Detection of Sum and Difference Squeezing

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Abstract: *Sum and difference squeezing was defined by Hillery who showed that these turn into normal squeezing in sum and difference frequency generation. We re-examine this using an intense coherent pump mode and with a much better approximation. Our results are valid for much larger interaction times and therefore enable detection of smaller sum and difference squeezings, in principle., Moreover, if both, our and Hillary's results are regarded holding, our results lead to more squeezing.*

Keywords: *Coherent state; squeezing; sum and difference squeezing.*
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I.   Introduction

Nonclassiclal phenomenon have important role in Quantum optics. Squeezing [1], antibunching [2], collapses- revivals [3] and quantum beats [4] were the earliest studied nonclassical phenomenon. There are some recent developments in higher order nonclassical states [5].   Earlier study of such non-classical phenomenon was largely in academic interest [6]. Now Squeezing has attracted considerable attention owing to application in the reducing noise in a signal [7], resonance fluorescence [8], detection of very weak forces such as gravitational waves [9], quantum teleportation [10] and others. Squeezing has been studied by some nonlinear optical process e.g. parametric amplification [11], multiwave mixing process [12-14], harmonic generation [15], Jaynes Cummings Model [16], and others.

Different type of squeezing was studied by authors [17-26]. Higher order squeezing in a single mode has been proposed by several authors. For a single mode this is introduced by Hong and Mandel [18] and by Hillery [19]. Hillery formulation was used to investigate the amplitude squared squeezing in spontaneous degenerate hyper Raman process [20]. Hillery also proposed [21] sum and difference squeezing for two-mode light. This was generalized to three modes [22, 23] and to even an arbitrary number of modes [24, 25]. Hillery [21] showed that detection of these are possible, in principle, using a process with a trilinear Hamiltonian involving three modes. Among these three modes two are those whose sum or difference squeezing is to be determined and the third is a pump mode. Perturbation expansions were used and it was shown that the ordinary squeezing of the pump mode after a nonzero interaction time gives sum or difference squeezing of the two input mode, i.e., the sum or difference squeezing before enter into the interaction described by the trilinear Hamiltonian. Giri and Gupta [26] reported the same for a Raman Process. They also discussed [26] sum and difference squeezing of the two modes after interaction for some time.

Detection of sum and difference squeezings of two modes by coupling them to an intense pump mode and then by detecting ordinary squeezing of the pump mode has been discussed by Hillery [19] and by Giri and Gupta [26].

In the present paper, we reexamine the sum and difference squeezings using an intense coherent pump mode with a much better approximation and our results are valid for larger interaction times so that even smaller sum and difference squeezing are detectable in principle. We make perturbation expansions in powers of reciprocal of the amplitude of intense pump mode and not in powers of coupling time as is done by others [21-23, 26]. Not only our results are valid for longer time intervals, at times, when our results are valid and Hillary's results are not valid, our results are seen to give a much larger squeezing then Hillary's.

II.   Sum squeezing of the two modes in terms of ordinary squeezing of an intense pump mode in three wave interaction

Sum frequency of two modes, say, modes B and C having annihilation operator b, c and creation operators $b^\dagger$, $c^\dagger$, is defined in terms of operators $W_{1,2}$ defined by

$$W_1 + i\, W_2 \equiv bc, \tag{1}$$

which obey

$$[W_1, W_2] = i\frac{1}{2}\,(N_b + N_c + 1), \quad \Delta W_1^2\,\Delta W_2^2 \geq \frac{1}{16}\langle N_b + N_c + 1\rangle^2. \tag{2}$$

A state is said to be sum squeezed in W_1 or in W_2 according as $(\Delta W_1)^2$ or $(\Delta W_2)^2$ is $< \frac{1}{4}\langle N_b + N_c + 1\rangle$.

To measure sum squeezing of modes B and C, we couple them to an intense pump mode A, which has operators a and $a^\dagger$ with a sum-frequency resonance, i.e., $\omega_A = \omega_B + \omega_C$. The 3-wave interaction is described by the interaction picture Hamiltonian,

$$H_I = g\,(a^\dagger bc + a\,b^\dagger c^\dagger), \tag{3}$$

where $a^\dagger(a)$, $b^\dagger(b)$ and $c^\dagger(c)$ are the interaction picture creation (annihilation) operators of the A, B and C modes respectively and g is the coupling constant of the interaction. We consider an intense pump mode initially in the coherent state $\left|xe^{i\theta}\right\rangle$ with $x \gg 1$, and write $a = (x + A_\theta)\,e^{i\theta}$.

