Irregularity and Density of Irregular Intuitionistic Fuzzy Graphs

[1]S. Yahya Mohamed and[2]R.JahirHussain

[1]P .G and Research Department of Mathematics, Govt. Arts College, Tiruchirappalli-620 022, India
[2]P .G and Research Department of Mathematics, Jamal Mohamed College,Tiruchirappalli-620 020, India

Abstract: *In this paper we introduce the notion of Irregularity and total Irregularity of any Irregular Intuitionistic Fuzzy graphs and some results are present. Also Density, Balanced Irregular Intuitionistic Fuzzy graphs are discussed and some of their properties are established.*
Keywords: *Intuitionistic fuzzy graph, degree, Irregularity, Density of IIFG, Balanced IIFG.*

I. Introduction

Albertson[1] defined measure of Irregularity of Crisp graph and discussed some properties. Atanassov [3] introduced the concept of intuitionistic fuzzy (IF) relations and intuitionistic fuzzy graphs (IFGs). Research on the theory of intuitionistic fuzzy sets (IFSs) has been witnessing an exponential growth in Mathematics and its applications. This ranges from traditional Mathematics to Information Sciences. This leads to consider IFGs and their applications. R. Parvathy and M.G.Karunambigai's paper [4] introduced the concept of IFG and analyzed its components. NagoorGani, A and Sajitha Begum, S [6] defined degree, Order and Size in intuitionistic fuzzy graphs and extend the properties. A. NagoorGani, et. al[13] defined types of Irregular Intuitionistic fuzzy graphs and discussed some properties.

In this paper, we define Irregularity and total Irregularity of any Irregular Intuitionistic fuzzy graphs and some results are attempted. Also Density of Irregular Intuitionistic fuzzy graphs and balanced Irregular Intuitionistic fuzzy graphs are discussed. Some of their properties are established and this concept is useful in Signal processing, artificial intelligence and computer networks.

II. Preliminaries

Definition 2.1: An Intuitionistic fuzzy graph is of the form G = (V, E) where
(i) V={$v_1,v_2,....,v_n$} such that μ_1: V$\rightarrow$[0,1]and γ_1: V $\rightarrow$[0,1] denote the degree ofmembership and non-membership of the element vi ϵV, respectively, and$0 \le \mu_1 (vi) + \gamma_1 (vi) \le 1$ for every vi ϵV, (i = 1,2, ……. n),
(ii) E ⊆ V x V where μ_2: VxV$\rightarrow$[0,1] and γ_2: VxV$\rightarrow$[0,1] are such that$\mu_2 (vi , vj) \le$ min [$\mu_1(vi), \mu_1(vj)$] and $\gamma_2 (vi , vj) \le$ max [$\gamma_1(vi), \gamma_1(vj)$] and $0 \le \mu_2 (vi, vj) + \gamma_2 (vi,vj) \le 1$ for every (vi ,vj) ϵE, (i, j = 1,2, ……. n)

Definition 2.2: An IFG H = (V', E') is said to be an Intuitionistic fuzzy subgraph (IFSG) of the IFG
G = (V, E), if V' ⊆V and E' ⊆ E. In other words, if $\mu_{1i}' \le \mu_{1i}$; $\gamma_{1i}' \ge \gamma_{1i}$ and $\mu_{2ij}' \le \mu_{2ij}$; $\gamma_{2ij}' \ge \gamma_{2ij}$
for every i, j = 1,2………n.

Definition 2.3: An Intuitionistic fuzzy graph is complete if $\mu_{2ij} =$ min (μ_{1i}, μ_{1j}) and $\gamma_{2ij} =$ max (γ_{2i} ,γ_{2j})
for all (vi , vj) Ϲ V.

Definition 2.4: Let G = (V,E) be an IFG. Then the degree of a vertex v is defined by d(v) = (dμ(v), dγ(v)) where
dμ(v) = $\Sigma u \ne v$ μ_2(v,u) and dγ(v) = $\Sigma u \ne v$ γ_2(v,u).

