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Some Results Concerning Memomorphic Matrix Valued Functions.

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Abstract: *In this paper we have extended some basic results of Nevanlinna theory to Matrix valued meromorphic functions.*
Key words*:Nevanlinna theory,Matrix valued meromorphic functions.*

I. Preliminaries:

By a meromorphic function we shall always mean a transcendental meromorphic function in the plane. If f is a meromorphic function, $a \in \overline{C}$ and r > 0, we use the following notations of frequent use in Nevanlinna theory with their usual meaning: $m(r,a,f) = m\left(r\frac{1}{f-a}\right)$, $n(r,a,f) = n\left(r,\frac{1}{f-a}\right)$,

$\bar{n}(r,a,f)$, $N(r,a,f)$, $\bar{N}(r,a,f)$, $T(r,f)$,
$\delta(a,f), \Delta(a,f), \Theta(a,f)$ etc..as in [2]

As usual, if a = ∞, then by a zero of f-a, we mean a pole of f.

We define a meromorphic matrix valued function as in [1].

By a matrix valued meromorphic function A(z) we mean a matrix all of whose entries are meromorphic on the whole (finite) complex plane.

A complex number z is called a pole of A(z) if it is a pole of one of the entries of A(z), and z is called a zero of A(z) if it is a pole of $[A(z)]^{-1}$.

For a meromorphic m × m matrix valued function A(z),

let $$m(r,A) = \frac{1}{2\pi}\int_0^{2\pi} \log \| A(re^{i\theta}) \| \, d\theta \tag{1}$$

where A has no poles on the circle $|z| = r$.

Here, $$\| A(z) \| = \max_{\substack{\|x\| = 1 \\ x \in C^n}} \| A(z)x \|$$

Set $$N(r,A) = \int_0^r \frac{n(t,A)}{t}\, dt \tag{2}$$

where n(t, A) denotes the number of poles of A in the disk $\{z : |z| \le t\}$, counting multiplicities.

Let T(r, A) = m(r, A) + N(r, A)

Definition : Let A = $[a_{ij}]_{i,j=1}^{m}$ be a memomorphic matrix valued function of finite order. Let a $\in$C.

Let $$\delta(a_{ij}) = \delta(a,a_{ij}) = \lim_{r\to\infty}\frac{m(r,a)}{T(r,a_{ij})} = \overline{\lim_{r\to\infty}}\frac{N(r,a)}{T(r,a_{ij})}$$

And $$\theta(a_{ij}) = \theta(a,a_{ij}) = 1 - \overline{\lim_{r\to\infty}}\frac{N(r,a)}{T(r,a_{ij})}$$

Also, Let $\delta(a,A) = \underset{1\le i,j\le m}{Max}\,\delta(a_{ij}) = \underset{1\le i,j\le m}{Max}\,\delta(a,a_{ij})$

and $\theta(a,A) = \underset{1\le i,j\le m}{Max}\,\theta(a_{ij}) = \underset{1\le i,j\le m}{Max}\,\theta(a,a_{ij})$

We wish to prove the following results.

Theorem 1 : Let A = $[a_{ij}]_{i,j=1}^{m}$ be a meromorphic matrix valued functions of finite order. If a1, a2, a3, ….. a_q are distinct numbers, then $\forall 1\le i,\ j\le m.$

$$\sum_{k=1}^{q} m(r,a_{k,}a_{ij}) + N\left(r,\frac{1}{a_{ij}}\right) \le T(r,a_{ij}^{'}) + S(r,a_{k,}a_{ij})$$

Where S(r, a_k, a_{ij})=S(r, a_{ij})= 0 { T(r, a_{ij}) } r $\to \infty$ through all values of a_{ij}.

Theorem 2 : Let A = $[a_{ij}]_{i,j=1}^{m}$ be a meromorphic matrix valued function of finite order. Then,

$$\lim_{r\to\infty}\frac{T(r,A^{'})}{T(r,A)} \ge \sum_{a\in C}\theta(a,A)$$

Theorem 3 Let A = $[a_{ij}]_{i,j=1}^{m}$ be meromorphoic matrix valued function of finite order.

Then, $\lim_{r\to\infty}\frac{T(r,A^{'})}{T(r,A)} \le 2-\theta(\infty,A)$

Proof of theorem 1:

Without loss of generality, let us assume that for q $\ge$ 2,

F(z) = $\sum_{k=1}^{q}\frac{1}{a_{ij}-a_k}$, $1\le i,j\le m$.

Then by a known result [Hayman, p.33], we have

$$M(r,F) \ge \sum_{k=1}^{q} m(r,a_{k,}a_{ij}) - q\log + \frac{3q}{\delta} - \log 2 \quad \text{, where } \delta = \min_{k_1\ne k_2}\left|a_{k_1}-a_{k_2}\right|$$

