

Design and Implementation of Sliding Mode Controller for Level Control

Parvat B. J.¹, Jadhav V. K.², Lokhande N. N.³

¹²³(Instrumentation & Control Engineering Department, P.R.E.C. Loni, India)

ABSTRACT : In this paper we illustrate the application of sliding mode controller (SMC) approach for level control of coupled tank system, which is non-linear and dynamic in nature. For the design of SMC constant relay gain approach (Signum function) is used. Mathematical model of coupled tank system is computed using material balance equation approach. Performances of the SMC and conventional PID controller are compared for theoretical analysis

Keywords -Coupled tank system, Modelling, Nonlinear system, PID, SMC

1.INTRODUCTION

The term sliding mode control comes from the theory of variable-structure systems, specifically relay systems, and became the principal operational model for this class of systems. Practically all design methods for variable-structure systems are based on the deliberate introduction of sliding modes. The concept of sliding mode control appeared in the Russian literature in the late fifties. However, the vibration control of a DC generator of an aircraft by V. Kulebakin (1932) and the use of relays for controlling the course of a ship by Nikolski (1934) can also be considered as contemporary sliding mode control [1]. It was Emel'yanov who first observed that due to altering the structure in the course of controlling a process, the properties could be attained which were not inherent in any of the individual structures [2]. The survey paper by Utkin [3] introduced this concept in the English literature. After this, the theory has been extended in various directions. The technique became popular because of its application to a wide class of systems containing discontinuous control elements such as relays.

Sliding Mode Control (SMC) is known to be a robust control method appropriate for controlling uncertain systems. High robustness are maintained against various kinds of uncertainties such as external disturbances and measurement error [4]. It is also straightforward to implement the resulting algorithms. Sliding mode control has long been considered for control of dynamic nonlinear systems. The need of SMC is to use a high speed switching control to move system's state trajectories onto specified and user chosen surface in the state space, known as the sliding surface or switching surface which keep the system's state trajectory along the surface. Once the state trajectory intercepts the sliding surface, it remains on the surface for all time and sliding along the surface, hence the term sliding mode. In the design of sliding mode controller the first stage is a design of sliding surface while the second is forces the state to approach the sliding surface from any other region of the state space, and remains on it [1] (Refer Fig.1).

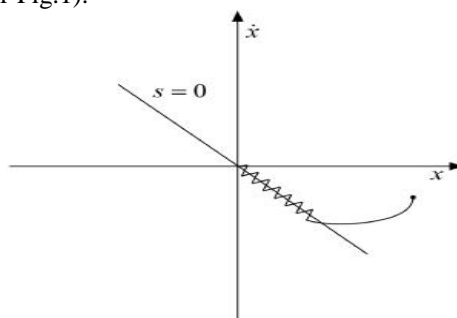


Fig.1 State trajectory and sliding surface in SMC

II.SLIDING MODE CONTROL

Consider a plant

$$\begin{aligned} x_1(t) &= x_2(t) \\ \dot{x}_2(t) &= -Ax_2(t) - Bx_1(t) + Cu(t) \\ y(t) &= x_1(t) \end{aligned} \quad (1)$$

Where A , B and C are nominal plant parameters, $u(t)$ is control input to the plant, $y(t)$ plant output and t is the independent time variable. The control problem is to find a suitable control input $u(t)$ such that the output tracks a desired command asymptotically in the presence of model uncertainties and disturbances. The tracking error $e(t)$, in terms of the command signal, $y_r(t)$ and measured output signal, $y_m(t)$, is defined as

$$e(t) = y_r(t) - y_m(t) \quad (2)$$

The crucial and the most important step of SMC design is the construction of the sliding surface $s(t)$, also called sliding function, $s(t) \in \mathbb{R}$ [5]. A sliding mode is said to exist if and only if $s(t)=0$ and $\dot{s}(t) < 0$. The inequality $\dot{s}(t) < 0$ is the fundamental condition for sliding mode [5]. The aim of the first-order sliding mode control is to force the state (error) to move on the switching surface $s(t)=0$. The sliding surface, $s(t)$ in the traditional SMC depends on the tracking error, $e(t)$ and derivative(s) of the tracking error as [5,6]:

$$s(t) = \left(\frac{d}{dt} + \lambda_c \right)^{n-1} e(t) \quad (3)$$

Where n indicates the order of uncontrolled system, λ_c is a positive constant. If the system concerned is assumed to be of second-order ($n=2$), and then, in a two-dimensional phase plane related to Eq. (3), is a straight line passing through the origin. Maintaining the error on $s(t)$ will result in $e(t)$ approaching zero. Therefore, $s(t)=0$ is a stable sliding surface and $e(t) \rightarrow 0$ as $t \rightarrow \infty$. If the error has reached the sliding surface $s(t)=0$, then it simultaneously slides into the origin $e(t)=0$. The error starting from a certain initial value will converge to the boundary layer and move inside the boundary layer towards the horizontal axis until it reaches $\dot{e}(t) = 0$ [6]. The derivative of the sliding surface, Eq. (3), is

