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Fuzzy Sub Lattice Ordered Ternary near Rings

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Abstract: *The concept of fuzzy sub ℓ-ternary near ring of a ℓ-ternary near ring and some examples are presented. Some of their properties are derived.*
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I. Introduction

The notion of fuzzy sets was introduced by Loffi.A. Zadeh [16] in 1965. Zadeh had initiated fuzzy set theory as a modification of the ordinary set theory. The notion of fuzzy subgroups was made by Rosenfeld [12] in 1971. In 1982, Fuzzy ideals in rings were introduced by W. Liu [8]. Abou-Zaid [1] introduced the notion of fuzzy sub near ring and studied fuzzy ideals of a near ring.

One of the structures that are most extensively used and discussed in the crisp mathematical theory is certainly the lattice structure. As it is well known it is considered as a relational ordered structure on one hand and as an algebra on the other hand. The concept of lattice was first defined by Dedekind in 1897 and then developed by G. Birkhoff [2] in 1933. Boole has also introduced Boolean algebra a special class of lattices. G.Brikhoff imposed an open problem "Is there a common abstraction which includes Boolean algebra and lattice ordered group as special cases?", where the lattice ordered group is an algebraic structure connecting lattice and group. In 1971 Piltz initiated the study of ordered near rings. Radhakrishna and Bhandari [11] introduced the concepts of lattice ordered near rings in 1977. In 1990, Yuan and Wu [15] applied the concept of fuzzy sets in lattice theory. After that Naseem and Thomas [10] developed the theory of fuzzy lattices.

The theory of ternary algebraic system was introduced by D.H.Lehmer [6] in 1932. He investigated certain ternary algebraic systems called triplexess which turn out to be commutative ternary groups. The notion of ternary semigroups was introduced by S. Banach [c.f [5]]. Dutta and kar [3] introduced the notion of ternary semiring which is a generalization of the ternary ring introduced by Lister [7]. To deal with the idea of near rings using ternary product Warud Nakkhaseen and Bundit Pibaljommee [14] have applied the concept of ternary semiring to defi-ned ternary near rings, ideals of ternary near rings, L-fuzzy sub near rings, L-fuzzy ideals of ternary near rings.

Keeping these in mind, we develop the algebraic nature of fuzzy subset especially fuzzy sub lattice ordered ternary near rings.

II. Preliminaries

Definition 2.1.

A nonempty set T together with a ternary operation [ ] : $T\times T\times T \rightarrow T$ is called a ternary semigroup if for every x, y, z, u, v $\in$T, [[xyz]uv] = [x[yzu]v] = [xy[zuv]].

Definition 2.2.

Let A,B,C be three nonempty subsets of a ternary semigroup. Then, [ABC] = $\{$[abc] : a$\in$A, b$\in$B, c$\in$ C$\}$.

Definition 2.3.

A nonempty subset I of a ternary semigroup (T, [ ]) is called an ideal of the ternary semigroup T if
(i) [TTI] $\subseteq$ I;
(ii) [ITT] $\subseteq$ I;
(iii) [TIT] $\subseteq$ I,
where [ABC] = $\{$[abc] : a$\in$A, b$\in$B, c$\in$C$\}$, for any nonempty subsets A,B,C$\in$T.

If I satisfies (i), then I is called a left ideal of T, if I satisfies (ii), then I is called a right ideal of T, and if I satisfies (iii), then I is called a lateral ideal of T.

Definition 2.4.

A triplet ($\aleph$,+, [ ]) consisting of a nonempty set $\aleph$, a binary operation "+" and a ternary operation [ ] on $\aleph$ is called a ternary near ring of $\aleph$ if,
(i) ($\aleph$, +) is a group;

(ii) $(\aleph, [\ ])$ is a ternary semigroup;
(iii) $[ab(c + d)] = [abc] + [abd]$, for all a, b, $c \in \aleph$.
Example 2.5.

Let $\aleph = \{a, b, c, d\}$ be a set with a binary operation as follows:

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

Define the ternary operation $[\ ] : \aleph^3 \rightarrow \aleph$ by $[xyz] = z$ for all $x, y, z \in \aleph$. Then $(\aleph, +, [\ ])$ is a ternary near ring.
Definition 2.6.

