An Appliaction of Generalized Fuzzy Soft Matrices in Decision Making Problem

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Abstract: *After development of fuzzy soft matrices, it has been applying in many fields of real life scenarios. The problems which are unable to solve by ordinary matrices can be solved by fuzzy soft matrices. In this paper our main aim is to define generalized fuzzy soft matrices and to study a few of its properties. Finally, we presented a decision making problem based on one of the operation of generalized fuzzy soft matrices.*
Keywords*: Fuzzy Soft Matrices, Generalized Fuzzy Soft Set, Generalized Fuzzy Soft Matrices.*

I. Introduction

In 1999, Molodtsov [9] introduced the theory of soft sets, which is a new approach to vagueness. In 2003, Maji et al. [11] studied the theory of soft sets initiated by Molodtsov and developed several basic notions of Soft Set Theory. Till now, researchers are contributing a lot on the extension of soft set theory. In 2005, Pei and Miao [6] and Hai Long et al. [6] studied and improved the Maji et al. [11]. Recently Cagman et al. [2, 3] introduced soft matrices and applied it in decision making problems. Researchers published several papers on fuzzy soft matrices and it has been applying in many fields of real life scenarios. B. Chetia and P.K. Das [1] defined Intuitionistic Fuzzy Soft Matrices with different products and properties on these products. In 2010, Majumdar et al. [10] generalized the concept of fuzzy soft set introduced by Maji et al. [11] to generalized fuzzy soft set and applied it in decision making problem[8]. Keeping in view, in 2010 Dinda et al. [5] gives the notion of Generalized Intuitionistic Fuzzy Soft Set .They continually work in this field and applied it in decision making problems [4].

In this paper we define generalized fuzzy soft matrix and a few of their operations. Then we study some of its properties. Subsequently based on the operation intersection we have applied generalized fuzzy soft matrices in a decision making problem in medical diagnosis.

II. Preliminaries

***Definition* 2.1 [10]**

Let U be an initial universe, $P(U)$ be the power set of U, E be the set of all parameters and $A \subseteq E$. A pair (f_A, E) is called a soft set over U is defined as the set of ordered pairs $(f_A, E) = \{(e, f_A(e)) : e \in E, f_A(e) \in P(U)\}$, where f_A is a mapping given by $f_A : E \to P(U)$ such that $f_A(e) = \phi$ if $e \notin A$.

Here f_A is called approximate function of the soft set (f_A, E). The set $f_A(e)$ is called *e-* approximate value set which consists of related objects of the parameter $e \in E$. In the other words, a soft set over U is a parameterized family of subsets of the universe U.

***Definition* 2.2**

Let (f_A, E) be a fuzzy soft set over U, where

$U = \{u_1, u_2, ..., u_m\}$ and $E = \{e_1, e_2, ..., e_n\}, \forall u_i \in U, \forall e_j \in E$ there exists the membership degree

$a_{ij} = f_{e_j}(u_i)$ and then we can present all membership degrees by a table as follows:

$$
\begin{array}{c|cccc}
 & e_1 & e_2 & \cdots & e_n \\
\hline
u_1 & a_{11} & a_{12} & \cdots & a_{1n} \\
u_2 & a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
u_m & a_{m1} & a_{m2} & \cdots & a_{mn}
\end{array}
$$

Then the fuzzy matrix

$$
\tilde{A}_{m \times n} = [a_{ij}]_{m \times n} = \begin{pmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{m1} & a_{m2} & \cdots & a_{mn}
\end{pmatrix}
$$

is called the fuzzy soft matrix of (f_A, E) over U.

According to this definition, a fuzzy soft set is uniquely characterized by the fuzzy matrix and vice versa.

