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A Variant Deterministic Model of Classical EOQ Formula

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Abstract: *Inventory management and transportation have been the principal areas of focus in industrial engineering and management for a long time. Inventory management attracts considerable attention in logistics and supply chain management today because new supply chain models have become more integrative and complex. New market forces have introduced many complex elements which affect the performance of the supply chain in general and inventory level in particular. Inventory decisions are high risk and high impact for supply chain management. Hence, this paper compiles all the derivations of classical deterministic lot size economic order quantity models and proposes a new method to verify the formula.*
Keyword: *Inventory Management, Supply Chain Management (SCM), Economic Order Quantity (EOQ)*

I. INTRODUCTION

At the very basic level any firm faces two main decisions concerning the management of inventory: When should new stock be ordered and in what quantities? With regard to the order quantity, that minimizes inventory related costs. The classical EOQ (economic order quantity) model remains the basic inventory model even when it is not applicable in real life business situations in most cases. In inventory related literature, the answer to the question of when to order is given with reference to the ROP (reorder point), and the point at which the replenishment order should be initiated so that the facility receives the inventory in time to maintain its target level of service. In the static and deterministic model, the ROP is the simple multiplication of the number of lead days and the daily demand. It means that every time the inventory falls to the ROP level, an order must be initiated. And the order quantity is given by the EOQ model which is based on cost minimization.

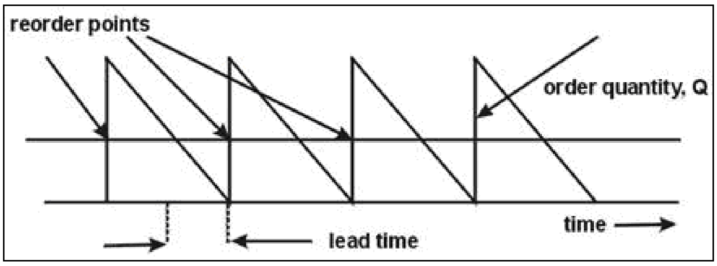

Figure-1:~ A simple inventory model based on fixed demand and fixed lead time [1].

The EOQ is the balance between order and holding costs attached with the inventory. The order cost is made up of fixed and variable costs, whereas the holding cost consist of costs of maintenance. The formula is:

$Q = \sqrt{[2CoD/Cc]}$

Q is the order quantity per order, D is the demand per year

Co is the fixed cost which the warehouse incurs every time it places an order

Cc is the inventory carrying or holding cost per unit per year, and

Notice that it highlights two important insights regarding the EOQ model. These are:

1. Optimum order size is a balance between the holding cost and the fixed order cost.
2. Total inventory cost is related with order size, but the relationship is not significant.

II. LITERATURE REVIEW

The aim of inventory management is to seize the inventory at lowest possible cost, given the objective to ensure uninterrupted supplies for ongoing operations. While making decisions on inventory, management has to find a trade-off between different cost components like, compromise between costs of inventory supply, inventory holding costs and costs resulting from insufficient inventory [2].

Economic order quantity is the order quantity that minimizes the sum value of total stock holding costs and total ordering costs. The framework used to determine this order quantity is known as Basic EOQ Model. It is the oldest classical production scheduling model. The model was developed by Ford W. Harris in 1913 [3] but R. H. Wilson is given the credit for his comprehensive analysis, who had applied it extensively [4].

A discussion of the EOQ model would remain incomplete if the inherent assumptions on which the model is based are ignored. The major assumptions are [5]:

a. All demand is satisfied as there are no limits on capital availability.
b. The rate of demand is continuous, constant and known.
c. Replenishment lead time is constant and known.
d. There is a constant price of product, independent of order quantity or time.
e. There is no interaction between multiple items of inventory and no inventory in transit.

The importance of this EOQ formula can be comprehended from a survey conducted on two hundred manufacturing firms in the USA, which states that the solutions contributed to a significant inventory reduction achieved by these firms. And the solutions were mainly such strategies as better forecasts in supply chain management, supplier managed inventory, the adoption of cycle counting and better training techniques [6].

III. VARIANT DETERMINISTIC MODEL

A model for a single-stage system in which we manage inventory of a single item is developed in this section. The purpose of this model is to determine how much to purchase (order quantity) and when to place the order (the reorder point). The common assumption is that, demand occurs continuously at a constant and known rate. In this simple model the demand is satisfied on time. Assumptions of the model are:

1) No shortages are allowed
2) Demand is known with certainty and is constant over time
3) Lead time for the receipt of orders is constant and known
4) Order quantity is received all at once and all the demand is satisfied on time
5) All the model parameters are unchanging over time.

