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Hydromagnetic Effects on Fluid Flow and Internal Flow Separation in a Linearly Diverging Channel

M. Saiful Islam Mallik[1], Tahmina Begum[2], Md. Minarul Haque[3], Rebeya Akter[4]

[1]*Assistant Professor, Department of Arts and Sciences, Ahsanullah University of Science and Technology, Dhaka.*
[2] *Assistant Professor, Department of Mathematics, Dhaka City College, Dhaka.*
[3]*Lecturer, Department of Mathematics, Dhaka City College, Dhaka.*
[4] *Lecturer, Department of Mathematics, Jagannath University, Dhaka.*

Abstract: *In this paper, we investigated the hydromagnetic steady flow of a viscous conducting fluid in a two-dimensional uniform width linearly diverging channel. For this investigation the effect of an externally applied homogeneous magnetic field on the development of velocity profiles and internal flow separation in the diverging channel are observed. The solution for the flow governing non-linear differential equation is found using perturbation method together with Pade´ approximation technique. The investigation results reveal that the requirement of flow Reynolds number for the development of internal flow separation increases with an increase in magnetic field intensity. Furthermore, the behavior of velocity profiles under the effect of magnetic field is discussed.*

Keywords: *Linear diverging channel, internal flow separation, magnetic field, Pade´ approximants.*

I. Introduction

The study of electrically conducting viscous fluid flowing through diverging channels under the influence of an external magnetic field is not only fascinating theoretically, but also finds applications in mathematical modeling of several industrial and biological systems such as magnetohydrodynamics (MHD) generators, plasma studies, nuclear reactors, industrial metal casting, controlling of molten metal flows, etc. The theoretical study also finds application in the area of motion of liquid metals or alloys in the cooling systems of advanced nuclear reactors. In the past few years, several simple flow problems associated with classical hydrodynamics have received new attention within the more general context of MHD. A survey of MHD studies in the technological fields can be found in (Moreau 1990). In modern times the theory of flow convergent-divergent channels has many applications in aerospace, chemical, civil, environmental, mechanical and bio-mechanical engineering as well as in understanding rivers and canals. A numerical investigation of the study of hydromagnetic flows in a slowly varying exponentially diverging channel under the effect of an externally applied homogeneous magnetic field was performed by Makinde and Mhone (2006). For this they have used perturbation method and Pade´ approximation technique (Baker Jr. 1975). The mathematical investigations of this type of problem have been studied by Rao and Deshikachar (1986) and (Makinde 1995).

In the present paper, the steady hydromagnetic flows in a two-dimensional uniform width linearly diverging channel under the influence of an externally applied homogeneous magnetic field have been investigated. The objective of this study is to analyze the fluid velocity profiles and to determine numerically the effect of the externally applied homogeneous magnetic field on the development of internal flow separation as flow Reynolds number increases using perturbation method together with Pade´ approximation technique.

II. Mathematical Formulation

Consider the steady flow of an incompressible viscous conducting fluid through an uniform width linearly diverging channel under the influence of an externally applied homogeneous magnetic field as shown in Fig. 1.

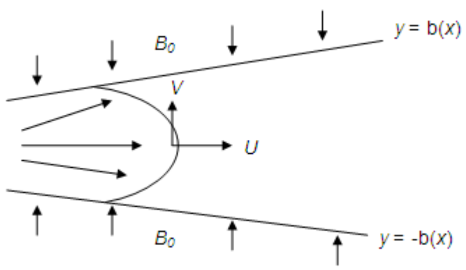

Fig. 1. Geometry of the problem.

It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. Take a Cartesian coordinate system (x, y) where $0x$ lies along the centre of the channel and y is the distance measured in the normal section. Let u and v be the velocity components in the directions of x and y increasing respectively and $b(x)$ defines the wall diverging geometrically. Then, the governing equations for the two-dimensional steady flow, in terms of vorticity (ω) and stream-function (ψ) as given by Rao and Deshikachar (1986) are

$$\frac{\partial(\omega,\psi)}{\partial(x,y)} = \upsilon\nabla^2\omega + \frac{\sigma_e B_0{}^2}{\rho}\frac{\partial^2\psi}{\partial y^2}, \qquad \omega = -\nabla^2\psi, \tag{1}$$

and the appropriate boundary conditions are

$$\psi = 0, \qquad \frac{\partial^2\psi}{\partial y^2} = 0 \quad \text{on } y = 0, \tag{2}$$

$$\psi = Q, \qquad \frac{\partial\psi}{\partial y} - \frac{\partial\psi}{\partial x}\frac{db}{dx} = 0 \quad \text{on } y = b(x). \tag{3}$$

where $Q = \int_0^{b(x)} u\,dy$ is the fluid flux rate across any section of the channel, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, B_0 = ($\mu_e.H$) the electromagnetic induction, μ_e the magnetic permeability, H the intensity of magnetic field, σ_e the conductivity of the fluid, ρ the fluid density and υ is the kinematic viscosity coefficient.