Example 2.3

Let $U = \{u_1, u_2, u_3\}$ and $A = \{e_1, e_2, e_3\}$. Then a fuzzy soft set (f_A, E) over U given as below:

$$f_A(e_1) = \{ u_1 / 0.3, u_2 / 0.5, u_3 / 0.2\}$$

$$f_A(e_2) = \{u_1 / 0.5, u_2 / 0.7, u_3 / 0.6\}$$

$$f_A(e_3) = \{u_1 / 0.4, u_2 / 0.6, u_3 / 0.5\}$$

Hence the corresponding fuzzy soft matrix $[a_{ij}]_{m \times n}$ is written by:

$$
\tilde{A} = \begin{pmatrix}
0.3 & 0.5 & 0.4 & 0 \\
0.5 & 0.7 & 0.6 & 0 \\
0.2 & 0.6 & 0.5 & 0
\end{pmatrix}
$$

Definition 2.4 [5]

Let $U = \{u_1, u_2, ..., u_m\}$ be the universal set of elements and $E = \{e_1, e_2, ..., e_n\}$ be the set of parameters. The pair (U, E) will be called soft universe. Let $F : E \to I^U$ and λ be a fuzzy subset of E i.e.,

$\lambda : E \to I = [0,1]$, I^U is the collection of all fuzzy subsets of U.

Let $F_\lambda : E \to I^U \times I$ be a function define as $F_\lambda(e) = (F(e), \lambda(e))$, where $F(e) \in I^U$.

Then F_λ is called the fuzzy soft set over the soft universe (U, E).

Here for each parameters e_i, $F_\lambda(e_i) = (F(e_i), \lambda(e_i))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$ but also the degree of possibility of such belongingness which is represented by $\lambda(e_i)$.

Example 2.5

Let $U = \{s_1, s_2, s_3, s_4\}$ be the set of students under consideration and $E = \{e_1, e_2, e_3, e_4, e_5\}$ be the set of parameters where e_1 = expert in 'Mathematics', e_2 =expert in 'Chemistry, e_3 =expert in 'Physics',

e_4 =expert in 'Biology' and e_5 =expert in 'English'.

Let $A = \{e_1, e_2, e_3, e_4\} \subseteq E$. Let $\lambda : A \rightarrow I = [0,1]$ be given as:

$\lambda(e_1) = 0.8, \ \lambda(e_2) = 0.6, \ \lambda(e_3) = 0.5$ and $\lambda(e_4) = 0.7$.

We define F_λ as follows:

$$F_\lambda(e_2) = (\{s_1 / 0.5, s_2 / 0.7, s_3 / 0.4, s_4 / 0.2\}, 0.6)$$
$$F_\lambda(e_3) = (\{s_1 / 0.2, s_2 / 0.3, s_3 / 0.5, s_4 / 0.1\}, 0.5)$$
$$F_\lambda(e_4) = (\{s_1 / 0.3, s_2 / 0.1, S_3 / 0.2, s_4 / 0.4\}, 0.7)$$

which is the generalized fuzzy soft set representing overall expert of the students.

III. Generalized Fuzzy Soft Matrix

***Definition* 3.1**

Let *U be* an initial universe, *E* be a set of parameters and $A \subseteq E$. Let (F_λ, E) be a generalized fuzzy soft set over (U, E) . Then a subset of $U \times E$ is uniquely defined by $R_A = \{(u,e) : e \in A, u \in F_\lambda(e)\}$ which called a relation form of (F_λ, E) . The membership function μ_{R_A} and the function λ_{R_A} are written by $\mu_{R_A} : U \times E \rightarrow [0,1]$ and $\lambda_{R_A} : U \times E \rightarrow [0,1]$, where $\mu_{R_A} : (u,e) \in [0,1]$ is the membership value of $u \in U$ for each $e \in E$ and $\lambda_{R_A} : (u,e) \in [0,1]$.

If $[\mu_{ij}, \lambda_j]_{m \times n} = (\mu_{R_A}(u_i, e_j), \lambda_j(u_i, e_j))$, we define a matrix

$$[\mu_{ij}, \lambda_j]_{m \times n} = \begin{bmatrix} (\mu_{11}, \lambda_1) & (\mu_{12}, \lambda_2) & \cdots & (\mu_{1n}, \lambda_n) \\ (\mu_{21}, \lambda_1) & (\mu_{22}, \lambda_2) & \cdots & (\mu_{2n}, \lambda_n) \\ \vdots & \vdots & \vdots & \vdots \\ (\mu_{m1}, \lambda_1) & (\mu_{m2}, \lambda_2) & \cdots & (\mu_{mn}, \lambda_n) \end{bmatrix}$$

which is called an $m \times n$ generalized fuzzy soft matrix (GFSM) of generalized fuzzy soft set(GFSS) (F_λ, E) .