The interaction Hamiltonian then, can, be written in the form,

$$H_I = G\,[\,(BC + B^\dagger C^\dagger)\,] + \frac{1}{x}G\,[A_\theta^\dagger BC + A_\theta B^\dagger C^\dagger\,], \tag{4}$$

where operators B, C and constant G are defined by

$$b = B\,e^{i\theta/2}, \quad c = C\,e^{i\theta/2} \text{ and } G = g\,x. \tag{5}$$

It may be noted that, for $x \gg 1$, Gt, may be appreciable even for a small value of gt. We shall do calculations exact in Gt but with a perturbation expansion in $1/x$, correct to first order only. We write subscript θ on operator of A mode as, later, we consider various values of θ, the phase of coherent pump state. Although operators B and C also depends on θ, we do not put a subscript on them as they do not appear in final results.

Equations of motion for operators A_θ, B, C are

$$\dot{A}_\theta = -i\frac{1}{x}GBC, \quad \dot{B} = -iG\,C^\dagger - i\frac{1}{x}G A C^\dagger, \quad \dot{C} = -i\,G\,B^\dagger - i\frac{1}{x}G A B^\dagger \tag{6}$$

These equations can be solved by perturbation to second order, the result for A_θ is

$$A_\theta(t) = A_{\theta S} + \frac{1}{4x}[-(B_S^\dagger B_S + C_S^\dagger C_S + 1)(\cosh 2Gt - 1) - i(B_S C_S - B_S^\dagger C_S^\dagger)\sinh 2Gt$$

$$- 2iGt(B_S C_S + B_S^\dagger C_S^\dagger)]$$

$$- \frac{1}{8x^2}(A_{\theta S} + A_{\theta S}^\dagger)[(B_S^\dagger B_S + C_S^\dagger C_S + 1)(2Gt\sinh 2Gt - \cosh 2Gt + 1)$$

$$+ i(B_S C_S - B_S^\dagger C_S^\dagger)(2Gt\cosh 2Gt - \sinh 2Gt)]$$

$$+ \frac{1}{8x^2}(A_{\theta S}^\dagger - A_{\theta S})[(B_S^\dagger B_S + C_S^\dagger C_S + 1)(\cosh 2Gt - 1) - 2i B_S^\dagger C_S^\dagger \sinh 2Gt$$

$$+ 4iGt B_S^\dagger C_S^\dagger], \tag{7}$$

Here, suffix S on an operator refers to Schrodinger picture which gives value of the operator at $t = 0$.

To examine the squeezing of the field amplitude of the pump mode A, we define general operator

$$X_{\theta\varphi}(t) = \frac{1}{2}[A_\theta e^{-i\varphi} + A_\theta^\dagger e^{i\varphi}]. \tag{8}$$

Using above equation we obtain

$$X_{\theta\varphi}(t) = X_{\theta\varphi S} + \frac{1}{2x}[-W_3\,f_1\cos\varphi + (-W_1\sin\theta + W_2\cos\theta)f_2\cos\varphi$$

$$- (W_1\cos\theta + W_2\sin\theta)f_3\sin\varphi]$$

$$- \frac{1}{2x^2}X_{\theta 0 S}\left[W_3(f_2 f_3 - f_1)\cos\varphi + (W_1\sin\theta - W_2\cos\theta)(f_1 f_3 + f_3 - f_2)\cos\varphi\right]$$

$$- \frac{1}{2x^2}X_{\theta\frac{\pi}{2}S}\left[W_3 f_1\sin\varphi - \{W_1\cos(\theta-\varphi) + W_2\sin(\theta-\varphi)\}(f_2 - f_3)\right], \tag{9}$$

where $X_{\theta\varphi S} = \frac{1}{2}(A_{\theta S}e^{-i\varphi} + A_{\theta S}^\dagger e^{i\varphi})$, and $f_{1,2,3}$ are defined by $f_1 = (\cosh 2Gt - 1)$, $f_2 = \sinh 2Gt$, $f_3 = 2Gt$ and $W_3 = \frac{1}{2}(N_b + N_c + 1)$.