Definition 2.5: The total degree of a vertex 'v' is defined as t(v) =(t_μ(v),t_γ(v)), where
t_μ(v) = Σ u$\ne$v μ_2(v,u) + μ_1(v) and t_γ(v) = Σ u$\ne$vγ_2(v,u) + γ_1(v)

Definition 2.6: The complement of an IFG G = (V, E) is denoted by $\overline{G} = (\overline{V}, \overline{E})$and is defined as
i) $\overline{\mu}_1$(v)= μ_1(v) and $\overline{\gamma}_1$(v) = γ_1(v)
ii) $\overline{\mu}_2$(u,v) = μ_1(u) Λ μ_1(v) - μ_2(u,v) and $\overline{\gamma}_2$(u,v) = γ_1(u) V γ_1(v) - γ_2(u,v) for u,v in V.

Definition 2.7: An Intuitionistic fuzzy graph G = (V, E) is said to be regular, if every vertex has same degree.

Definition 2.8: Let G = (V,E) be IFG. Then G is irregular, if there is a vertex which is adjacent to vertices with distinct degrees.

Definition 2.9: Let G = (V, E) be a IFG. Then G is totally irregular, if there is a vertex which is adjacent to vertices with distinct total degrees.

Definition 2.10: Let G = (V, E) be a connected IFG. Then, G is said to be a highly irregular IFG if every vertex of G is adjacent to vertices with distinct degrees

Definition2.11:
A homomorphism of Intuitionistic fuzzy graphs h: G→G' is a map h: S→S' which satisfies
(i) $\mu_1(x) \le \mu_1'(h(x))$; $\gamma_1(x) \ge \gamma_1'(h(x))$ for every x∈S.
(ii) $\mu_2(x,y) \le \mu_2'(h(x), h(y))$; $\gamma_2(x,y) \ge \gamma_2'(h(x), h(y))$ for every x∈S.

Definition 2.12:
A weak isomorphism of Intuitionistic fuzzy graphs h: G→G' is a map h: S→S' which is a bijective homomorphism that satisfies $(\mu_1(x), \gamma_1(x)) = (\mu_1'(h(x)), \gamma_1'(h(x))$ for every x∈S.

Definition 2.13:
A co-weak isomorphism of intuitionistic fuzzy graphs h: G→G' is a map h: S→S' which is a bijective homomorphism that satisfies $\mu_2(x,y), \gamma_2(x,y)) = (\mu_2'(h(x), h(y)), \gamma_2'(h(x), h(y)))$ for every x,y∈S.

Definition 2.14:
An isomorphism of intuitionistic fuzzy graphs h: G→G' is a map h: S→S' which is a bijective homomorphism that satisfies
(i) $(\mu_1(x), \gamma_1(x)) = (\mu_1'(h(x)), \gamma_1'(h(x))$ for every x∈S.
(ii) $(\mu_2(x,y), \gamma_2(x,y)) = (\mu_2'(h(x), h(y)), \gamma_2'(h(x), h(y)))$ for every x,y∈S.
  This will denote as G≅G' .
An IntuitonisticFuzzy graph G is said to be self complementary if G≅ $\overline{G}$.

III.   Irregularity of Intuitionistic Fuzzy Graphs

Definition 3.1: Let G = (σ, μ) be any Irregular fuzzy graph   then the Irregularity of
G is defined as   Irreg(G) = $\sum_{xy \in E} |d(x) - d(y)|$ for all x ,y ∈ V.

      The total Irregularity of the Irregular fuzzy graph is defined as Irreg$_t$(G) = $\frac{1}{2} \sum_{x,y \in V} |d(x) - d(y)|$