Therefore, we can say that a generalized fuzzy soft set (F_λ, E) is uniquely characterized by the matrix $[\mu_{ij}, \lambda_j]_{m \times n}$ and both concepts are interchangeable. The set of all ' $m \times n$ ' fuzzy soft matrices over U will be denoted by $GFSM_{m \times n}$.

***Example* 3.2**

Let $U = \{u_1, u_2, u_3\}$ be the set of the students under consideration for the best academic year with respect the given parameters $E = \{e_1, e_2, e_3, e_4\}$ where e_1 = result, e_2 = conduct, e_3 = attendance and e_4 = games and sports. Consider $A = \{e_1, e_2, e_4\} \subseteq E$. Let $\lambda : A \rightarrow [0,1]$ be given as $\lambda(e_1) = 0.8, \ \lambda(e_2) = 0.7$ and $\lambda(e_4) = 0.6$.

We define F_λ as follows:

$F_\lambda(e_1) = (\{u_1 / 0.4, u_2 / 0.5, u_4 / 0.6\}, 0.8), \quad F_\lambda(e_2) = (\{u_1 / 0.5, u_2 / 0.7, u_4 / 0.2\}, 0.7)$

$F_\lambda(e_3) = (\{u_1 / 0.2, u_2 / 0.3, u_4 / 0.5\}, 0.6)$

We can write the above set as follows:

R_A	e_1	e_2	e_3	e_4
u_1	(u_{11},λ_1)	(u_{12},λ_2)	(u_{13},λ_3)	(u_{14},λ_4)
u_2	(u_{21},λ_1)	(u_{22},λ_2)	(u_{23},λ_3)	(u_{24},λ_4)
u_3	(u_{31},λ_1)	(u_{32},λ_2)	(u_{33},λ_3)	(u_{34},λ_4)

Then the generalized fuzzy soft matrix can be written as follows:

$$[\mu_{ij},\lambda_j]_{3\times4} = \begin{bmatrix} (0.4,0.8) & (0.5,0.7) & (0,0) & (0.2,0.6) \\ (0.5,0.8) & (0.7,0.7) & (0,0) & (0.3,0.6) \\ (0.6,0.8) & (0.2,0.7) & (0,0) & (0.5,0.6) \end{bmatrix}$$

Definition 3.3

Let $[\mu_{ij},\lambda_j] \in GFSM_{m\times n}$. Then $[\mu_{ij},\lambda_j]$ is called

    (a) a zero matrix, denoted by [0], if $\mu_{ij}=0, \lambda_j=0$ for all i and j.

    (b) a universal $GFSM$, denoted by $[\mu_{ij}^{/},\lambda_j]$ if $\mu_{ij}=1, \lambda_j=1$ for all i and j.

Definition 3.4

Let $[a_{ij},\lambda_j], [b_{ij},\lambda_j^{/}] \in GFSM_{m\times n}$. Then $GFSM$ $[c_{ij},\gamma_j]$ is called

    (a) union of $[a_{ij},\lambda_j]$ and $[b_{ij},\lambda_j^{/}]$ denoted $[a_{ij},\lambda_j] \tilde{\cup} [b_{ij},\lambda_j^{/}]$, if $c_{ij} = \min\{a_{ij},b_{ij}\}$ and

        $\gamma_j = \min\{\lambda_j,\lambda_j^{'}\}$ for all i and j.

    (b) intersection of $[a_{ij},\lambda_j]$ and $[b_{ij},\lambda_j^{/}]$, denoted $[a_{ij},\lambda_j] \tilde{\cap} [b_{ij},\lambda_j^{/}]$, if $c_{ij} = \min\{a_{ij},b_{ij}\}$ and

        $\gamma_j = \max\{\lambda_j,\lambda_j^{/}\}$ for all i and j.

    (c) complement of $[a_{ij},\lambda_j]$, denoted by $[a_{ij},\lambda_j]^o$, if $c_{ij}=1-a_{ij}$ and $\gamma_j=1-\lambda_j$ for all i and j.

Definition 3.5

Let $[a_{ij},\lambda_j], [b_{ij},\lambda_j^{/}] \in GFSM_{m\times n}$. Then $[a_{ij},\lambda_j]$ and $[b_{ij},\lambda_j^{/}]$ are disjoint, if $[a_{ij},\lambda_j] \tilde{\cap} [b_{ij},\lambda_j^{/}] = [0]$ for all i and j.