A nonempty subset S of a ternary near ring $\aleph$ is called a ternary sub near ring of $\aleph$ if
(i) $(S,+)$ is a subgroup of $(\aleph,+)$;
(ii) $[SSS] \subseteq S$.
Remark 2.7.

(1) Arbitrary intersection of ternary sub near rings of a ternary near ring is a ternary sub near ring.

(2) The arbitrary intersection of ternary ideals of a ternary near ring is again a ternary ideal of the ternary near ring.
Definition 2.8.

A nonempty set $\mathcal{L}_{\aleph}$ together with three binary operations $+, \vee, \wedge$ and a ternary operation [ ] defined on it is called a lattice ordered ternary near ring (or ℓ-ternary near ring) if the following conditions are satisfied:

(i) $(\mathcal{L}_{\aleph}, +, [\ ])$ is a ternary near ring;
(ii) $(\mathcal{L}_{\aleph}, \vee, \wedge)$ is a lattice;
(iii) $x + (y \vee z) = (x + y) \vee (x + z)$;
$(y \vee z) + x = (y + x) \vee (z + x)$;
$x + (y \wedge z) = (x + y) \wedge (x + z)$;
$(y \wedge z) + x = (y + x) \wedge (z + x)$, for all $x, y, z \in \mathcal{L}_{\aleph}$.
(iv) $[xy(z \vee z_1)] = [xyz] \vee [xyz_1]$;
$[x(y \vee z)z_1] = [xyz_1] \vee [xzz_1]$;
$[(x \vee y)zz_1] = [xzz_1] \vee [yzz_1]$;
$[xy(z \wedge z_1)] = [xyz] \wedge [xyz_1]$;
$[x(y \wedge z)z_1] = [xyz_1] \wedge [xzz_1]$;
$[(x \wedge y)zz_1] = [xzz_1] \wedge [yzz_1]$, for all $x, y, z, z_1 \in \mathcal{L}_{\aleph}$.
Example 2.9

Consider the set $\mathbb{R}$ of all real number. Define three binary operations "+", "$\vee$" and "$\wedge$" on $\mathbb{R}$ by "+" the usual addition, $a \vee b = \max\{a, b\}$and $a \wedge b = \min\{a, b\}$and define a ternary operation [ ] on $\mathbb{R}$ by $[abc] = c$, for all $a, b, c \in \mathbb{R}$. Then $(\mathbb{R}, +, \vee, \wedge, [\ ])$ is a ℓ-ternary near ring.
Definition 2.10.

Let $(\mathcal{L}_{\aleph}, +, [\ ], \vee, \wedge)$ be a ℓ-ternary near ring. A nonempty subset S of $\mathcal{L}_{\aleph}$ is called a sub ℓ-ternary near ring of $\mathcal{L}_{\aleph}$ if $x-y$, $[xyz]$, $x\vee y$, $x\wedge y \in S$, for all $x, y, z \in S$.
Example 2.11.

Consider the set $\mathbb{Z}$ of all integer, Define three binary operations "+", "$\vee$" and "$\wedge$" on $\mathbb{Z}$ by " + " the usual addition, $a \vee b = \max\{a, b\}$ and $a \wedge b = \min\{a, b\}$ and define a ternary operation [ ] on $\mathbb{Z}$ by $[abc] = c$, for all $a, b, c \in \mathbb{Z}$. Then $(\mathbb{Z}, +, [\ ], \vee, \wedge)$ is a sub ℓ-ternary near ring.

Now, we review some fuzzy logic concepts: Let X be a nonempty set. A fuzzy subset α of a set X is a function $\alpha : X \rightarrow [0,1]$. For any $t \in [0; 1]$, the set $\alpha_t = \{x \in X : \alpha(x) \geq t\}$ is called a level subset of α. Let α_1 and α_2 be any two fuzzy subsets of X. Then α_1 is said to be contained in α_2, denoted by $\alpha_1 \subseteq \alpha_2$, if $\alpha_1(x) \leq \alpha_2(x)$ for all $x \in X$. If $\alpha_1(x) = \alpha_2(x)$, for all $x \in X$, then α_1 and α_2 are said to be equal and we write $\alpha_1 = \alpha_2$. If $(\alpha_1 \cup \alpha_2)(x)$ and$(\alpha_1 \cap \alpha_2)(x)$ are fuzzy subsets of X defined by $(\alpha_1 \cup \alpha_2)(x) = \max\{\alpha_1(x),\alpha_2(x)\}$ and $(\alpha_1 \cap \alpha_2)(x) = \min\{\alpha_1(x), \alpha_2(x)\}$.
Lemma 2.12.