The axial and normal fluid velocity components in terms of the stream function can be written as $u = \frac{\partial\psi}{\partial y}$ and $\text{v} = -\frac{\partial\psi}{\partial x}$.

The inner surface of the walls is given by $y = \pm b(x/L)$. Let $b(x) = S(\varepsilon x/a)$ where S is the function of x, a is the characteristic half-width of the channel, ε is a small dimensionless parameter that specifies the slow variation in the cross-section of the channel which is defined as $0 < \varepsilon = a/L << 1$ and L defines the channel characteristic length. In the limit $\varepsilon \to 0$, the channel is of constant width. The introduced dimensionless variables and hence the reduced dimensionless governing equations together with the appropriate boundary conditions, (neglecting the bars for clarity) can be written as follows,

$$\bar{\omega} = \frac{a^2\omega}{Q},\ \bar{x} = \frac{\varepsilon x}{a},\ \bar{y} = \frac{y}{a},\ \bar{\psi} = \frac{\psi}{Q},\ H^2 = \frac{La\sigma_e B_0{}^2}{\rho Q}. \tag{4}$$

$$\frac{\partial^2\omega}{\partial y^2} = \text{Re}\left[\frac{\partial(\omega,\psi)}{\partial(x,y)} - H^2\frac{\partial^2\psi}{\partial y^2}\right], \quad \omega = -\frac{\partial^2\psi}{\partial y^2}, \tag{5}$$

$$\psi = 0, \qquad \frac{\partial^2\psi}{\partial y^2} = 0 \qquad \text{on } y = 0, \tag{6}$$

$$\psi = 1, \qquad \frac{\partial\psi}{\partial y} = 0 \qquad \text{on } y = S. \tag{7}$$

where the flow is considered in the boundary layer approximation or for channel with a small aspect ratio ε, $\text{Re} = \varepsilon Q/\nu$ is the effective flow Reynolds number and H is the magnetic field intensity parameter or Hartmann number. For the geometry of the channel under consideration, S is defined as $S = 1 + x$.

III. Perturbation Expansion

The equations (5)-(7) are non-linear in nature and therefore not possible to find its solution exactly. However, the solution can be found in the form of power series in Re i.e.,

$$\psi = \sum_{i=0}^{\infty} \mathrm{Re}^i \psi_i, \qquad \omega = \sum_{i=0}^{\infty} \mathrm{Re}^i \omega_i . \tag{8}$$

Now substitute the above expressions (8) into (5)-(7) and collect the coefficients of like powers of Re. The resulting equations are:

Zeroth Order :

$$\frac{\partial^2 \omega_0}{\partial y^2} = 0, \ \omega_0 = -\frac{\partial^2 \psi_0}{\partial y^2}, \tag{9}$$

$$\psi_0 = 0, \ \frac{\partial^2 \psi_0}{\partial y^2} = 0, \quad \text{on } \ y = 0, \tag{10}$$

$$\psi_0 = 1, \ \frac{\partial \psi_0}{\partial y} = 0, \quad \text{on } \ y = S . \tag{11}$$

Higher Order (n$\geq$1):

$$\frac{\partial^2 \omega_n}{\partial y^2} = \sum_{i=0}^{n-1} \frac{\partial(\omega_i, \psi_{n-i-1})}{\partial(x,y)} - H^2 \frac{\partial^2 \psi_{n-1}}{\partial y^2}, \quad \omega_n = -\frac{\partial^2 \psi_n}{\partial y^2}, \tag{12}$$

$$\psi_n = 0, \ \frac{\partial^2 \psi_n}{\partial y^2} = 0, \quad \text{on } \ y = 0, \tag{13}$$

$$\frac{\partial \psi_n}{\partial y} = 0, \ \ \psi_n = 0, \quad \text{on } \ y = S . \tag{14}$$