Example 3.6

Let $[a_{ij},\lambda_j] = \begin{bmatrix} (0.4,0.5) & (0.5,0.7) & (0.2,0.6) \\ (0.5,0.5) & (0.7,0.7) & (0.3,0.6) \\ (0.6,0.5) & (0.2,0.7) & (0.4,0.6) \end{bmatrix}$ and $[b_{ij},\lambda_j^{/}] = \begin{bmatrix} (0.7,0.3) & (0.2,0.5) & (0.3,0.4) \\ (0.4,0.3) & (0.3,0.5) & (0.2,0.4) \\ (0.5,0.3) & (0.6,0.5) & (0.5,0.4) \end{bmatrix}$

Then $[a_{ij},\lambda_j] \tilde{\cup} [b_{ij},\lambda_j^{/}] = \begin{bmatrix} (0.7,0.3) & (0.5,0.5) & (0.3,0.4) \\ (0.5,0.3) & (0.7,0.5) & (0.3,0.4) \\ (0.6,0.3) & (0.6,0.5) & (0.5,0.4) \end{bmatrix}$

$[a_{ij},\lambda_j] \tilde{\cap} [b_{ij},\lambda_j^{/}] = \begin{bmatrix} (0.4,0.5) & (0.2,0.7) & (0.3,0.6) \\ (0.4,0.5) & (0.3,0.7) & (0.2,0.6) \\ (0.5,0.5) & (0.2,0.7) & (0.4,0.6) \end{bmatrix}$

$$\text{and} \quad [a_{ij}, \lambda_j]^0 = \begin{bmatrix} (0.6, 0.5) & (0.5, 0.3) & (0.8, 0.4) \\ (0.5, 0.5) & (0.3, 0.3) & (0.7, 0.4) \\ (0.4, 0.5) & (0.8, 0.3) & (0.6, 0.4) \end{bmatrix}$$

Proportion 3.7

Let $[a_{ij}, \lambda_j]$ and $[b_{ij}, \lambda_j^/] \in GFSM_{m \times n}$. Then

(a) $[[a_{ij}, \lambda_j]^0]^0 = [a_{ij}, \lambda_j]$

(b) $([a_{ij}, \lambda_j] \tilde{\cup} [b_{ij}, \lambda_j^/])^0 = [a_{ij}, \lambda_j]^0 \tilde{\cap} [b_{ij}, \lambda_j^/]^0$

(c) $([a_{ij}, \lambda_j] \tilde{\cap} [b_{ij}, \lambda_j^/])^0 = [a_{ij}, \lambda_j]^0 \tilde{\cup} [b_{ij}, \lambda_j^/]^0$

Proof: (a) $[[a_{ij}, \lambda_j]^0]^0 = [1 - a_{ij}, 1 - \lambda_j]^0 = [1 - (1 - a_{ij}), 1 - (1 - \lambda_j)] = [a_{ij}, \lambda_j]$

(b) $([a_{ij}, \lambda_j] \tilde{\cap} [b_{ij}, \lambda_j^/])^0 = [\min\{a_{ij}, b_{ij}\}, \max\{\lambda_j, \lambda_j^/\}]^0$

$$= [1 - \min\{a_{ij}, b_{ij}\}, 1 - \max\{\lambda_j, \lambda_j^/\}]$$

$$= [\max\{1 - a_{ij}, 1 - b_{ij}\}, \min\{1 - \lambda_j, 1 - \lambda_j^/\}]$$

$$= [1 - a_{ij}, 1 - \lambda_j] \tilde{\cup} [1 - b_{ij}, 1 - \lambda_j^/]$$

$$= [a_{ij}, \lambda_j]^0 \tilde{\cup} [b_{ij}, \lambda_j^/]^0$$

$(c) ([a_{ij}, \lambda_j] \tilde{\cup} [b_{ij}, \lambda_j^/])^0 = [\max\{a_{ij}, b_{ij}\}, \min\{\lambda_j, \lambda_j^/\}]^0$