Example 3.2: Let G = (σ, μ) be any Irregular fuzzy graph with V = { u, v, w, x} defined by $\sigma(u) = 0.6$,
 $\sigma(v) = 0.5$, $\sigma(w) = 0.8$, $\sigma(x) = 1$ and $\mu(u,v) = 0.4$, $\mu(u,x) = 0.6$, $\mu(v, w) = 0.3$, $\mu(w, x) = 0.5$.
Then, d(u) = 1.0, d(v) = 0.7, d(w) = 0.8, d(x) = 1.1 and Irreg(G) = 0.3+0.1+0.3+0.1 = 0.8 and
Irreg$_t$(G) = $\frac{1}{2}$ [0.3+0.2+0.1+0.1+0.4+0.3] = $\frac{1.4}{2}$ = 0.7
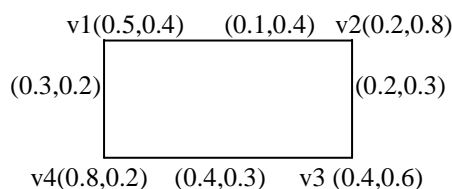
Definition 3.3: Let G = (V,E) be a any type of Irregular Intuitionistic fuzzy graph then the Irregularity of
G is defined as Irreg(G) = (Irreg$_\mu$(G), Irreg$_\gamma$(G)) Where Irreg$_\mu$(G) = $\sum_{xy \in E}|d\mu(x) - d\mu(y)|$ and
Irreg$_\gamma$(G) = $\sum_{xy \in E}|d\gamma(x) - d\gamma(y)|$ for all x ,y ∈ V

      The total Irregularity of the Irregular Intuitionistic fuzzy graph is defined as

Irreg$_t$(G) = $\frac{1}{2} \sum_{x,y \in V} |d(x) - d(y)|$

Example 3.4:   v1(0.5,0.4)   (0.1,0.4) v2(0.2,0.8)

(0.3,0.2)   (0.2,0.3)

v4(0.8,0.2)   (0.4,0.3)   v3 (0.4,0.6)
Fig -1: Irregular Intuitionistic fuzzy graph

Here d(v1) = (0.4,0.6) , d(v2) = (0.3,0.7), d(v3) = (0.6,0.6) and d(v4)=(0.7,0.5).
Then, Irreg(G) = (0.1,0.1) + (0.3,0.1)+(0.1,0.1) + (0.3,0.1) = (0.8,0.4).
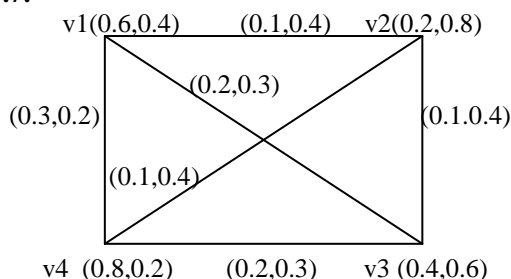
But the total Irregularity is Irreg$_t$(G) = $\frac{1}{2}$[(0.1,0.1)+(0.2,0.0) + (0.3,0.1) + (0.3,0.1)+(0.4,0.2) + (0.1,0.1)
                                        = (0.7,0.3)
Remark 3.5 :

 (1) If G is regular Intuitionistic fuzzy graph then Irreg(G) = 0

(2) For any Irregular Intuitionistic fuzzy graph Irreg$_t$(G) ≤ Irreg (G)

(3) Let G = (V, E) be the Irregular Intuitionistic fuzzy graph which is complete then Irreg(G) =2 [Irreg$_t$(G)]

Theorem 3.6: Let G = (V, E) be the Irregular IFG with all vertices having edges with $\mu_2(x, y) = \frac{1}{2} (\mu_1 (x) \wedge \mu_1 (y))$ and $\gamma_2 (x,y) = \frac{1}{2} (\gamma_1(x) \vee \gamma_1(y))$ and $\mu_1 + \gamma_1 = 1$ then $\text{Irreg}_\mu(G) = \text{Irreg}_\gamma(G)$ for all edges.

Example 3.7:

v1(0.6,0.4)   (0.1,0.4)   v2(0.2,0.8)

(0.2,0.3)

(0.3,0.2)   (0.1.0.4)

(0.1,0.4)

v4 (0.8,0.2)   (0.2,0.3)   v3 (0.4,0.6)

