{(2)})^2 + ((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}) (\lambda)^{(2)} \right) (q_{18})^{(2)}G_{18} \\
 & + ((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}) ((a_{18})^{(2)}(q_{17})^{(2)}G_{17}^* + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}G_{16}^*) \\
 & \left. \left(((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})s_{(17),(18)}T_{17}^* + (b_{17})^{(2)}s_{(16),(18)}T_{16}^* \right) \right\} = 0 \\
 & + \\
 & \left((\lambda)^{(3)} + (b'_{22})^{(3)} - (r_{22})^{(3)} \right) \{ (\lambda)^{(3)} + (a'_{22})^{(3)} + (p_{22})^{(3)} \} \\
 & \left[\left(((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)})(q_{21})^{(3)}G_{21}^* + (a_{21})^{(3)}(q_{20})^{(3)}G_{20}^* \right) \right] \\
 & \left(((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)})s_{(21),(21)}T_{21}^* + (b_{21})^{(3)}s_{(20),(21)}T_{21}^* \right) \\
 & + \left(((\lambda)^{(3)} + (a'_{21})^{(3)} + (p_{21})^{(3)})(q_{20})^{(3)}G_{20}^* + (a_{20})^{(3)}(q_{21})^{(1)}G_{21}^* \right) \\
 & \left(((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)})s_{(21),(20)}T_{21}^* + (b_{21})^{(3)}s_{(20),(20)}T_{20}^* \right) \\
 & \left(((\lambda)^{(3)})^2 + ((a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}) (\lambda)^{(3)} \right) \\
 & \left(((\lambda)^{(3)})^2 + ((b'_{20})^{(3)} + (b'_{21})^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)}) (\lambda)^{(3)} \right) \\
 & + \left(((\lambda)^{(3)})^2 + ((a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}) (\lambda)^{(3)} \right) (q_{22})^{(3)}G_{22} \\
 & + ((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)}) ((a_{22})^{(3)}(q_{21})^{(3)}G_{21}^* + (a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G_{20}^*) \\
 & \left. \left(((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)})s_{(21),(22)}T_{21}^* + (b_{21})^{(3)}s_{(20),(22)}T_{20}^* \right) \right\} = 0 \\
 & + \\
 & \left((\lambda)^{(4)} + (b'_{26})^{(4)} - (r_{26})^{(4)} \right) \{ (\lambda)^{(4)} + (a'_{26})^{(4)} + (p_{26})^{(4)} \} \\
 & \left[\left(((\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)})(q_{25})^{(4)}G_{25}^* + (a_{25})^{(4)}(q_{24})^{(4)}G_{24}^* \right) \right] \\
 & \left(((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)})s_{(25),(25)}T_{25}^* + (b_{25})^{(4)}s_{(24),(25)}T_{25}^* \right)
 \end{aligned}$$