$$= [1 - \max\{a_{ij}, b_{ij}\}, 1 - \min\{\lambda_j, \lambda_j^/\}]$$

$$= [\min\{1 - a_{ij}, 1 - b_{ij}\}, \max\{1 - \lambda_j, 1 - \lambda_j^/\}]$$

$$= [1 - a_{ij}, 1 - \lambda_j] \tilde{\cap} [1 - b_{ij}, 1 - \lambda_j^/]$$

$$= [a_{ij}, \lambda_j]^0 \tilde{\cap} [b_{ij}, \lambda_j^/]^0$$

Proportion 3.8

Let $[a_{ij}, \lambda_j], [b_{ij}, \gamma_j]$ and $[c_{ij}, \varphi_j] \in GFSM_{m \times n}$. Then

(i) $[a_{ij}, \lambda_j] \tilde{\cap} [b_{ij}, \gamma_j] = [b_{ij}, \lambda_j] \tilde{\cap} [a_{ij}, \gamma_j]$

(ii) $[a_{ij}, \lambda_j] \tilde{\cup} [b_{ij}, \gamma_j] = [b_{ij}, \lambda_j] \tilde{\cup} [a_{ij}, \gamma_j]$

(iii) $[a_{ij}, \lambda_j] \tilde{\cap} ([b_{ij}, \gamma_j] \tilde{\cap} [c_{ij}, \varphi_j]) = ([a_{ij}, \lambda_j] \tilde{\cap} [b_{ij}, \gamma_j]) \tilde{\cap} [c_{ij}, \varphi_j]$

(iv) $[a_{ij}, \lambda_j] \tilde{\cup} ([b_{ij}, \gamma_j] \tilde{\cup} [c_{ij}, \varphi_j]) = ([a_{ij}, \lambda_j] \tilde{\cup} [b_{ij}, \gamma_j]) \tilde{\cup} [c_{ij}, \varphi_j]$

(v) $[a_{ij}, \lambda_j] \tilde{\cup} ([b_{ij}, \gamma_j] \tilde{\cap} [c_{ij}, \varphi_j]) = ([a_{ij}, \lambda_j] \tilde{\cup} [b_{ij}, \gamma_j]) \tilde{\cap} ([a_{ij}, \lambda_j] \tilde{\cup} [c_{ij}, \varphi_j])$

(vi) $[a_{ij}, \lambda_j] \tilde{\cap} ([b_{ij}, \gamma_j] \tilde{\cup} [c_{ij}, \varphi_j]) = ([a_{ij}, \lambda_j] \tilde{\cap} [b_{ij}, \gamma_j]) \tilde{\cup} ([a_{ij}, \lambda_j] \tilde{\cap} [c_{ij}, \varphi_j])$

IV. Product of Generalized Fuzzy Soft Matrices

In these section we define four different product of Generalized Intuitionistic Fuzzy Soft Matrices. Cagman et al. [2] defined four different types of product of fozzy soft matrices. We extend these four products to Generalized Intuitionistic Fuzzy Soft Matrices and study some of its properties.

Definition 4.1

Let $[a_{ij}, \lambda_j]$ and $[b_{ik}, \gamma_k] \in GFSM_{m \times n}$. Then $And - product \ of \ [a_{ij}, \lambda_j]$ and $[b_{ik}, \gamma_k]$ is defined by

$$\wedge : GFSM_{m \times n} \times GFSM_{m \times n} \rightarrow GFSM_{m \times n^2} \text{ and } [a_{ij}, \lambda_j] \wedge [b_{ik}, \gamma_k] = [c_{ip}, \psi_p]$$

where $c_{ip} = \min\{a_{ij}, b_{ik}\}$ and $\psi_p = \max\{\lambda_j, \gamma_k\}$ such that $p = n(j-1) + k$.

Definition 4.2

Let $[a_{ij}, \lambda_j]$ and $[b_{ik}, \gamma_k] \in GFSM_{m \times n}$. *Then Or $-$ product of* $[a_{ij}, \lambda_j]$ and $[b_{ik}, \gamma_k]$ is defined by

$$\vee : GFSM_{m \times n} \times GFSM_{m \times n} \to GFSM_{m \times n^2} \text{ and } [a_{ij}, \lambda_j] \vee [b_{ik}, \gamma_k] = [c_{ip}, \psi_p]$$

where $c_{ip} = \max\{a_{ij}, b_{ik}\}$ and $\psi_p = \min\{\lambda_j, \gamma_k\}$ such that $p = n(j-1) + k$.