$$\dot{s}(t) = \lambda_c \dot{e}(t) + \ddot{e}(t) \quad (4)$$

$\ddot{e}(t)$ can be written in terms of the plant parameters as

$$\ddot{e}(t) = \ddot{y}_r(t) + Ax_2(t) + Bx_1(t) - Cu(t) \quad (5)$$

Substituting in Eq. (7), we get,

$$\dot{s}(t) = \lambda_c \dot{e}(t) + \ddot{y}_r(t) + Ax_2(t) + Bx_1(t) - Cu(t) \quad (6)$$

The control input can be given as [5],[6]

$$u(t) = u_{eq}(t) + u_{sw}(t) \quad (7)$$

where $u_{eq}(t)$ and $u_{sw}(t)$ are the equivalent control and the switching control, respectively. The equivalent control, $u_{eq}(t)$, proposed by Utkin [7] is based on the nominal (estimated) plant parameters and provides the main control action, while the switching control, $u_{sw}(t)$, ensures the discontinuity of the control law across sliding surface. If the initial error is not on the sliding surface $s(t)$, or there is a deviation of the representative point from $s(t)$ due to parameter variations and disturbances, the controller must be designed such that it can drive the error to the sliding surface. The error under the condition that will move toward and reach the sliding surface is said to be on the reaching phase. The switching control law $u_{sw}(t)$, is introduced as [5]:

$$u_{sw}(t) = K \text{sign}(s) \quad (8)$$

Where K is a positive constant and sign function is defined as:

$$\text{sign}(s) = \begin{cases} 1 & s > 0 \\ 0 & s = 0 \\ -1 & s < 0 \end{cases} \quad (9)$$

III. MODELLING OF COUPLED TANK SYSTEM

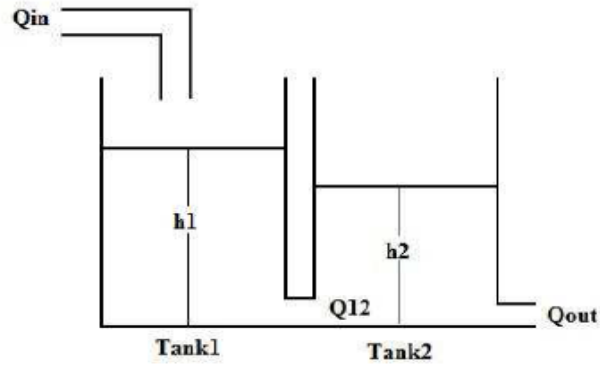


Fig.2 Coupled Tank system

The coupled tank system consists of two small tanks of equal areas and are coupled with an orifice. A pump is used to supply water to the first tank providing flow rate Q_{in} . In this case pump is only applicable for increases the liquid level and not responsible for pumping out water. The arrangement is shown in Fig.2. In the system Q_{in} , Q_{12} and Q_{out} are the inlet volumetric flow rate to tank₁, volumetric flow rate from tank₁ to tank₂ and outlet flow rate from tank₂ respectively in cm^3/sec whereas h_1 and h_2 are the levels of tank₁ and tank₂ respectively in cm. Let A is the cross section area of tank₁ and tank₂, a_{12} is the area of the coupling orifice between tank₁ and tank₂, a_2 is the cross section area of the outlet pipe of tank₂ and g is the gravitational constant. Then the system dynamical equations can be written as [8, 9]:

$$\frac{dh_1}{dt} = \frac{(Q_{in} - Q_{12})}{A} \quad (10)$$

$$\frac{dh_2}{dt} = \frac{(Q_{12} - Q_{out})}{A} \quad (11)$$

Where

$$Q_{out} = a_2\sqrt{2gh_2} \quad (12)$$

$$Q_{12} = a_{12}\sqrt{2g(h_1 - h_2)} \quad (13)$$

By using above equations we can write simplified form of equations as:

$$\frac{dh_1}{dt} = \frac{Q_{in}}{A} - k_1\sqrt{(h_1 - h_2)} \quad (14)$$

$$\frac{dh_2}{dt} = k_1\sqrt{(h_1 - h_2)} - k_2\sqrt{h_2} \quad (15)$$

Where,

$$k_1 = \frac{a_{12}\sqrt{2g}}{A} \quad (16)$$

$$k_2 = \frac{a_2\sqrt{2g}}{A} \quad (17)$$

By using all above equations, model of coupled tank system is developed in SIMULINK. The main aim is to regulate the water level h_2 of tank₂ when

1. set point value changed by user
2. external disturbances added to anyone or both the tanks simultaneously and

IV. RESULTS

The sliding mode controller (SMC) is developed and applied to the mathematical model of coupled tank system. The SMC is the latest and advanced controller in control theory. The step response of the system with SMC controller and conventional PID controller are shown in Fig.3. For simulation $\lambda_c=10$, $K=50$, $K_p=12$ and $K_i=0.2$ are chosen. dynamic performance specifications obtained from results are given in table 1.