It may be noted that for $Gt \ll 1$, $f_1 \approx 2G^2 t^2$, $f_2 - f_3 \approx \frac{4}{3}G^3 t^3$ and $\Delta X_{\theta\frac{\pi}{2}}^2(t) - \frac{1}{4} \propto t^2$.

The variance of $X_{\theta\frac{\pi}{2}}(t)$ is obtained as

$$\Delta X^2_{\theta\frac{\pi}{2}}(t) = \frac{1}{4} + \frac{1}{4x^2}\Big[\ \Delta W_1^2 cos^2\theta + \Delta W_2^2 sin^2\theta + (\langle W_1 W_2\rangle + \langle W_2 W_1\rangle$$

$$-2\langle W_1\rangle\langle W_2\rangle)\ sin\theta\, cos\theta\ \Big]f_3^2$$

$$-\frac{1}{4x^2}\Big[\langle W_3\rangle f_1 - (f_2 - f_3)\langle W_1 sin\theta - W_2 cos\theta\rangle\Big]. \tag{10}$$

The above equation can be solved for $(\Delta W_1)^2$ and $(\Delta W_2)^2$ and gives

$$(\Delta W_1)^2 = \frac{x^2}{2G^2 t^2}[\Delta X^2_{0\frac{\pi}{2}} + \Delta X^2_{\pi\frac{\pi}{2}} - \frac{1}{2}] + \frac{1}{8G^2 t^2}\ \langle N_b + N_c + 1\rangle(cosh\,2Gt - 1), \tag{11}$$

$$(\Delta W_2)^2 = \frac{x^2}{2G^2 t^2}[\Delta X^2_{\frac{\pi}{2}\frac{\pi}{2}} + \Delta X^2_{\frac{3\pi}{2}\frac{\pi}{2}} - \frac{1}{2}] + \frac{1}{8G^2 t^2}\ \langle N_b + N_c + 1\rangle(cosh\,2Gt - 1). \tag{12}$$

These equations give us the relation between sum squeezing of the two given input modes in terms of normal squeezing of the intense pump modes after 3-wave interaction for some time, for a fixed φ and different θ.

III. Difference squeezing of the two modes in terms of ordinary squeezing of an intense pump mode in three wave interaction

Difference frequency of two modes, say, modes B and C having annihilation operator b, c and creation operators $b^\dagger$, $c^\dagger$ is defined in terms of operators $V_{1,2}$ defined by

$$V_1 + i\,V_2 \equiv b^\dagger c, \tag{13}$$

which obey

$$[V_1, V_2] = i\frac{1}{2}(N_b - N_c), \qquad \Delta V_1^2 \Delta V_2^2 \geq \frac{1}{16}\langle N_b - N_c\rangle^2. \tag{14}$$

A state is said to be difference squeezed in V_1 or in V_2 according as $(\Delta V_1)^2$ or $(\Delta V_2)^2$ is $< \frac{1}{4}\langle N_b - N_c\rangle$.

To measure difference squeezing of modes B and C, we couple them to an intense pump mode A which has operators a and $a^\dagger$ with a difference frequency resonance $\omega_A = \omega_C - \omega_B$. Note that we call that input mode as C which has larger frequency without loss of generality. The 3-wave interaction is described by the interaction picture Hamiltonian,

$$H_I = g\,(a^\dagger b^\dagger c + abc^\dagger), \tag{15}$$

We write $a = (x + A_\theta)\,e^{i\theta}$, $b = B\,e^{-i\theta/2}$ and $c = C\,e^{i\theta/2}$ and the interaction Hamiltonian takes the form,

$$H_I = G\,[(B^\dagger C + BC^\dagger) + \frac{G}{x}(A_\theta^\dagger B^\dagger C + A_\theta B C^\dagger)], \quad G = g\,x. \tag{16}$$

We do calculations exact in Gt but with perturbation expansion in $1/x$, correct to first order only exactly as we did for sum squeezing.