Definition 4.3

Let $[a_{ij}, \lambda_j]$ and $[b_{ik}, \gamma_k] \in GFSM_{m \times n}$. *Then And $-$ Not $-$ product of* $[a_{ij}, \lambda_j]$ and $[b_{ik}, \gamma_k]$ is defined by

$$\overline{\wedge} : GFSM_{m \times n} \times GFSM_{m \times n} \to GFSM_{m \times n^2} \text{ and } [a_{ij}, \lambda_j] \overline{\wedge} [b_{ik}, \gamma_k] = [c_{ip}, \psi_p]$$

where $c_{ip} = \min\{a_{ij}, 1 - b_{ik}\}$ and $\psi_p = \max\{\lambda_j, 1 - \gamma_k\}$ such that $p = n(j-1) + k$.

Definition 4.4

Let $[a_{ij}, \lambda_j]$ and $[b_{ik}, \gamma_k] \in GFSM_{m \times n}$. *Then Or $-$ Not $-$ product of* $[a_{ij}, \lambda_j]$ and $[b_{ik}, \gamma_k]$ is defined by

$$\overline{\vee} : GFSM_{m \times n} \times GFSM_{m \times n} \to GFSM_{m \times n^2} \text{ and } [a_{ij}, \lambda_j] \overline{\vee} [b_{ik}, \gamma_k] = [c_{ip}, \psi_p]$$

where $c_{ip} = \max\{a_{ij}, 1 - b_{ik}\}$ and $\psi_p = \min\{\lambda_j, 1 - \gamma_k\}$ such that $p = n(j-1) + k$.

Example 4.5

Assume that $[a_{ij}, \lambda_j]$, $[b_{ik}, \gamma_k] \in GFSM_{2 \times 3}$ are given as follows:

$$[a_{ij}, \lambda_j] = \begin{bmatrix} (0.4, 0.5) & (0.5, 0.7) & (0.2, 0.6) \\ (0.5, 0.5) & (0.7, 0.7) & (0.3, 0.6) \\ (0.6, 0.5) & (0.2, 0.7) & (0.4, 0.6) \end{bmatrix} \text{ and } [b_{ik}, \gamma_k] = \begin{bmatrix} (0.7, 0.3) & (0.2, 0.5) & (0.3, 0.4) \\ (0.4, 0.3) & (0.3, 0.5) & (0.2, 0.4) \\ (0.5, 0.3) & (0.6, 0.5) & (0.5, 0.4) \end{bmatrix}$$

Then

$[a_{ij}, \lambda_j] \wedge [b_{ij}, \gamma_k]$

$$= \begin{bmatrix} (0.4,0.5) & (0.2,0.5) & (0.3,0.5) & (0.5,0.7) & (0.2,0.7) & (0.2,0.7) & (0.2,0.6) & (0.2,0.6) & (0.2,0.6) \\ (0.4,0.5) & (0.3,0.5) & (0.2,0.5) & (0.4,0.7) & (0.3,0.7) & (0.2,0.7) & (0.3,0.6) & (0.3,0.6) & (0.2,0.6) \\ (0.5,0.5) & (0.6,0.5) & (0.5,0.5) & (0.2,0.7) & (0.2,0.7) & (0.2,0.7) & (0.4,0.6) & (0.4,0.6) & (0.4,0.6) \end{bmatrix}$$

Similarly, we can also find the other products $[a_{ij}, \lambda_j] \vee [b_{ij}, \gamma_k], [a_{ij}, \lambda_j] \overline{\vee} [b_{ij}, \gamma_k]$ *and* $[a_{ij}, \lambda_j] \overline{\wedge} [b_{ij}, \gamma_k]$ of

generalized fuzzy soft matrices.

Note that commutativity is not valid for the product of generalized fuzzy soft matrices.