$$\begin{aligned}
 &+ \left(((\lambda)^{(4)} + (a'_{25})^{(4)} + (p_{25})^{(4)})(q_{24})^{(4)} G_{24}^* + (a_{24})^{(4)}(q_{25})^{(4)} G_{25}^* \right) \\
 &\quad \left(((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)})s_{(25),(24)}T_{25}^* + (b_{25})^{(4)}s_{(24),(24)}T_{24}^* \right) \\
 &\left(((\lambda)^{(4)})^2 + ((a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)}) (\lambda)^{(4)} \right) \\
 &\quad \left(((\lambda)^{(4)})^2 + ((b'_{24})^{(4)} + (b'_{25})^{(4)} - (r_{24})^{(4)} + (r_{25})^{(4)}) (\lambda)^{(4)} \right) \\
 &+ \left(((\lambda)^{(4)})^2 + ((a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)}) (\lambda)^{(4)} \right) (q_{26})^{(4)} G_{26} \\
 &+ ((\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)}) ((a_{26})^{(4)}(q_{25})^{(4)} G_{25}^* + (a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)} G_{24}^*) \\
 &\left(((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)})s_{(25),(26)}T_{25}^* + (b_{25})^{(4)}s_{(24),(26)}T_{24}^* \right) \} = 0 \\
 &+ \\
 &((\lambda)^{(5)} + (b'_{30})^{(5)} - (r_{30})^{(5)}) \{ ((\lambda)^{(5)} + (a'_{30})^{(5)} + (p_{30})^{(5)}) \\
 &\left[((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)})(q_{29})^{(5)} G_{29}^* + (a_{29})^{(5)}(q_{28})^{(5)} G_{28}^* \right] \\
 &\left(((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)})s_{(29),(29)}T_{29}^* + (b_{29})^{(5)}s_{(28),(29)}T_{29}^* \right) \\
 &+ \left(((\lambda)^{(5)} + (a'_{29})^{(5)} + (p_{29})^{(5)})(q_{28})^{(5)} G_{28}^* + (a_{28})^{(5)}(q_{29})^{(5)} G_{29}^* \right) \\
 &\quad \left(((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)})s_{(29),(28)}T_{29}^* + (b_{29})^{(5)}s_{(28),(28)}T_{28}^* \right) \\
 &\left(((\lambda)^{(5)})^2 + ((a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)}) (\lambda)^{(5)} \right) \\
 &\quad \left(((\lambda)^{(5)})^2 + ((b'_{28})^{(5)} + (b'_{29})^{(5)} - (r_{28})^{(5)} + (r_{29})^{(5)}) (\lambda)^{(5)} \right) \\
 &+ \left(((\lambda)^{(5)})^2 + ((a'_{28})^{(5)} + (a'_{29})^{(5)} + (p_{28})^{(5)} + (p_{29})^{(5)}) (\lambda)^{(5)} \right) (q_{30})^{(5)} G_{30} \\
 &+ ((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)}) ((a_{30})^{(5)}(q_{29})^{(5)} G_{29}^* + (a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)} G_{28}^*) \\
 &\left(((\lambda)^{(5)} + (b'_{28})^{(5)} - (r_{28})^{(5)})s_{(29),(30)}T_{29}^* + (b_{29})^{(5)}s_{(28),(30)}T_{28}^* \right) \} = 0 \\
 &+ \\
 &((\lambda)^{(6)} + (b'_{34})^{(6)} - (r_{34})^{(6)}) \{ ((\lambda)^{(6)} + (a'_{34})^{(6)} + (p_{34})^{(6)}) \\
 &\left[((\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)})(q_{33})^{(6)} G_{33}^* + (a_{33})^{(6)}(q_{32})^{(6)} G_{32}^* \right] \\
 &\left(((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)})s_{(33),(33)}T_{33}^* + (b_{33})^{(6)}s_{(32),(33)}T_{33}^* \right) \\
 &+ \left(((\lambda)^{(6)} + (a'_{33})^{(6)} + (p_{33})^{(6)})(q_{32})^{(6)} G_{32}^* + (a_{32})^{(6)}(q_{33})^{(6)} G_{33}^* \right)
 \end{aligned}$$

$$\begin{aligned} & \left(((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)})s_{(33),(32)}T_{33}^* + (b_{33})^{(6)}s_{(32),(32)}T_{32}^* \right) \\ & \left(((\lambda)^{(6)})^2 + (a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)} \right) (\lambda)^{(6)} \\ & \left(((\lambda)^{(6)})^2 + (b'_{32})^{(6)} + (b'_{33})^{(6)} - (r_{32})^{(6)} + (r_{33})^{(6)} \right) (\lambda)^{(6)} \\ & + \left(((\lambda)^{(6)})^2 + (a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)} \right) (\lambda)^{(6)} (q_{34})^{(6)} G_{34} \\ & + ((\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)}) \left((a_{34})^{(6)} (q_{33})^{(6)} G_{33}^* + (a_{33})^{(6)} (a_{34})^{(6)} (q_{32})^{(6)} G_{32}^* \right) \\ & \left. \left(((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)})s_{(33),(34)}T_{33}^* + (b_{33})^{(6)}s_{(32),(34)}T_{32}^* \right) \right\} = 0 \end{aligned}$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

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