Fig -2:Irregualr but not highly Irregular IFG
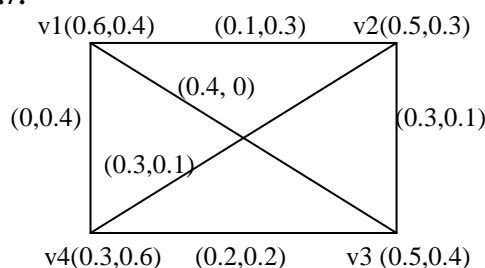
Here d(v1) = (0.6,0.9) , d(v2) = (0.3,1.2), d(v3) =(0.5,1.0) and d(v4)=(0.6, 0.9).
So Irreg(G) = (0.3,0.3) + (0.2,0.2) + (0.1,0.1)+ (0,0) + (0.3,0.3) + (0.1 ,0.1) = (1.0,1.0).
i.e) $\text{Irreg}_\mu(G) = \text{Irreg}_\gamma(G)$

Theorem3.8: Let G = (V, E) be the Irregular IFG with $\mu_2 (v_i, v_j) + \gamma_2 (v_i,v_j)$ is same for every $(v_i ,v_j) \in E$, (i, j = 1,2, ……. n) then $\text{Irreg}_\mu(G) = \text{Irreg}_\gamma(G)$.

Example 3.7:

v1(0.6,0.4)   (0.1,0.3)   v2(0.5,0.3)

(0.4, 0)

(0,0.4)   (0.3,0.1)

(0.3,0.1)

v4(0.3,0.6)   (0.2,0.2)   v3 (0.5,0.4)

Fig - 3

Here d(v1) = (0.5,0.7) , d(v2) = (0.7,0.5), d(v3) = (0.9,0.3) and d(v4)=(0.5, 0.7). So Irreg(G) = (0.2,0.2) + (0.2,0.2) + (0.4,0.4)+ (0.4,0.4)+ ((0.0,0.0) + (0.2 ,0.2) = (1.4,1.4).
i.e) $\text{Irreg}_\mu(G) = \text{Irreg}_\gamma(G)$.

Theorem 3.9: Let G = (V, E) be the Irregular IFG with $d\mu(V_i) + d\gamma(V_i) = d\mu(V_j) + d\gamma(V_j)$ for all $V_i, V_j \in V$ then $\text{Irreg}_\mu(G) = \text{Irreg}_\gamma(G)$.

Proof: Given $d\mu(V_i) + d\gamma(V_i) = d\mu(V_j) + d\gamma(V_j)$ then
$d\mu(V_i) - d\mu(V_j) = d\gamma(V_i) - d\gamma(V_j)$ for all $V_i, V_j \in V$
$\sum_{ViVj}|d\mu(V_i) - d\mu(V_j)|$ $= \sum_{ViVj}|d\gamma(V_i) - d\gamma(V_j)|$
i.e) $\text{Irreg}_\mu(G) = \text{Irreg}_\gamma(G)$.

Theorem 3.10 : Let G = (V,E) and $G^1 = (V^1,E^1)$ be two Irregular IFG and G is co-weak isomorphic with G^1 then $\text{Irreg}(G) = \text{Irreg}(G^1)$.

Proof: Since G is co-weak isomorphic with G^1 then $(\mu_2(x,y), \gamma_2 (x,y)) = (\mu_2'(h(x), h(y)) , \gamma_2'(h(x), h(y)))$ for every x,y$\in$S.
Now $d\mu(x) = \Sigma_{x\neq y} \mu_2(x,y) = \Sigma_{x\neq y} \mu_2' ((h(x),h(y)) = d\mu(h(x)$
Therefore $\sum_{xy\in E}|d\mu(x) - d\mu(y)|$ $= \sum_{xy\in E}|d\mu(h(x)) - d\mu(h(y))|$
Similarly $\sum_{xy\in E}|d\gamma(x) - d\gamma(y)|$ $= \sum_{xy\in E}|d\gamma(h(x)) - d\gamma(h(y))|$ for every x,y$\in$S.
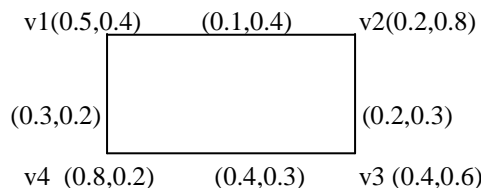i.e) Irreg (G) = Irreg (G^1)