Proposition 4.6

Let $[a_{ij}, \lambda_j]$ and $[b_{ik}, \gamma_k] \in GFSM_{m \times n}$. *Then the following are true*

$$(i)\ ([a_{ij}, \lambda_j] \vee [b_{ij}, \gamma_k])^0 = [a_{ij}, \lambda_j]^0 \wedge [b_{ij}, \gamma_k]^0$$

$$(ii)\ ([a_{ij}, \lambda_j] \wedge [b_{ij}, \gamma_k])^0 = [a_{ij}, \lambda_j]^0 \vee [b_{ij}, \gamma_k]^0$$

$$(iii)\ ([a_{ij}, \lambda_j] \overline{\vee} [b_{ij}, \gamma_k])^0 = [a_{ij}, \lambda_j]^0 \overline{\wedge} [b_{ij}, \gamma_k]^0$$

$$(iv)\ ([a_{ij}, \lambda_j] \overline{\wedge} [b_{ij}, \gamma_k])^0 = [a_{ij}, \lambda_j]^0 \overline{\vee} [b_{ij}, \gamma_k]^0$$

V. A Generalized Fuzzy Matrices Based Decision Making Problem

Let more than one decision makers want to take a decision or to select an object jointly from the 'm' number of objects which have 'n' number of features i.e., parameters (E). Suppose that each decision maker has freedom to take his decision of inclusion and evaluation of parameters associated with the selected object or may choice the same set of parameters i.e., each decision maker has his own choice parameters belonging to the parameter set E and has his own view of evaluation. Here it is assumed that the parameter evaluation of the objects by the decision makers must be generalized fuzzy and may be presented in linguistic form or generalized fuzzy soft set format, alternatively, in the form of generalized fuzzy soft matrices. Now the aim of the decision makers is to find out the object out of 'm' object jointly as far as possible.

Definition 5.1

Let $[c_{ij}, \gamma_j] \in \tilde{A} \tilde{\cap} \tilde{B}$ where $\tilde{A}$ and $\tilde{B}$ are two generalized fuzzy soft matrices. Then the set

$$W(u_i) = \{u_i \in U \mid \sum c_{ij}\}$$ is called weight for each $u_i \in U$.

Procedure of solving problem:

A real life problem can be solved by using different Mathematical methods. The decision maker can choose the easiest method from the alternative. After taking decision by the decision makers that, they solve a particular problem by using the operation of generalized soft matrices, the decision makers go through the following algorithm.

Algorithm:

Step 1: To construct the generalized fuzzy soft matrices with respect to their own choice parameters of the decision makers.

Step 2: To compute the union or intersection of generalised fuzzy soft matrices.

Step 3: To compute the weight of each object (O_i) by adding the membership values of the entries of its concerned row (i^{th}-row) of the union or intersection of generalised fuzzy soft matrices.

Step 4: The object having the highest weight becomes the optimal choice object.

To illustrate the basic idea of the GFSM-algorithm, now we apply it to the following generalized fuzzy soft set (or generalized fuzzy soft matrix) based decision making problems.

Case Study:

Generally in medical science a patient suffering from a disease may have multiple symptoms. Again it is also observed that there are certain symptoms which may be common to more than one diseases leading to diagnostic dilemma. Sometimes the doctor has to face many problems when an area is largely affected new disease. Then the doctor has detected the disease commencing the common symptoms of the patients.

Let $U = \{p_1, p_2, p_3, p_4\}$ be the set of patients and $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$ be the set of parameters of symptoms of dengue where

$e_1 =$ high fever (104 F, 40°C), $e_2 =$ headache, $e_3 =$ extreme fatigue

$e_4 =$ red eyes and pain in the eyes, $e_5 =$ enlarged lymph nodes,

$e_6 =$ deep muscle and joint pains (during first hours of illness)

$e_7 =$ nausea and vomiting, $e_8 =$ low pressure and heart rate.

Suppose two Dr. X and Dr. Y examines the patients on based on the same set of parameters.
Let $\lambda : E \rightarrow [0,1]$. They consider the function on the parameters as follows:
Dr. X grade on the parameters as

$\lambda(e_1) = 0.7, \lambda(e_2) = 0.4, \lambda(e_3) = 0.3, \lambda(e_4) = 0.5, \lambda(e_5) = 0.6, \lambda(e_6) = 0.5, \lambda(e_7) = 0.7, \lambda(e_8) = 0.2$

and the Dr. Y grade on the parameters as

$\lambda(e_1) = 0.7, \lambda(e_2) = 0.5, \lambda(e_3) = 0.4, \lambda(e_4) = 0.3, \lambda(e_5) = 0.6, \lambda(e_6) = 0.4, \lambda(e_7) = 0.7, \lambda(e_8) = 0.3.$

