IOSR Journal of Electrical and Electronics Engineering (IOSR-JEEE)
e-ISSN: 2278-1676,p-ISSN: 2320-3331, Volume 10, Issue 6 Ver. I (Nov – Dec. 2015), PP 112-116
www.iosrjournals.org

Relative Study of Measurement Noise Covariance R and Process Noise Covariance Q of the Kalman Filter in Estimation.

[1]Yashpal Singh, [2]Rajesh Mehra
[1]M.E. Scholar, [2]Associate Professor
[1,2](Department of Electronics and Communication, National Institute of Technical Teachers Training and Research, Chandigarh, India)

Abstract: *In this paper is study the role of Measurement noise covariance R and Process noise covariance Q. As both the parameter in the Kalman filter is a important parameter to decide the estimation closeness to the True value , Speed and Bandwidth [1] . First of the most important work in integration is to consider the realistic dynamic model covariance matrix Q and measurement noise covariance matrix R for work in the Kalman filter. The performance of the methods to estimate and calculate both of these matrices depends entirely on the minimization of dynamic and measurement update errors that lead the filter to converge[2]. This paper evaluates the performances of Kalman filter method with different adaptations in Q and in R. This paper perform the estimation for different value of Q and R and make a study that how Q and R affects the estimation and how much it differ to true value to the estimated value by plot in graph for different value of Q and R as well as we give a that give the brief idea of error in estimation and true value by the help of the MATLAB.*
Keywords: *Covariance matrix, Kalman filter, Optimal estimation*

I. Introduction

The optimality of Kalman filter depends on the quality of prior assumptions about the process noise covariance matrix Q and the measurements noise covariance R. The quality of prior assumptions which are estimated by certain prior knowledge about the measurements and analysis are important factors that lead to the optimality of the Kalman filtering. Inadequacy of prior assumptions to calculate the real noise level could be lead to unexpected results and filter divergence. A Kalman filter has been used to estimate the measurement and process noise covariance matrices R and Q respectively. Determination of the suitable values of R and Q plays a crucial role to obtain a converged filter [2, 3]. Unexpected results will be yielded in case of determining small values of Q and R, the other factor is big diagonal element values of Q and R that could produce filter divergence [2]. The much attention has to paid on determine the disturbance matrices in order to obtain optimal Kalman filter parameters for integration applications [2-7]. The term optimal estimating filtering refers to method used for estimating the state of a time varying system, from which we can observed indirect noisy measurements. The state is the physical state, which can be defined by dynamic variables like position, velocity and acceleration of a moving object. The noise in the measurements means that there is a certain degree of uncertainty in them. The dynamic system define as a function of time and there is also noise in the dynamics of system, process noise and white additive Gaussian noise, meaning that the dynamic system cannot be modeled entirely deterministically[8]. The term filtering basically tells that the process of filtering is to process out the noise in the measurements values and give an optimal estimate for the state given the observed measurements and the assumptions made about the dynamic system. Kalman filters have been developed using three different scenarios of adaptation. These adaptation scenarios are adapting dynamic noise covariance matrix Q, measurement noise covariance matrix R, and the initial values of the error covariance matrix P . One of the philosophies for the Kalman filtering adaptation is to fix P and Q and vary R by trial and error to find the smallest value that gives stable state estimates, if this design does not give satisfactory performance, P and Q should also be varied [9, 10]. Various approaches have already been proposed for estimating Q and R matrices.

II. Kalman Filter

The Kalman Filter is a state estimator filter which gives result as optimal estimator in the sense of that the mean value of the sum of the estimation errors gets a minimal value [1, 2]. The Kalman filter estimated the sum of squared errors. The filter contain a model of dynamic state process that perform a function and as a feedback correction function method there are no requirement of store a data and other all associate matrices like measurement system, dynamics and noise are consider to be known .

Kalman filter have two steps [11]
1. Prediction
2. Correction

Dynamic model define prediction and Observation model define correction. The process is iterate each time so as define a recursive filter[11]. The Kalman filter contains the propagation of state estimation and the error covariance matrices at one time sample to next one.