Therefore, $\sum_{k=1}^{q} m(r,a_{k,}a_{ij}) \le m(r,F) + O(1)$

$$= m\left(r,\frac{1}{a_{ij}^{'}}.F.a_{ij}^{'}\right) + O(1)$$

$$\le m\left(r,\frac{1}{a_{ij}^{'}}\right) + m(r,F.a_{ij}^{'}) + O(1)$$

$$\le m\left(r,\frac{1}{a_{ij}^{'}}\right) + m\left(r,\sum_{k=1}^{q}\frac{a_{ij}^{'}}{a_{ij-}a_k}\right) + O(1)$$

$$\le m\left(r,\frac{1}{a_{ij}^{'}}\right) + \sum_{k=1}^{q} m\left(r,\frac{a_{ij}^{'}}{a_{ij}-a_k}\right) + O(1)$$

$$= m\left(r,\frac{1}{a_{ij}^{'}}\right) + S(r,a_{ij})$$

Thus, $\sum_{k=1}^{q} m(r, a_{k,} a_{ij}) + N\left(r, \frac{1}{a_{ij}^{'}}\right) \le T\left(r, \frac{1}{a_{ij}^{'}}\right) + S(r, a_{ij})$

$$\le T\left(r, \frac{1}{a_{ij}^{'}}\right) + S(r, a_{ij})$$

Hence, the result.

Proof of theorem 2:

Let $\{a_{ij}\}_{i=1}^{\infty}$ be an infinite sequence of distinct elements of C which includes for each a $\in$ C, θ (a, a_{ij}) > 0.

Then, $\sum_{a\in c}^{q} \theta(a_{i,} a_{ij}) = \sum_{a\in c} \theta(a, a_{ij})$

Let q be a positive integer. Then by previous theorem,

$$\sum_{k=1}^{q} m(r, a_{k,} a_{ij}) + N\left(r, \frac{1}{a_{ij}}\right) \le T(r, a_{ij}^{'}) + S(r, a_{k,} a_{ij})$$

Adding $\sum_{k=1}^{q} N(r, a_k, a_{ij})$ both sides, we get

$$\sum_{k=1}^{q} T\left(\frac{1}{a_{ij} - a_k}\right) \le T(r, a_{ij}^{'}) + \sum_{k=1}^{q} N(r, a_{k,} a_{ij}) - N\left(r, \frac{1}{a_{ij}^{'}}\right) + S(r, a_{ij})$$

$$= T(r, a_{ij}^{'}) + \sum_{k=1}^{q} N(r, a_{k,} a_{ij}) - N_0\left(r, \frac{1}{a_{ij}^{'}}\right) + S(r, a_{ij})$$

where $N_0\left(r, \frac{1}{a_{ij}^{'}}\right)$ is formed with the zeros of $a_{ij}^{'}$ which are not zeros of any of $a_{ij} - a_k$.

Since $N_0\left(r, \frac{1}{a_{ij}^{'}}\right) \ge 0$, We have

$$\sum_{k=1}^{q} T\left(r, \frac{1}{a_{ij} - a_k}\right) \le T(r, a_{ij}^{'}) + \sum_{k=1}^{q} \overline{N}(r, a_{k,} a_{ij}) + S(r, a_{ij}) \qquad (*)$$

Now, $T\left(r, \frac{1}{a_{ij} - a_k}\right) \le T(r, a_{ij}) + O(\text{Log } r)$

$= T(r, a_{ij}) + o\{T(r, a_{ij})\}$ as $r \to \infty$

Thus (*) takes the form

$$q\, T(r, a_{ij}) < T(r, a_{ij}^{'}) + \sum_{k=1}^{q} \overline{N}(r, a_{k,} a_{ij}) + S(r, a_{ij})$$

Hence, $q \le \lim_{r\to\infty} \frac{T(r, a_{ij}^{'})}{T(r, a_{ij})} + \sum_{k=1}^{q} \overline{\lim_{r\to\infty}} \frac{\overline{N}(r, a_{k,} a_{ij})}{T(r, a_{ij})} + \overline{\lim_{r\to\infty}} \frac{S(r, a_{ij})}{T(r, a_{ij})}$

Therefore, $q \le \lim_{r\to\infty} \frac{T(r, a_{ij}^{'})}{T(r, a_{ij})} + \sum_{k=1}^{q} \{1 - \theta(a_k, a_{ij})\} + \overline{\lim_{r\to\infty}} \frac{S(r, a_{ij})}{T(r, a_{ij})}$

Taking maximum over $1 \le i, j \le m$, we get

$$\sum_{k=1}^{q} \theta(a_k, A) \le \lim_{r\to\infty} \frac{T(r, A^{'})}{T(r, A)}$$

Hence the result.

Proof of theorem 3:

From [1],We know that $m\left(r, \frac{f'}{f}\right)$ = S(r, f)

Therefore, m(r, a'_{ij}) ≤ m(r, a_{ij}) + S(r, a_{ij})

Also, N(r, a'_{ij}) = N(r, a_{ij}) + $\overline{N}$ (r, a_{ij})

Hence, T(r, a'_{ij}) = T(r, a_{ij}) + $\overline{N}$ (r, a_{ij}) + S(r, a_{ij})

Therefore, $\frac{T(r, a'_{ij})}{T(r, a_{ij})} \leq 1 + \overline{N}(r, a_{ij}) + O(1)$

Therefore, $\lim\limits_{r \to \infty} \frac{T(r, a'_{ij})}{T(r, a_{ij})} \leq 2 - \theta(\infty, a_{ij})$

Taking maximum over 1 ≤ i, j ≤ m, we get $\overline{\lim\limits_{r \to \infty}} \frac{T(r, A')}{T(r, A)} \quad 2 - \theta(\infty, A)$

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