IV.   Density of Irregular Intuitionistic Fuzzy Graphs

Definition 4.1: Let G = (V, E) be a Irregular IFG. The Density of G is defined as D(G) = $(D_\mu(G), D_\gamma(G))$
Where $D_\mu(G) = \frac{2 \sum_{x,y\in V} (\mu_2(x,y))}{\sum_{xy\in E} (\mu_1(x)\wedge\mu_1(y))}$ for all x,y$\in$Vand $D_\gamma(G) = \frac{2 \sum_{x,y\in V} (\gamma_2(x,y))}{\sum_{xy\in E} (\gamma_1(x)\vee\gamma_1(y))}$ for all x, y$\in$V

Example 4.2:

v1(0.5,0.4) (0.1,0.4) v2(0.2,0.8)

(0.3,0.2)   (0.2,0.3)

v4 (0.8,0.2) (0.4,0.3) v3 (0.4,0.6)

Fig – 4: Highly, neighbourly and totally Irregular IFG.

$$D (G) = (D_\mu(G), D_\gamma(G)) = (\frac{2\,[\,0.1+0.2+0.4+0.3]}{0.2+0.2+0.4+0.5}, \frac{2\,[\,0.4+0.3+0.3+0.2]}{0.8+0.8+0.6+0.4}) = (1.54, 0.923)$$

Theorem 4.3: Let $G = (V, E)$ be the Irregular Intuitionistic fuzzy graph with $\mu_2(x, y) = \frac{1}{2} (\mu_1(x) \wedge \mu_1(y))$ and $\gamma_2(x,y) = \frac{1}{2} (\gamma_1(x) \vee \gamma_1(y))$ then $D(G) = (1,1)$

Proof:

$$D(G) = (\frac{2\sum_{x,y\in V}(\mu_2(x,y))}{\sum_{xy\in E}(\mu_1(x)\wedge\mu_1(y))}, \frac{2\sum_{x,y\in V}(\gamma_2(x,y))}{\sum_{xy\in E}(\gamma_1(x)\vee\gamma_1(y))}) \quad \text{for all } x, y \in V$$

$$= (\frac{2\sum_{x,y\in V}\frac{1}{2}(\mu_1(x)\wedge\mu_1(y))}{\sum_{xy\in E}(\mu_1(x)\wedge\mu_1(y))}, \frac{2\sum_{x,y\in V}\frac{1}{2}(\gamma_1(x)\vee\gamma_1(y))}{\sum_{xy\in E}(\gamma_1(x)\vee\gamma_1(y))})$$

$$= (\frac{2\left(\frac{1}{2}\right)\sum_{x,y\in V}(\mu_1(x)\wedge\mu_1(y))}{\sum_{xy\in E}(\mu_1(x)\wedge\mu_1(y))}, \frac{2\left(\frac{1}{2}\right)\sum_{x,y\in V}(\gamma_1(x)\vee\gamma_1(y))}{\sum_{xy\in E}(\gamma_1(x)\vee\gamma_1(y))}) = (1, 1).$$

Theorem 4.4: Let $G = (V, E)$ be Irregular and complete IFG then $D(G) = (2,2)$

Proof:

Given G is Irregular and complete IFG then

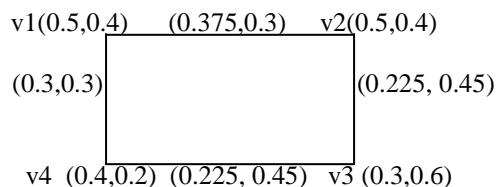$\mu_2(x, y) = \mu_1(x) \wedge \mu_1(y)$ and $\gamma_2(x, y) = \gamma_1(x) \vee \gamma_1(y)$ for all $x, y \in V$

Now $\sum_{x,y\in V}(\mu_2(x,y)) = \sum_{xy\in E}(\mu_1(x)\wedge\mu_1(y))$, Also $\sum_{x,y\in V}(\gamma_2(x,y)) = \sum_{xy\in E}(\gamma_1(x)\vee\gamma_1(y))$