Kalman dynamic system can be represent as [1-7].

$$x(k+1) = A(k) * x(k) + B(k) * w(k) \quad (1)$$
$$y(k+1) = C(k+1) * x(k+1) + v(k+1) \quad (2)$$

$A(k)$ = n × n state transition matrix, define dynamic system
$x(k)$=state vector with n dimension
$w(k)$=p -dimensional system noise
$B(k)$=n × p matrix defines impact on w (k)
$y(k)$ =m vector measurement vector.
$v(k)$ = m vector measurement noise sequence.
the noise $w(k)$ and $v(k)$ are consider independent, zero mean and white noise with covariance Q and R, respectively .Filter describe on the matrices A,B,C,Q,R as matrix notations.

$$P1(k) = A * P(k+1) * A^{T} + B * Q * B^{T} \quad (3)$$

$$K(k) = P1(k) * C^{T} * [C * P1(k) * C^{T} + R]^{-1} \quad (4)$$

The Kalman Filter was developed for systems to be calculated with a linear state-space model. But, usually applications are nonlinear state model. Furthermore the linear model is just a special case of a nonlinear model. Therefore it has decided to represent the Kalman Filter as nonlinear state models, but comments are given here about the linear case. The trivial method to consider the values for two matrices is through empirical analysis on the system and measurement errors or manually it by ad hoc fashion [2].

III. Relation between Q and R.

As by for the Kalman gain, variation in K (k) is related to the P1(k) and R, as uncertainty in the model can be characterized by process noise covariance Q .For large Q ,P1(k) ,shows large uncertainty or inadequate model so we conceive of the Kalman gain as a ratio of dynamic process to measurement noise. for small value of Q faster and accurate the response and for R small leads to slow response large value that is large noise covariance ,error is also large as it similar function is also found in case of R. It is assumed the that the system for which the states are to be estimated is excited by random ("white") disturbances (or process noise, there must be at least one real measurement in a Kalman Filter) contain random ("white") measurement noise [1-7].

K propositional to Q/R;

So K determine the time vary band width of the filter, larger the band width shorter the time for the tracking.

As R increase, K(k) decrease so bandwidth decrease system tracking remain slow [1] as it has been observed also that the error that is true value and estimated value difference also depends on that parameter as Q have[1-4] .

$$\tilde{x}p(k) = A * \tilde{x}(k-1) \quad (5)$$
$$\tilde{x}p(k) = \tilde{x}p(k) + K(k) * [y(k) - C * \tilde{x}p(k)] \quad (6)$$
$$P(k) = P1(k) - K(k) * C * P1(k) \quad (7)$$

k =1, 2, 3......
Assume initial condition are given P (0) and $\tilde{x}$ (0)
$\tilde{x}$ (0)=optimal estimation of the state$x(k)$.

The measurement noise covariance matrix R is a importance factor to the optimality and significance parameter for the Kalman filter output. It describes "How well the measurement model is and how good the measurements are". The lacking of the mathematical model is due to either non-modeling of the system or mis-modeling of the system or ignoring the non-white (colored or correlated) measurement errors. This type of error propagates in the Kalman filter algorithm through the measurement covariance matrix. The non-whiteness of the measurement errors, which contradicts the Kalman filter assumptions, results in a sub-optimal behavior of the filter [12].

IV. Measurement And Observation

The motive of this paper to study the error i.e. the difference of estimated value and true value on change of the process noise covariance (Q) when measurement noise covariance remain (R) same.
On changing the value of Q in the matrix we can observe the 'err' vector,

Take Q as
Process noise covariance

$$Q(q) = \begin{bmatrix} q & 0 \\ 0 & 0 \end{bmatrix}$$

q=[0.1, 0.01, 0.001, 0.00001]

on R constant

$$R = \begin{bmatrix} 0.0001 & 0 \\ 0 & 0.001 \end{bmatrix}$$

On second it is interested to vary the measurement noise covariance R and put the Q as constant as

$$Q = \begin{bmatrix} 0.00001 & 0 \\ 0 & 0 \end{bmatrix}$$

and R define as

Measurement noise covariance

$$R\ (r\ ,w) = \begin{bmatrix} r & 0 \\ 0 & w \end{bmatrix}$$

r=(0.1,0.01,0.0001,0.00001,) when w=0.001

$$R(r) = \begin{bmatrix} r & 0 \\ 0 & 0.001 \end{bmatrix}$$

For given R the estimation of states with different value of the Q (q)