It is difficult to find many terms of the solution series manually. So a MAPLE program has been written that calculates successively the coefficients of the solution series. It consists of the following segments:

(i) Declaration of arrays for the solution series coefficients; ψ = array(0……..25), ω = array (0……25).
(ii) Input the leading order term and their derivatives *i.e.* ψ_0, ω_0.
(iii) Input the modeled channel geometry slope (*i.e.* dS/dx).
(iv) Using a MAPLE loop procedure, iterate to solve equations (12)-(14) for the higher order terms *i.e.* ψ_n, ω_n, n = 1,2,3,…… .
(v) Compute the wall shear stress and the axial pressure gradient.

The first two terms of the solution for stream-function and vorticity are obtained as

$$\psi = -\frac{1}{2}\eta\left[\eta^2 - 3\right] - \frac{\mathrm{Re}}{280}\eta\left[3\left(-5 + 11\eta^2 - 7\eta^4 - \eta^6\right) + 7S^2H^2\left(1 - 2\eta^2 + \eta^4\right)\right] + O\left(\mathrm{Re}^2\right) \tag{15}$$

$$\omega = 3\frac{\eta}{S^2} + \frac{\mathrm{Re}}{140S^2}\eta\left[3\left(33 - 70\eta^2 + 2\eta^4\right) + 14S^2H^2\left(-3 + 5\eta^2\right)\right] + O\left(\mathrm{Re}^2\right) \tag{16}$$

where $\eta = y/S$. The shear stress at the boundary of the channel is given by

$$\tau_w = -\frac{1}{1 + {b_x}^2}\left[\left(\sigma_{yy} - \sigma_{xx}\right)b_x + \left(1 - {b_x}^2\right)\sigma_{xy}\right] \quad \text{on } y = b(x), \tag{17}$$

where σ_{yy} , σ_{xx} , σ_{xy} are the usual stress components, *i.e.*,

$$\begin{aligned} \sigma_{xy} &= \mu\left[\psi_{yy} - \psi_{xx}\right] \\ \sigma_{yy} - \sigma_{xx} &= -4\mu\psi_{xy} . \end{aligned} \tag{18}$$

The subscripts (x, y) denote partial differentiation with respect to (x, y), respectively. The dimensionless form of wall shear stress can be written as

$$G = \frac{a^2 S^2}{\mu Q}\tau_w = -\frac{S^2}{\left(1+\varepsilon^2 S_x{}^2\right)}\left[\left(\psi_{yy} - \varepsilon^2 \psi_{xx}\right)\left(1-\varepsilon^2 S_x{}^2\right) - 4\varepsilon^2 S_x \psi_{xy}\right] \quad \text{on } y = S \tag{19}$$

and for $0 < \varepsilon << 1$ obtain

$$G = 3 + \frac{\mathrm{Re}}{35 S^2}\left(-6 + 7S^2H^2\right) + O\left(\mathrm{Re}^2\right) \tag{20}$$

IV. Internal Flow Separation

We have investigated the solution behavior by algebraic programming language (MAPLE). The first 19 coefficients for the above solution series have been obtained which represent the flow characteristics. The above series are reformed into several diagonal Pade´ approximants of order $N = M + M$ as

$$G = \sum_{i=0}^{N} f_i \,\mathrm{Re}^i = \frac{\sum_{i=0}^{M} a_i \,\mathrm{Re}^i}{\sum_{i=0}^{M} c_i \,\mathrm{Re}^i} \tag{21}$$

This method fails when the denominator of the fraction is evaluated near the zeros. By equating the numerator of equation (21) to zero we have computed the Reynolds number at which separation occurs in the flow field (*i.e.* $G \to 0$) for different values of H at position $S = 1$ on the channel, as shown in Table 1.

Table 1. Computations showing the Reynolds number for internal flow separation Development at S = 1 in the diverging channel.

H	0	0.5	1	1.5	2
Re	4.86321	5.15031	5.35771	6.16032	7.53121

V. Results And Discussion

Since the fluid is incompressible and viscous, the above mathematical analysis is very suitable for liquid. Flow separations have been observed in the diverging channel and computed numerically as shown in Table 1, which represents that the flow Reynolds number increases as the magnetic field intensity increases in magnitude. But the increasing rate of the flow Reynolds numbers is not so high as given in Makinde and Mhone (2006).