Equations of motion for operators A_θ, B, C are

$$\dot{A}_\theta = -i\frac{1}{x}G B^\dagger C, \quad \dot{B} = -iGC - i\frac{1}{x}G A^\dagger C^\dagger, \quad \dot{C} = -iGB - i\frac{1}{x}G A B, \tag{17}$$

and they lead to

$$A_\theta(t) = A_{\theta S} + \frac{1}{4x}[(B_S^\dagger B_S - C_S^\dagger C_S)(cos\,2Gt - 1) - i(B_S^\dagger C_S - B_S C_S^\dagger)sin\,2Gt$$

$$- 2iGt(B_S^\dagger C_S + B_S C_S^\dagger)]$$

$$- \frac{1}{8x^2}(A_{\theta S} + A_{\theta S}^\dagger)[(B_S^\dagger B_S - C_S^\dagger C_S)(2Gt\,sin\,2Gt + cos\,2Gt - 1)$$

$$+ i(B_S^\dagger C_S - B_S C_S^\dagger)(2\,Gt\,cos\,2Gt - sin\,2Gt)]$$

$$- \frac{1}{8x^2}(A_{\theta S}^\dagger - A_{\theta S})[(B_S^\dagger B_S - C_S^\dagger C_S)(cos\,2Gt - 1) + 2i\,B_S C_S^\dagger sin\,2Gt$$

$$- 4\,i\,Gt\,B_S C_S^\dagger]. \tag{18}$$

Using equation (8) and (18) we obtain

$$X_{\theta\varphi}(t) = X_{\theta\varphi S} + \frac{1}{2x}\left[V_3 \, k_1 cos \, \varphi + \left(-V_1 \, sin \, \theta + V_2 \, cos \, \theta\right)k_2 \, cos \, \varphi\right.$$

$$\left. - \left(V_1 \, cos \, \theta + V_2 \, sin \, \theta\right)k_3 \, sin \, \varphi\right]$$

$$- \frac{1}{2x^2}X_{\theta 0 S}\left[V_3(k_2 \, k_3 + k_1) \, cos \, \varphi + \left(V_1 sin \, \theta - V_2 \, cos \, \theta\right)(k_1 \, k_3 + k_3 - k_2) \, cos \, \varphi\right]$$

$$- \frac{1}{2x^2}X_{\theta \frac{\pi}{2} S}\left[-V_3 \, k_1 \, sin \, \varphi + \left\{V_1 cos \, (\theta - \varphi) + V_2 \, sin \, (\theta - \varphi)\right\}(k_2 - k_3)\right]. \tag{19}$$

where $k_{1,2,3}$ are defined by $k_1 = (cos \, 2Gt - 1)$, $k_2 = sin \, 2Gt$, $k_3 = 2Gt$ and $V_3 = (N_b - N_c)/2$.

Here, we note that, for Gt<<1, $k_1 \approx 2G^2t^2$, $k_2 - k_3 \approx \frac{4}{3}G^3t^3$ and $\Delta X^2_{\theta\frac{\pi}{2}}(t) - \frac{1}{4} \propto t^2$.

Variance of $X_{\theta\frac{\pi}{2}}(t)$ is obtained as

$$\Delta X^2_{\theta\frac{\pi}{2}}(t) = \frac{1}{4} + \frac{1}{4x^2}[\Delta V_1^2 cos^2\theta + \Delta V_2^2 \, sin^2\theta + (\langle V_1 V_2\rangle + \langle V_2 V_1\rangle$$

$$- \langle V_1\rangle\langle V_2\rangle - \langle V_2\rangle\langle V_1\rangle) \, sin \, \theta \, cos\theta]k_3^2$$

$$- \frac{1}{4x^2}\left[-\langle V_3\rangle \, k_1 + (k_2 - k_3)\langle V_1 \, sin\theta - V_2 cos\theta\rangle\right]. \tag{20}$$

The above equation can be solved for $(\Delta V_1)^2$ and $(\Delta V_2)^2$, and gives

$$(\Delta V_1)^2 = \frac{x^2}{2G^2t^2}[\Delta X^2_{0\frac{\pi}{2}} + \Delta X^2_{\pi\frac{\pi}{2}} - \frac{1}{2}] - \frac{1}{8G^2t^2}\langle N_b - N_c\rangle(cos \, 2Gt - 1), \tag{21}$$

$$(\Delta V_2)^2 = \frac{x^2}{2G^2t^2}[\Delta X^2_{\frac{\pi}{2}\frac{\pi}{2}} + \Delta X^2_{\frac{3\pi}{2}\frac{\pi}{2}} - \frac{1}{2}] - \frac{1}{8G^2t^2}\langle N_b - N_c\rangle(cos \, 2Gt - 1). \tag{22}$$

These equations give us the relation between difference squeezing of the two given input modes in terms of normal squeezing of the intense pump modes after 3-wave interaction for some time, for a fixed ϕ and different θ.