Therefore $D(G) = (\frac{2\sum_{x,y\in V}(\mu_2(x,y))}{\sum_{xy\in E}(\mu_1(x)\wedge\mu_1(y))}, \frac{2\sum_{x,y\in V}(\gamma_2(x,y))}{\sum_{xy\in E}(\gamma_1(x)\vee\gamma_1(y))})$ for all $x, y \in V$

$$= (\frac{2\sum_{xy\in E}(\mu_1(x)\wedge\mu_1(y))}{\sum_{xy\in E}(\mu_1(x)\wedge\mu_1(y))}, \frac{2\sum_{xy\in E}(\gamma_1(x)\vee\gamma_1(y))}{\sum_{xy\in E}(\gamma_1(x)\vee\gamma_1(y))}) \text{for all } x, y \in V$$
$$= (2, 2)$$

Definition 4.5: An Irregular IFG G is said to be balanced if $D_\mu(H) \le D_\mu(G)$ and $D_\gamma(H) \le D_\gamma(G)$ for all subgraphs H of G.

Example 4.6

v1(0.5,0.4) (0.375,0.3) v2(0.5,0.4)

(0.3,0.3)   (0.225, 0.45)

v4 (0.4,0.2) (0.225, 0.45) v3 (0.3,0.6)

Fig – 5: Balanced Irregular IFG (G)

Here $D(G) = (D_\mu(G), D_\gamma(G)) = (1.5,1.5)$ and sub graphs of G are
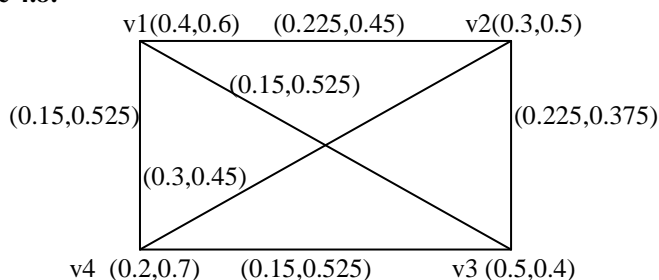
H1 = {v1, v2}, H2 = {v1, v3}, H3 = {v1, v4}, H4 = {v2, v3}, H5 = {v2, v4}, H6 = {v3, v4, H7= {v1, v2, v3}, H8 = {v1, v3, v4}, H9 = {v1, v2, v4}, H10 ={v2, v3, v4}, H11 = {v1, v2, v3, v4}.

Now Density $(D\mu(H),Dv(H))$ is D(H1) = (1.5, 1.5), D(H2) = (0, 0), D(H3) = (1.5, 1.5), D(H4) = (1.5, 1.5), D(H5) = (0,0), D(H6) = (15,1.5), D(H7) = (1.5, 1.5),D(H8) = (1.5, 1.5), D(H9) = (1.5,1.5), D(H10) = (15, 1.5), D(H11) = (1.5, 1.5).

So $D(H) \le D(G)$ for all subgraphs H of G. Hence G is balanced Irregular IFG.

Definition 4.7: An Irregular IFG G is said to be strictly balanced Irregular IFG if $D_\mu(H) = D_\mu(G)$ and $D_\gamma(H) = D_\gamma(G)$ for all non empty subgraphs H of G.i.e) $D(H) = D(G)$

Example 4.8:

v1(0.4,0.6) (0.225,0.45) v2(0.3,0.5)

(0.15,0.525)

(0.15,0.525) (0.225,0.375)

(0.3,0.45)

v4 (0.2,0.7) (0.15,0.525) v3 (0.5,0.4)

Fig – 6: Strictly Balanced Irregular IFG

In this Graph $D(G) = (\frac{2\,[\,0.225+0.225+0.15+0.15+0.15+0.3\,]}{03+0.3+0.2+0.2+0.2+0.4} \,,\, \frac{2\,[\,0.45+0.375+0.525+0.525+0.525+0.45\,]}{0.6+0.5+0.7+0.7+0.6+0.7}) = (1.5, 1.5)$
And for all the non-empty subgraphs H of G, $D(H) = (1.5,1.5)$.
i.e) $D(H) = D(G)$. Therefore G is Strictly Balanced Irregular IFG.

Theorem 4.9: Let $G = (V,E)$ be strictly balanced totally Irregular IFG then $D(G) + D(\overline{G}) = (2,2)$ and the complement also totally Irregular IFG.