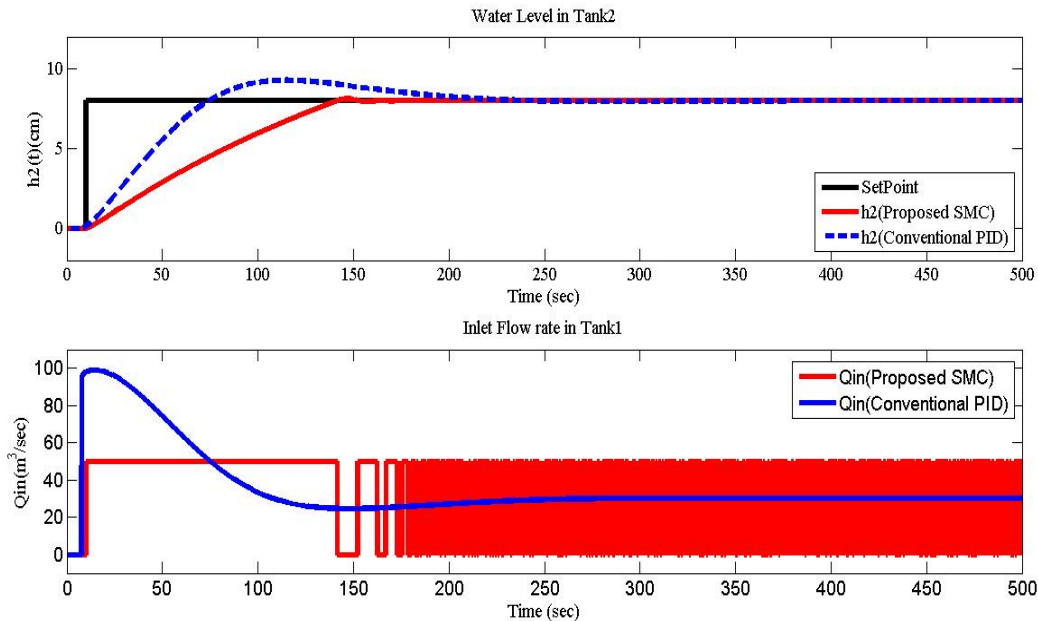


Fig.3 Simulation Results

Characteristics	SMC	PID
Overshoot	1.33	15.88
Peak	8.10	9.27
Settling Time	167.3	241.3

Table 1. Performances summery

v. CONCLUSION

The coupled tank system model development presented in the paper is nothing but the representation of the system in mathematical form. However, the complexity increases with increase in number of variables to be controlled. Further the SMC controller is designed for the process and closed loop performance is compared with conventional PID controllers. It can be concluded that the SMC controller can yield a better dynamic performance than the PID controllers in terms of rising time, setting time, maximum overshoot etc.

REFERENCES

[1] V. Utkin, G. Guldner, and J. Shi, Sliding Mode Control in Electromechanical Systems, Taylor & Francis, 1999. *Note that the journal title, volume number and issue number are set in italics*
 [2] V.I. Utkin. First stage of VSS: people and events. In Xinghuo Yu and Jian-Xin Xu, editors, Variable structure systems: towards the 21st century, volume 274 of Lecture Notes in Control and Optimization, pages 1.32. Springer, London, 2002.
 [3] V.I. Utkin. Variable structure systems with sliding mode. IEEE Trans. on Automatic Control, AC-22:212.222, 1977.
 [4] C. Edwards and S. Spurgeon, Sliding mode control: Theory and Applications. Taylor & Francis, 1998.
 [5] Slotine JJ, Li W. Applied nonlinear control. Englewood Cliffs (New Jersey): Prentice Hall Inc.; 1991.
 [6] Eker •, Ak_nal .A. Sliding mode control with integral augmented sliding surface: design and experimental application to an electromechanical system. Electr Eng 2008;90:189_97.
 [7] Utkin VI. Sliding modes in optimization and control problems. New York: Springer-Verlag; 1992.
 [8] M. Khan, and S. Spurgeon, Robust MIMO water level control in interconnected twin-tanks using second order sliding mode control, Control Engineering Practice 14, 375-386, 2006.
 [9] H. Delavari, R. Ghaderi, A. Ranjbar, and S. Momani, Fuzzy fractional order sliding mode controller for nonlinear systems, Commun Nonlinear Sci Numer Simulat 15, 963-978, 2010.