Denotations:

Co = Cost of Placing an order
Cc = Annual per-unit carrying cost
D = Annual Demand
Q = Order Quantity
R = Re-Order Point (level of inventory at which order is placed)
L = Lead Time (the order arrival time after placement of an order)
d = Demand Rate (say, daily demand) = R/L
So, Number of orders placed in that year = D/Q
And Annual Ordering cost = CoD/Q
Average inventory = [Area of the triangle (**Figure-1**)/Length of the cycle] = ½QT/T = ½Q;
Or, Average inventory = [(Maximum Inventory + Minimum Inventory)/2] = (Q+0)/2 = ½Q
So, Annual Carrying Cost = ½CcQ
Annual Inventory Cost = Annual Carrying Cost + Annual Ordering Cost
So, TC = ½CcQ + CoD/Q
And TC* = Minimum Inventory Cost
The following diagram (**Figure-2**) illustrates the pattern of the cost curves with respect to lot size.

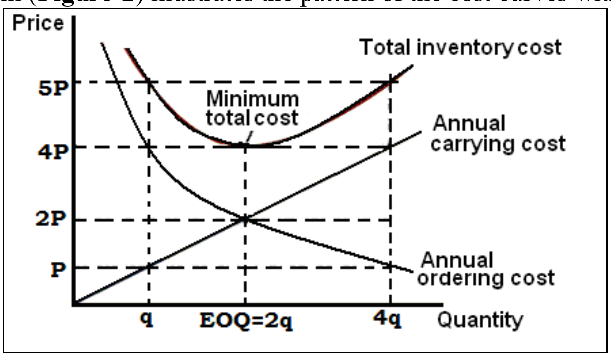

Figure-2:~ Pattern of the "Cost-Curves" [price vs. quantity] - based on the decided equations [1].

***Ford's Demonstration (1913)* [3]:**

Differentiating [TC = ½CcQ+CoD/Q] this equation w.r.t. Q;
ie, d/dQ of TC = Zero.
So, $½Cc + CoD/(Q)^2 = 0$
Or, $Q^{*2} = 2CoD/Cc$
ie, Q* = √ [2CoD/Cc]

***Wilson's Demonstration (1934)* [4]:**

From **Figure-2** it is evident at EOQ;
Ordering Cost = Carrying Cost
ie, CoD/Q = ½CcQ;
From this equation we can say that:
$Q^{*2} = 2CoD/Cc$
So, Q* = √ [2CoD/Cc]

***Proposed Variant Demonstration*:**

There is another alternative derivation of this formula;

Say for Q_1 order quantity and Q_2 order quantity the total inventory cost is same. TC (Q_1) = TC (Q_2)

Which means, $½CcQ_1 + CoD/Q_1 = ½CcQ_2 + CoD/Q_2$

Therefore, $½CcQ_1 - ½CcQ_2 = CoD/Q_2 – CoD/Q_1$

That is, $½Cc [Q_1 - Q_2] = [Q_1 - Q_2].CoD/Q_1Q_2$

So, $Q_1.Q_2 = 2CoD/Cc$

Now at EOQ Q_1 and Q_2 should always be same or equal [$Q_1 = Q_2$]

So, L.H.S. = $Q_1.Q_2 = Q^{*2}$

ie, $Q^{*2} = 2CoD/Cc$

So, $Q^* = \sqrt{[2CoD/Cc]}$

***Formula for Minimum Inventory Cost*:**

Again, $TC = ½CcQ + CoD/Q$; and at EOQ, $½CcQ = CoD/Q$

So, $TC^* = ½CcQ^* + ½CcQ^* = 2. [½CcQ^*]$; [at EOQ they are equal]

So, $TC^* = CcQ^*$

Now, applying the formula of Q^* we can find that,

$TC^* = Cc \sqrt{[2CoD/Cc]}$

So, $TC^{*2} = Cc^2. [2CoD/Cc]$

So, $TC^{*2} = 2CoD.Cc$

ie, $TC^* = \sqrt{[2Co.D.Cc]}$

IV. CONCLUSION

The model we have presented is the basic economic order quantity (EOQ) model. This model is studied first owing to its simplicity. Simplicity and restrictive modeling assumptions usually go together, and the EOQ model is not an exception. However, the presence of these modeling assumptions does not mean that the model cannot be used in practice. There are many situations in which this model will produce good results. Another advantage is that the model gives the optimal solution in closed form. This allows us to gain insights about the behavior of the inventory system. And the closed-form solution is also easy to compute.

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