IV. Discussion, Conclusions and suggestion for experimental verification

Validity of first order perturbation theory results may be represented by $\langle[\int H_I(t)dt]\rangle$ <<1. For Hillery's case it gives gt<< $1/\sqrt{n_a n_b n_c}$ which is equivalent to Gt<< $1/\sqrt{n_b n_c}$. Here, n_a, n_b, n_c are the number of photons in the pump and the two given modes. Our results for sum squeezing are however seen to be valid for Sinh2Gt<< $\sqrt{n_a/n_b n_c}$ by replacing H_I above by the part of H_I in which we make first order perturbation expansion. Our results obviously hold for much larger durations as compared to Hillary's results. We plot a graph with Gt vs $\Delta X^2_{\theta\frac{\pi}{2}}(t) - \frac{1}{4}$ for sum squeezing for $\theta = 0$ and π for some typical values of $\Delta W_1^2, \Delta W_2^2, \langle W_1 W_2\rangle, \langle W_1\rangle, \langle W_2\rangle, N_b, N_c, \Delta V_1^2, \Delta V_2^2, \langle V_1 V_2\rangle, \langle V_1\rangle, \langle V_2\rangle$ and also show Hillery's result in Figure (1). It shows that our results bring much larger squeezing .We compare these graphical results with Hillery's results.

Similarly, for difference squeezing our results are valid for 2Gt<< $\sqrt{n_a/n_b n_c}$. Then thus hold for much larger duration than the Hillary's results which holds for Gt<< $1/\sqrt{n_b n_c}$. In Figure (2), we show variation of our results for $\theta = 0$ and π and the Hillery's results for comparison. Our results give larger squeezing.

We note that our results for both sum and difference squeezing give larger squeezing and hold for larger duration as compared to Hillery's results. These should, therefore, enable detection of smaller sum and difference squeezing, in principle

Since sum and difference squeezing of modes B and C is connected to ordinary squeezing of intense pump mode, it can be detected using the usual homodyning method [27]. The sum (or difference) squeezed light

may be mixed by a beam splitter with the coherent light of a strong local pump oscillator, whose phase θ can be varied by a delay line used before mixing with input modes.

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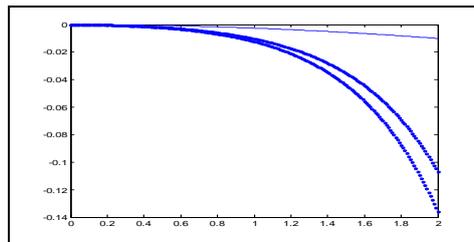
Figure1. Graph showing variation of $\Delta X^2_{\theta\frac{\pi}{2}}(t) - \frac{1}{4}$ with Gt in case of sum squeezing.

$\Delta W_1{}^2 = \Delta W_2{}^2 = \langle W_1 W_2 \rangle = \langle W_1 \rangle = \langle W_2 \rangle = 1/4, N_b = N_c = 1/2$. Dotted line shows our result (upper for $\theta = 0$ and lower for $\theta = \pi$) and solid line shows Giri and Gupta's result.
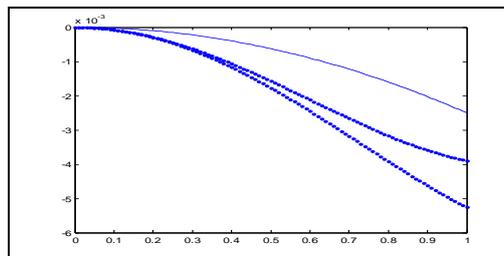
Figure2. Graph showing variation of $\Delta X^2_{\theta\frac{\pi}{2}}(t) - \frac{1}{4}$ with Gt in case of difference squeezing.

$\Delta V_1{}^2 = \Delta V_2{}^2 = \langle V_1 V_2 \rangle = \langle V_1 \rangle = \langle V_2 \rangle = 1/4, N_b = 4, N_c = 2$. Dotted line shows our result (upper for $\theta = 0$ and lower for $\theta = \pi$) and solid line shows Giri and Gupta's result.