Let α be a fuzzy set in a lattice X. Then for all $x, y \in X$ the followings are equivalent:
(i) $\alpha(x) \geq \alpha(y)$ whenever $x \leq y$;
(ii) $\alpha(x \vee y) \leq \min\{\alpha(x), \alpha(y)\}$;

(iii) $\alpha(x \wedge y) \geq \max\{\alpha(x), \alpha(y)\}$.

III. Fuzzy sub ℓ-ternary near ring

In this section, we see the notion of fuzzy sub ℓ-ternary near ring in a ℓ-ternary near ring.

Definition 3.1.

Let α be a fuzzy subset on a ℓ-ternary near ring $\mathcal{L}_\aleph$. Then α is called a fuzzy sub ℓ-ternary near ring of $\mathcal{L}_\aleph$, if it satisfies the following conditions:

(i) $\alpha(x-y) \geq \min\{\alpha(x), \alpha(y)\}$;

(ii) $\alpha[xyz]) \geq \min\{\alpha(x), \alpha(y), \alpha(z)\}$;

(iii) $\alpha(x \vee y) \geq \min\{\alpha(x), \alpha(y)\}$;

(iv) $\alpha(x \wedge y) \geq \min\{\alpha(x), \alpha(y)\}$, for all $x, y, z \in \mathcal{L}_\aleph$.

Example 3.2.

Consider the ℓ-ternary near ring $\mathcal{L}_\aleph = (\mathbb{R}, +, [\ ], \vee, \wedge)$ defined as in example 2.9. We define the fuzzy subset on $\mathcal{L}_\aleph$ by $\alpha(x) = \begin{cases} 0.7 & \text{if } x \in 3\mathbb{Z} \\ 0.2 & \text{if } x \notin 3\mathbb{Z} \end{cases}$. Then α is a fuzzy sub ℓ-ternary near ring of $\mathcal{L}_\aleph$.

Lemma 3.3.

If α is a fuzzy sub ℓ-ternary near ring of a ℓ-ternary near ring $\mathcal{L}_\aleph$, then $\alpha(x) \leq \alpha(0)$, for all $x \in \mathcal{L}_\aleph$.

Lemma 3.4.

If α is a fuzzy sub ℓ-ternary near ring of a ℓ-ternary near ring $\mathcal{L}_\aleph$, then $\alpha(x) = \alpha(-x)$, for all $x \in \mathcal{L}_\aleph$.

Lemma 3.5.

If α is a fuzzy sub ℓ-ternary near ring of a ℓ-ternary near ring $\mathcal{L}_\aleph$, then $\alpha(x + y) \geq \min\{\alpha(x), \alpha(y)\}$ for all $x, y \in \mathcal{L}_\aleph$.

Theorem 3.6.

Let S be any nonempty proper subset of a ℓ-ternary near ring $\mathcal{L}_\aleph$, and let α be a fuzzy subset of $\mathcal{L}_\aleph$, defined by $\alpha(x) = \begin{cases} g & \text{if } x \in S \\ h & \text{if } x \in \mathcal{L}_\aleph - S \end{cases}$ where $g, h \in [0; 1]$ with $0 \neq g > h \neq 1$. Then α is a fuzzy sub ℓ-ternary near ring of $\mathcal{L}_\aleph$ if and only if S is a sub ℓ-ternary near ring of $\mathcal{L}_\aleph$.

Proof:

Assume that α is a fuzzy sub ℓ-ternary near ring of $\mathcal{L}_\aleph$ and let $x, y, z \in S$ be arbitrary. Then $\alpha(x) = \alpha(y) = \alpha(z) = g$ and $\min\{\alpha(x), \alpha(y)\} = \min\{\alpha(x), \alpha(y), \alpha(z)\}= g$. Therefore $\alpha(x-y)$, $\alpha([xyz])$, $\alpha(x \vee y)$ and $\alpha(x \wedge y)$ are greater than or equal to g. But α has only two values g and h with $g > h$. So all the values of $\alpha(x-y)$, $\alpha([xyz])$, $\alpha(x \vee y)$ and $\alpha(x \wedge y)$ are equal to g. Therefore $x-y$, $[xyz]$, $x \vee y$ and $x \wedge y \in S$. Hence S is a sub ℓ-ternary near ring of $\mathcal{L}_\aleph$. Conversely, assume that S is a sub ℓ-ternary near ring of $\mathcal{L}_\aleph$. To prove that α is a fuzzy sub ℓ-ternary near ring of $\mathcal{L}_\aleph$, it is satisfy the following axioms:

(i) $\alpha(x-y) \geq \min\{\alpha(x), \alpha(y)\}$;

(ii) $\alpha([xyz]) \geq \min\{\alpha(x), \alpha(y), \alpha(z)\}$;

(iii) $\alpha(x \vee y) \geq \min\{\alpha(x), \alpha(y)\}$;

(iv) $\alpha(x \wedge y) \geq \min\{\alpha(x), \alpha(y)\}$, for all $x, y, z \in \mathcal{L}_\aleph$.

Let $x, y, z \in \mathcal{L}_\aleph$ be arbitrary.

Case(i)

Let $x, y, z \in S$. Then $\alpha(x) = \alpha(y) = \alpha(z) = g$, $\min\{\alpha(x), \alpha(y)\} = \min\{\alpha(x), \alpha(y), \alpha(z)\} = g$. Since S is a sub ℓ-ternary near ring, we have $x-y$, $[xyz]$, $x \vee y$, $x \wedge y \in S$ and so $\alpha(x-y) = \alpha([xyz]) = \alpha(x \vee y) = \alpha(x \wedge y) = g$. Therefore $\alpha(x-y) \geq \min\{\alpha(x), \alpha(y)\}$, $\alpha([xyz]) \geq \min\{\alpha(x), \alpha(y), \alpha(z)\}$, $\alpha(x \vee y) \geq \min\{\alpha(x), \alpha(y)\}$ and $\alpha(x \wedge y) \geq \min\{\alpha(x), \alpha(y)\}$. Hence all the inequalities are satisfied in this case.

Case(ii)

Let $x, y, z \in \mathcal{L}_\aleph - S$. Then $\alpha(x) = \alpha(y) = \alpha(z) = h$ and $\min\{\alpha(x), \alpha(y)\} = \min\{\alpha(x), \alpha(y), \alpha(z)\} = h$. Here $x-y$, $[xyz]$, $x \vee y$, $x \wedge y$ may either belongs to S or to $\mathcal{L}_\aleph - S$, so their images under α will either be g or h.

If $\alpha(x-y) = g$, then $\alpha(x-y) = g > h = \min\{\alpha(x), \alpha(y)\} \geq \min\{\alpha(x), \alpha(y)\}$. If $\alpha(x-y) = h$, then $\alpha(x-y) = h = \min\{\alpha(x), \alpha(y)\} \geq \min\{\alpha(x), \alpha(y)\}$. If $\alpha([xyz]) = g$, then $\alpha([xyz]) = g > h = \min\{\alpha(x), \alpha(y), \alpha(z)\} \geq \min\{\alpha(x), \alpha(y), \alpha(z)\}$. If $\alpha([xyz]) = h$, then $\alpha([xyz]) = h = \min\{\alpha(x), \alpha(y), \alpha(z)\} \geq \min\{\alpha(x), \alpha(y), \alpha(z)\}$.

If $\alpha(x \vee y) = g$, then $\alpha(x \vee y) = g > h = \min\{\alpha(x), \alpha(y)\} \geq \min\{\alpha(x), \alpha(y)\}$. If $\alpha(x \vee y) = h$, then $\alpha(x \vee y) = h = \min\{\alpha(x), \alpha(y)\} \geq \min\{\alpha(x), \alpha(y)\}$. If $\alpha(x \wedge y) = g$, then $\alpha(x \wedge y) = g > h = \min\{\alpha(x), \alpha(y)\} \geq \min\{\alpha(x), \alpha(y)\}$. If $\alpha(x \wedge y) = h$, then $\alpha(x \wedge y) = h = \min\{\alpha(x), \alpha(y)\} \geq \min\{\alpha(x), \alpha(y)\}$. Thus all the inequalities are satisfied in this case.