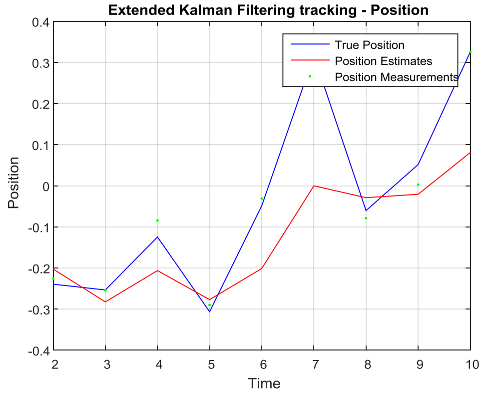
Measurement noise covariance

$$R = \begin{bmatrix} 0.0001 & 0 \\ 0 & 0.001 \end{bmatrix}$$

Where

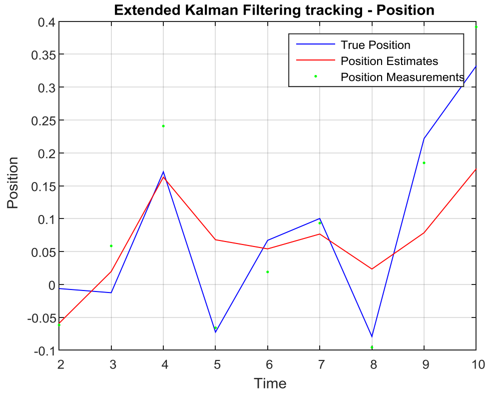
$$Q(q) = \begin{bmatrix} q & 0 \\ 0 & 0 \end{bmatrix}$$

fig-1.



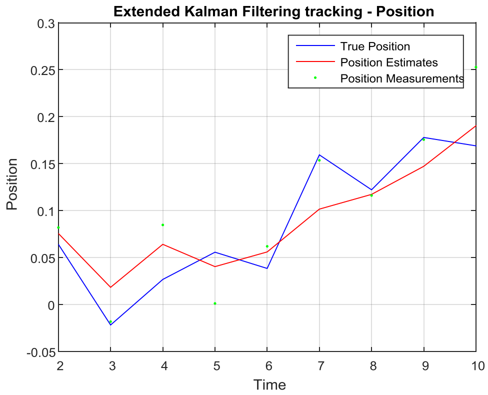
for q=0.1

fig-2



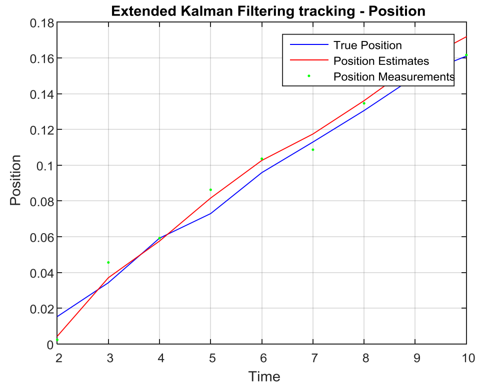
for q=0.01

fig-3



for q=0.001

fig-4



for q=0.00001

For R given the estimation of states with different value of the R (r), for given

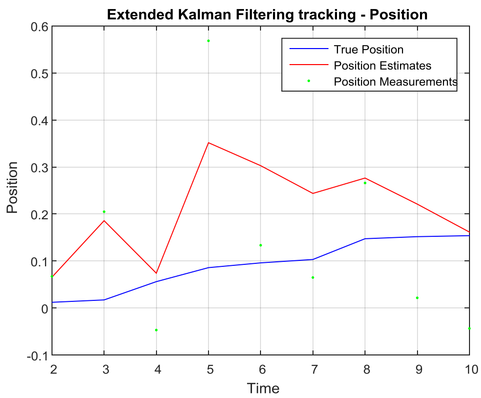
Process noise covariance

$$Q = \begin{bmatrix} 0.0001 & 0 \\ 0 & 0 \end{bmatrix}$$

where

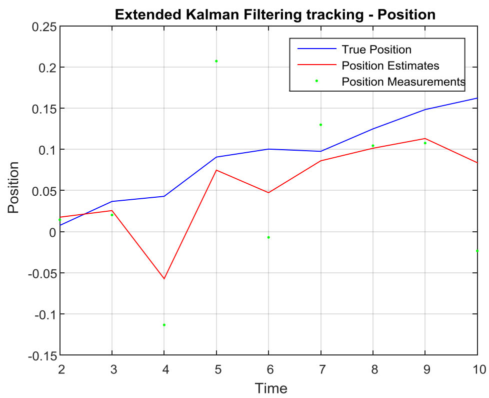
$$R = \begin{bmatrix} r & 0 \\ 0 & 0.001 \end{bmatrix}$$