Proof: Let G be the totally Irregular IFG. Also G is Strictly balanced ➔ $D(H) = D(G)$
Let $\overline{G} = (\overline{V}, \overline{E})$ be complement of G then
(i) $\overline{\mu}_1(x) = \mu_1(x)$ and $\overline{\gamma}_1(x) = \gamma_1(x)$ for all x in V and (ii) $\overline{\mu}_2(x, y) = \mu_1(x) \wedge \mu_1(y) - \mu_2(x,y)$ ➔(1)
and $\overline{\gamma}_2(x,y) = \gamma_1(x) \vee \gamma_1(y) - \gamma_2(x, y)$ for x, y in V ➔(2)
Dividing (1) by $\mu_1(x) \wedge \mu_1(y)$ on both sides

$$\frac{\overline{\mu}_2(x,y)}{\mu 1(x) \wedge \mu 1(y)} = 1 - \frac{\mu_2(x,y)}{\mu 1(x) \wedge \mu 1(y)}$$

Then $\sum_{x,y\in V} \frac{\overline{\mu}_2(x,y)}{\mu 1(x) \wedge \mu 1(y)} = 1 - \sum_{x,y\in V} \frac{\mu_2(x,y))}{\mu 1(x) \wedge \mu 1(y)}$

$2 \sum_{x,y\in V} \frac{\overline{\mu}_2(x,y)}{\mu 1(x) \wedge \mu 1(y)} = 2 - 2 \sum_{x,y\in V} \frac{\mu_2(x,y))}{\mu 1(x) \wedge \mu 1(y)}$ for all x, y $\in V$

$2 \sum_{x,y\in V} \frac{\overline{\mu}_2(x,y)}{\overline{\mu}_1(x) \wedge \overline{\mu}_1(y)} = 2 - 2 \sum_{x,y\in V} \frac{\mu_2(x,y))}{\mu 1(x) \wedge \mu 1(y)}$ by (i)

i.e) $D_\mu(\overline{G}) = 2 - D_\mu(G)$

Similarly Dividing (2) by $\gamma_1(x) \vee \gamma_1(y)$ on both sides

$$\frac{\overline{\gamma}_2(x,y)}{\gamma_1(x) \vee \gamma_1(y)} = 1 - \frac{\gamma_2(x,y)}{\gamma_1(x) \vee \gamma_1(y)}$$

$2 \sum_{x,y\in V} \frac{\overline{\gamma}_2(x,y)}{\overline{\gamma}_1(x) \vee \overline{\gamma}_1(y)} = 2 - 2 \sum_{x,y\in V} \frac{\gamma_2(x,y)}{\gamma_1(x) \vee \gamma_1(y)}$ for all x, y $\in V$

i.e) $D_\gamma(\overline{G}) = 2 - D_\gamma(G)$

Now, $D(G) + D(\overline{G}) = (D_\mu(G), D_\gamma(G)) + (D_\mu(\overline{G}), D_\gamma(\overline{G}))$
$= (D_\mu(G) + D_\mu(\overline{G}), D_\gamma(G) + D_\gamma(\overline{G})) = (D_\mu(G) + 2 - D_\mu(G), D_\gamma(G) + 2 - D_\gamma(G))$
$= (2, 2)$

Theorem 4.10: Let $G = (V,E)$ and $G^1 = (V^1, E^1)$ be two isomorphic totally Irregular IFGs . If G^1 is balanced then G is balanced. Also the subgraphs are also totally Irregular IFGs.

Proof: Let h: G➔G' is a map h: S➔S' which is a bijective homomorphism that satisfies
(i) $(\mu_1(x), \gamma_1(x)) = (\mu_1'(h(x)), \gamma_1'(h(x)))$ for every x $\in$S.
(ii) $(\mu_2(x, y), \gamma_2(x, y)) = (\mu_2'(h(x), h(y)), \gamma_2'(h(x), h(y)))$ for every x,y$\in$S.
Then $\sum_{x\in V} \mu_1(x) = \sum_{x\in V^1} \mu_1^1(x)$ and $\sum_{x\in V} \gamma_1(x) = \sum_{x\in V^1} \gamma_1^1(x)$
Also $\sum_{x,y\in V} \mu_2(x,y) = \sum_{x,y\in V^1} \mu_2^1(x,y)$ and $\sum_{x,y\in V} \gamma_2(x,y) = \sum_{x,y\in V^1} \gamma_2^1(x,y)$
Let $H_2 = G(V(H_2), E(H_2))$ is an Intuitionistic fuzzy subgraph of G^1.
Since G^1 is balanced then $D\mu(H_2) \le D\mu(G^1)$ and $D\gamma(H_2) \le D\gamma(G^1)$