Case(iii)

Let x, y ∈ S and z ∈ $\mathcal{L}_{\aleph}$− S. Then α(x) = α(y) = g, α(z) = h and min{α(x), α(y)} = g, min{α(x), α(y), α(z)}= h. Here x−y, [xyz], x ∨ y and x ∧ y may either belongs to S or to $\mathcal{L}_{\aleph}$−S. So their images under α will either be g or h. By case(ii) all the inequalities are satisfied. Hence α is a fuzzy sub ℓ-ternary near ring of $\mathcal{L}_{\aleph}$.

Corollary 3.7.

If a nonempty proper subset S of a ℓ-ternary near ring $\mathcal{L}_{\aleph}$, is a sub ℓ-ternary near ring of $\mathcal{L}_{\aleph}$, then λ_S is a fuzzy sub ℓ-ternary near ring, where $\lambda_S = \begin{cases} 1 & \text{if } x \in S \\ 0 & \text{if } x \in \mathcal{L}_{\aleph} - S \end{cases}$ is the characteristic function of the subset S.

Lemma 3.8.

Let α and β be two fuzzy sub ℓ-ternary near rings of $\mathcal{L}_{\aleph}$. Then α ∩ β is a fuzzy sub ℓ-ternary near ring of $\mathcal{L}_{\aleph}$.

Proof:

For any x, y, z ∈ $\mathcal{L}_{\aleph}$, we have (α ∩ β)(x) = min{α(x), β(x)}. Then,

$$(\alpha \cap \beta)(x - y) = \min\{\alpha(x - y), \beta(x - y)\} \geq \min\{\min\{\alpha(x), \beta(y)\}; \min\{\alpha(x), \beta(y)\}\}$$
$$= \min\{\min\{\alpha(x), \beta(x)\}, \min\{\alpha(y), \beta(y)\}\}$$
$$= \min\{(\alpha \cap \beta)(x), (\alpha \cap \beta)(y)\}.$$

Similarly, we can prove

$(\alpha \cap \beta)([xyz]) \geq \min\{(\alpha \cap \beta)(x), (\alpha \cap \beta)(y), (\alpha \cap \beta)(z)\}$.

$(\alpha \cap \beta)(x \vee y) \geq \min\{(\alpha \cap \beta)(x), (\alpha \cap \beta)(y)\}$.

$(\alpha \cap \beta)(x \wedge y) \geq \min\{(\alpha \cap \beta)(x), (\alpha \cap \beta)(y)\}$.

Lemma 3.9.

Let α be any fuzzy sub ℓ-ternary near ring of $\mathcal{L}_{\aleph}$. Then each level subset α_t, t∈Im(α) ⊂ [0,1] is a sub ℓ-ternary near ring of $\mathcal{L}_{\aleph}$.

Proof:

Given α is any fuzzy sub ℓ-ternary near ring of $\mathcal{L}_{\aleph}$. Consider the level subset α_t= {x∈X : α(x) ≥ t}, where t∈Im(α). By lemma 3.3, we have α(x) ≤ α(0), for all x α $\mathcal{L}_{\aleph}$, so α(0) ≥ t. Therefore 0∈α_t, for all t and hence $\alpha_t \neq \emptyset$. Let x, y, z∈α_t be arbitrary. Then α(x) ≥ t, α(y) ≥ t, α(z) ≥ t, and min{α(x, α(y)} ≥ t, min{α(x), α(y), α(z)}≥ t and so α(x−y)≥ t, α([xyz])≥ t, α(x ∨ y) ≥ t, α(x ∧ y) ≥ t. Therefore x−y, [xyz], x ∨ y, x ∧y ∈α_t. Hence each level subset α_t, t∈Im(α) are sub ℓ-ternary near ring of $\mathcal{L}_{\aleph}$.

Theorem 3.10. (Characterization Theorem)

A fuzzy subset α of a ℓ-ternary near ring $\mathcal{L}_{\aleph}$ is a fuzzy sub ℓ-ternary near ring of $\mathcal{L}_{\aleph}$ if and only if each level subset α_t, t∈ Im(α) is a sub ℓ-ternary near ring of $\mathcal{L}_{\aleph}$.