fig -5



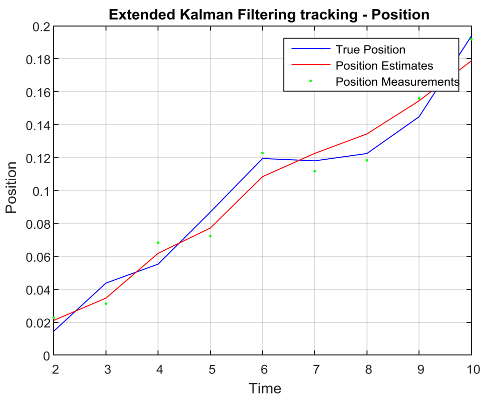
r=0.1

fig-6



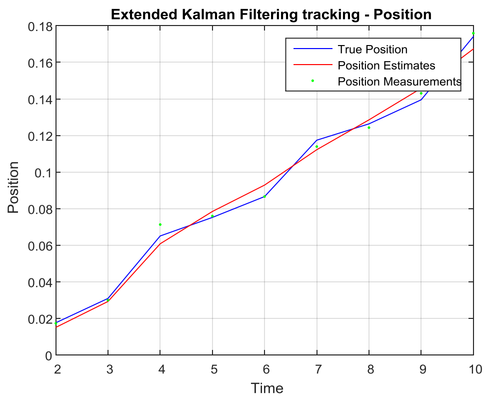
r=0.01

fig-7



r=0.0001

fig-8



r=0.00001

V. Conclusion

This paper demonstrated the behavior of the Kalman filter in different values of the Q process noise covariance and the R measurement noise covariance. We take different values of the Q (q) and R(r) and find out the error in estimation. We have observed that in both parameters Q (q) and the R(r) the value of argument q and r have a relation that on decease of that value leads to the error decrease i.e estimation leads to true value. As we discussed that small value of R make slow the response and Q small value, make it fast, so we will take a intermediate value that satisfied our estimation correction and time constraint both.

References

[1] Hen-Geul-Yen, Senior Member IEEE,"Real Time Implementation of a narrow-band Kalman Filter with a Floating-Point Processor".IEEE transaction on industrial electronics",Vol.37. pp 13-18.
[2] Ali Almagbile, Jinling Wang, and Weidong Ding "Evaluating the Performances of Adaptive Kalman Filter Methods in GPS/INS Integration" Journal of Global Positioning Systems (2010) Vol.9, No.1 , pp 33-40.
[3] Mohamed A. and Schwarz K. (1999) "Adaptive Kalman Filtering for INS/GPS". Journal of Geodesy, Vol.73, pp 193-203
[4] Ding W., Wang J. and Rizos, C. (2007) "Improving Adaptive Kalman Estimation in GPS/INS Integration". The Journal of Navigation, Vol.60, pp 517-529
[5] Hu C., Chen W., Chen Y. and Liu D. (2003) "Adaptive Kalman Filterinmg for Vehicle Navigation". Journal of Global Poaitioning System, Vol. 2, No. 1, pp 42-47.
[6] Li D. and Wang, J. (2006) "System Design and Performance Analysis of Extended Kalman Filter-Based Ultra-Tight GPS/INS Integration", IEEE/ION Position, Location, and Navigation Symposium, pp 291-199.
[7] Wang J., Stewart M. and Tsakiri M. (1999) " Online Stochastic Modeling for INS/GPS Integration" . ION GPS-99 proceedings, Nashville, Tennessee, pp 1887–1895.
[8] Jouni Hartikainen, Arno Solin, and Simo Särkkä "Optimal Filtering with Kalman Filters and Smoothers a Manual for the Matlab toolbox EKF/UKF", version 1.3, August 16, 2011, pp 4-7.
[9] Groves P. (2008), "Principles of GNSS, Inertial, and Multi sensor Integrated Navigation Systems". Artech House, Boston, London, pp 1-507.
[10] Mehra R. (1972) "Approaches to Adaptive Filtering". IEEE S-CS, Adaptive and learning systems. Pattern Recognition Committee, Austin,Tex, pp 693-698.
[11] Rachel Kleinbaur (2004) "Kalman filtering with implementation of MATLAB", pp 5-12.
[12] Ahmed H. Mohamed,"Optimizing the Estimation Procedure in INS/GPS Integration for Kinematic Applications", pp 73-84.
[13] Mohinder Singh Grewal, Angus P. Andrews, "Kalman filtering : Theory and Practice using Matlab" , John Wiley & Sons, 2007,pp 8-32.
[14] System Identification Toolbox User's Guide - MathWorks.
[15] Vikrant Vij , Rajesh Mehra ," FPGA Based Kalman Filter for Wireless Sensor Networks" Int. J. Comp. Tech. Appl., Vol 2 (1), 155-159.
[16] Shilpa Saxena, Rajesh Mehra , " Performance Comparision of Adaptive FIR Filter using Different Algorithm " Advanced Research in Electrical and Electronic Engineering, Vol – 1, pp 59-64.
[17] S.A.Quadri and Othman Sidek, "Error and Noise Analysis in an IMU using Kalman Filter" , International Journal of Hybrid Information Technology Vol.7, No.3 (2014), pp.39-48.
[18] Alejandro Ribeiro , Ioannis D. Schizas, Stergios I. "Kalman filtering in Wireless Sensor Networks " IEEE Control System Magazine , April 2010.pp 66-87.