$$\left(\frac{2 \sum_{x,y \in V(H_1)} \mu_2{}^1(h(x),h(y))}{\sum_{xy \in E(H_1)} \left(\mu_1{}^1(h(x)) \wedge \mu_1{}^1(h(y)) \right)} \,, \, \frac{2 \sum_{x,y \in V(H_1)} \gamma_2{}^1(h(x),h(y))}{\sum_{xy \in E(H_1)} (\gamma_1{}^1(h(x)) \vee \gamma_1{}^1(h(y)))} \right)$$

$$\leq \left(\frac{2 \sum_{x,y \in V^1} \mu_2{}^1(h(x),h(y))}{\sum_{xy \in E^1} (\mu_1{}^1(h(x)) \wedge \mu_1{}^1(h(y)))} \,, \, \frac{2 \sum_{x,y \in V^1} \gamma_2{}^1(h(x),h(y)}{\sum_{xy \in E^1}(\gamma_1{}^1(h(x)) \vee \gamma_1{}^1(h(y)))} \right)$$

$$\left(\frac{2 \sum_{x,y \in V(H_1)} \left(\mu_2(x,y) \right)}{\sum_{xy \in E(H_1)} \left(\mu_1(x) \wedge \mu_1(y) \right)} \,, \, \frac{2 \sum_{x,y \in V(H_1)} (\gamma_2(x,y))}{\sum_{xy \in E(H_1)} (\gamma_1(x) \vee \gamma_1(y)))} \right) \leq \left(\frac{2 \sum_{x,y \in V} (\mu_2(x,y))}{\sum_{xy \in E} (\mu_1(x) \wedge \mu_1(y))} \,, \, \frac{2 \sum_{x,y \in V} (\gamma_2(x,y))}{\sum_{xy \in E} (\gamma_1(x) \vee \gamma_1(y))} \right)$$

i.e) $(D\mu(H_1) , D\gamma(H_1)) \leq (D\mu(G) , D\gamma(G))$
Hence G is balanced.

Theorem 4.11: Let G = (V,E) be Irregular IFG and all the edges of G are strong the G is balanced
Proof: Since all the edges are strong then $\mu_2(x, y) = \mu_1(x) \wedge \mu_1(y)$ and $\gamma_2(x, y) = \gamma_1(x) \vee \gamma_1(y)$
Now, by theorem 3.4 D(G) = (2,2).
For the subgraphs H, ($D\mu(H)$, $D\gamma(G)$) = (2,2) if the vertices of H having edges,
otherwise ($D\mu(H)$, $D\gamma(G)$) = (0,0).
i.e) $D\mu(H) \leq D\mu(G)$ Which implies G is Balanced.

Theorem 4.12: Every Irregular IFG which is complete is balanced
Proof:
    Let G be Irregular IFG and complete then by theorem 3.4, D(G) = (2,2). Let H be a non emptysubgraphs of G and G is complete then, all the edges are Strong. Therefore, D(H) = (2,2) for every H in G. Also the subgraphs are totally Irregular IFGs.
i.e) G is balanced.

Remark: The converse need not be true. That is Every balanced Irregular IFG need not be Complete.
 Example: Fig- 5 is Balanced Irregular IFG, But is not complete.

V.   Conclusion

        Here we just define the new concept Irregularity and total Irregularity of Irregular Intuitionistic fuzzy graphs of any type and discussed some results. Also some properties of Density of Irregular Intuitionistic fuzzy graphs are derived . Finally, we studied balanced Irregular Intuitionistic fuzzy graphs and its properties. Further, we going to study more results on Irregularity of all the types of Irregular IFGs.

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