Proof:

Assume that α is a fuzzy sub ℓ-ternary near ring of $\mathcal{L}_{\aleph}$.The proof of this part follows from the lemma 3.9. Conversely, assume that the level subset α_t, t ∈ Im(α) is a sub ℓ-ternary near ring of $\mathcal{L}_{\aleph}$, let min{α(x), α(y)} = r. Then either α(x) = r and α(y) ≥ α(x) = r or α(y) = r and α(x) ≥ α(y) = r. So α(x) ≥ r and α(y) ≥ r and thus x,y∈α_r. Therefore x−y, x ∨ y, x ∧ y∈α_r, since α_r is a sub ℓ-ternary near ring of $\mathcal{L}_{\aleph}$. Hence α(x−y) ≥ r,
α(x ∨ y) ≥ r and α(x ∧ y) ≥ r. Let α(x−y) = r_1. To prove α(x−y) ≥ min{α(x), α(y)}. That is to prove r_1≥ r. Suppose that $r_1 < r$. Then α(x− y) = $r_1 < r$. which is a contradiction. Therefore $r_1 \geq r$ and hence α(x− y) ≥ min{α(x), α(y)}. Similarly, we can prove α(x ∨ y) > min{α(x), α(y)} and α(x ∧ y) > min{α(x), α(y)}. Next, let s = min{α(x), α(y), α(z)}. Then α(x) ≥ s, α(y) ≥ s, α(z) ≥ s and so x, y, z ∈ α_s. But by assumption, we have [xyz]∈α_s. Therefore α([xyz]) ≥ s = min{α(x), α(y), α(z)}. Hence α is a fuzzy sub ℓ-ternary near ring of $\mathcal{L}_{\aleph}$.

Theorem 3.11.

Two level subsets α_{t_1} and α_{t_2} (with $t_1 < t_2$) of a fuzzy sub ℓ-ternary near ring of $\mathcal{L}_{\aleph}$ are equal if and only if there is no x∈$\mathcal{L}_{\aleph}$ such that $t_1 \leq \alpha(x) < t_2$.

Proof:

Assume that α_{t_1} and α_{t_2} are equal with $t_1 < t_2$. Suppose that there is an x ∈ $\mathcal{L}_{\aleph}$ such that $t_1 \leq \alpha(x) < t_2$. Then $\alpha(x) < t_2$ and $\alpha(x) \geq t_1$and so x ∉ α_{t_2} and x ∈ α_{t_1}, which is a contradiction to α_{t_1}= α_{t_2}. Therefore there is no x∈$\mathcal{L}_{\aleph}$ such that t_1≤ α(x) < t_2. Conversely assume that there is no x ∈ $\mathcal{L}_{\aleph}$ such that t_1≤ α(x) < t_2, let $\alpha_{t_1} = \{x \in \mathcal{L}_{\aleph} : \alpha(x) \geq t_1\}$ and $\alpha_{t_2} = \{x \in \mathcal{L}_{\aleph} : \alpha(x) \geq t_2\}$ with $t_1 < t_2$. Then clearly, $\alpha_{t_2} \subseteq \alpha_{t_1}$. It is enough to prove that $\alpha_{t_1} \subseteq \alpha_{t_2}$. Let x ∈ α_{t_1}. Then α(x) ≥ t_1. Suppose that α(x) < t_2. Then $t_1 \leq \alpha(x) < t_2$. which is a contradiction to our assumption. Therefore $\alpha(x) \geq t_2$ and hencex ∈ α_{t_2}. Thus $\alpha_{t_1} \subseteq \alpha_{t_2}$. .

Theorem 3.12.

Let $\mathcal{L}_{\aleph}$ be a ℓ-ternary near ring and A be a nonempty subset of $\mathcal{L}_{\aleph}$. If A is a sub ℓ-ternary near ring of $\mathcal{L}_{\aleph}$, then there exists a fuzzy sub ℓ-ternary near ring α of $\mathcal{L}_{\aleph}$ such that α_t = A, for some t∈[0, 1].

Proof:

Give that $\mathcal{L}_{\aleph}$ is a ℓ-ternary near ring and let A be a sub ℓ-ternary near ring. Let t ∈ [0; 1] and define a fuzzy subset on $\mathcal{L}_{\aleph}$ by $\alpha(x)= \begin{cases} t & \text{if } x \in A \\ 0 & \text{otherwise} \end{cases}$. Then clearly α_{t1}= A. Now, we prove that α is a fuzzy sub ℓ-ternary near ring. It is clear that the level subset α_t is a sub ℓ-ternary near ring of $\mathcal{L}_{\aleph}$. Hence by the theorem 3.10, α is a fuzzy sub ℓ-ternary near ring of $\mathcal{L}_{\aleph}$.

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