Step 1: Based on these functions the two doctors construct generalized matrices for the patients $\tilde{A}$ and $\tilde{B}$ respectively given as follows:

$$\tilde{A} = \begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{bmatrix} (0.7,0.8) & (0.6,0.6) & (0.8,0.3) & (0.6,0.5) & (0.7,0.4) & (0.5,0.5) & (0.7,0.7) & (0.4,0.2) \\ (0.6,0.8) & (0.5,0.6) & (0.3,0.3) & (0.1,0.5) & (0.1,0.4) & (0.6,0.5) & (0.2,0.7) & (0.5,0.2) \\ (0.5,0.8) & (0.4,0.6) & (0.2,0.3) & (0.2,0.5) & (0.2,0.4) & (0.3,0.5) & (0.4,0.7) & (0.3,0.2) \\ (0.3,0.8) & (0.7,0.6) & (0.5,0.3) & (0.3,0.5) & (0.4,0.4) & (0.2,0.5) & (0.3,0.7) & (0.5,0.2) \end{bmatrix}$$

$$\tilde{B} = \begin{array}{c} p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{bmatrix} (0.7,0.7) & (0.6,0.5) & (0.8,0.4) & (0.6,0.3) & (0.7,0.6) & (0.6,0.4) & (0.8,0.7) & (0.5,0.3) \\ (0.5,0.7) & (0.2,0.5) & (0.4,0.4) & (0.4,0.3) & (0.2,0.6) & (0.5,0.4) & (0.3,0.7) & (0.4,0.3) \\ (0.4,0.7) & (0.3,0.5) & (0.5,0.4) & (0.1,0.3) & (0.3,0.6) & (0.2,0.4) & (0.2,0.7) & (0.2,0.3) \\ (0.2,0.7) & (0.4,0.5) & (0.3,0.4) & (0.2,0.3) & (0.2,0.6) & (0.1,0.4) & (0.4,0.7) & (0.3,0.3) \end{bmatrix}$$

$$\tilde{A} \tilde{\cap} \tilde{B} = \begin{bmatrix} (0.7,0.8) & (0.6,0.6) & (0.8,0.4) & (0.6,0.5) & (0.7,0.6) & (0.5,0.5) & (0.7,0.7) & (0.4,0.3) \\ (0.5,0.8) & (0.2,0.6) & (0.3,0.4) & (0.1,0.5) & (0.1,0.6) & (0.5,0.5) & (0.2,0.7) & (0.4,0.3) \\ (0.4,0.8) & (0.3,0.6) & (0.2,0.4) & (0.1,0.5) & (0.2,0.6) & (0.2,0.5) & (0.2,0.7) & (0.2,0.3) \\ (0.2,0.8) & (0.4,0.6) & (0.3,0.4) & (0.2,0.5) & (0.2,0.6) & (0.1,0.5) & (0.3,0.7) & (0.3,0.3) \end{bmatrix}$$

Step 3: Now the weights of the patients are respectively,

$$W(p_1) = 0.7 + 0.6 + 0.8 + 0.6 + 0.7 + 0.5 + 0.7 + 0.4 = 5.0$$
$$W(p_2) = 0.5 + 0.2 + 0.3 + 0.1 + 0.1 + 0.5 + 0.2 + 0.3 = 2.2$$
$$W(p_3) = 0.4 + 0.3 + 0.2 + 0.1 + 0.2 + 0.2 + 0.2 + 0.2 = 1.8$$
$$W(p_4) = 0.2 + 0.4 + 0.3 + 0.2 + 0.2 + 0.1 + 0.3 + 0.3 = 2.0$$

Step 4: The maximum weight (5.0) obtains by patient is p_1.

Hence the patient p_1 has suffered in dengue.

VI. Conclusion

In this paper, we have introduced generalized fuzzy soft matrices and some of their properties. We also defined product of generalized fuzzy soft matrices and some of their properties. Then we defined weighted generalized fuzzy soft fuzzy soft sets. Finally we presented an application of generalized soft matrices in medical diagnosis.

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