Fig. 2(a) below shows the axial velocity component (u) profile at S = 1 in the diverging channel. A parabolic axial velocity profile is observed with maximum value at the channel centerline and minimum value at the boundaries. Around the centerline of the channel it is observed that the effect of increasing values of the magnetic field intensity is to decrease the magnitude of the axial fluid velocity. Fig. 2(b) shows the normal velocity component (v) profile in the diverging channel where the maximum value is observed near the middle position of the channel centerline and the boundary. But at the channel centerline and boundary the minimum value of the normal velocity profile is observed. However, a general increase in the magnitude of normal velocity profile is noticed with an increase in magnetic field intensity.

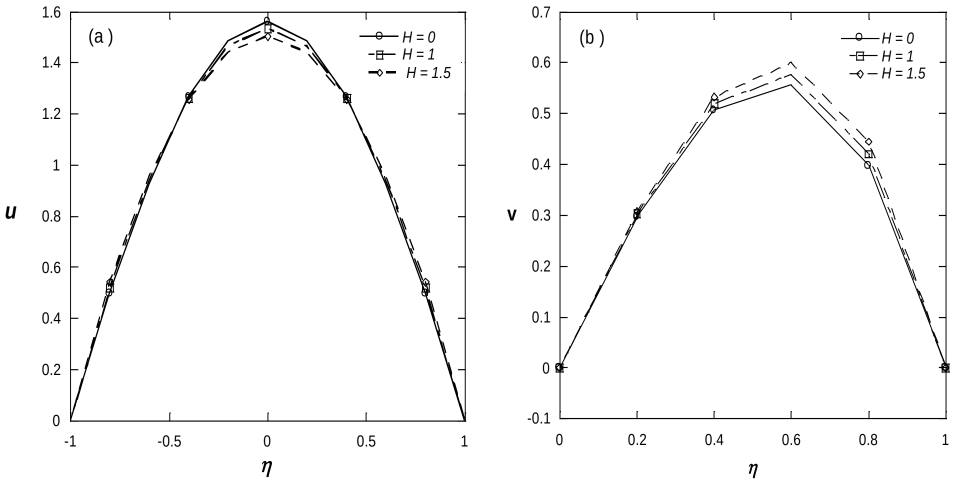

Fig. 2. Axial and normal velocity profiles for different values of *H*; *S* = 1 and Re = 1.

Fig. 3 represents the wall shear stress (*G*) with respect to flow Reynolds number at *S* = 1 in the diverging channel. The magnitude of wall shear stress decrease with an increase in flow Reynolds number for every value of magnetic field intensity. Here it is observed that the requirement of flow Reynolds number for the development of internal flow separation increases as the magnetic field intensity increases in magnitude. This situation is identical to those of Makinde and Mhone (2006), but in this case the separation occurs more earlier.

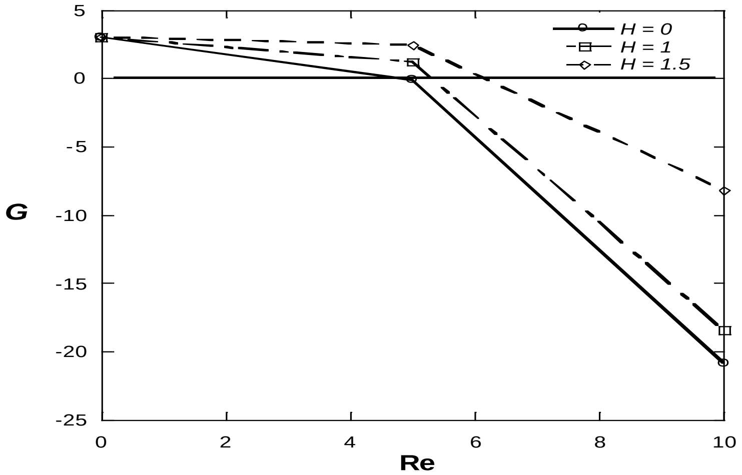

Fig. 3. Wall shear stress at *S* = 1 for different values of *H*.

Since the flow Reynolds number increases gradually with an increase in magnetic field intensity, so for the further development of internal flow separation it is possible to get reasonable values of flow Reynolds number for some higher values of *H* such as *H* = 2, 2.5. But a further increase in magnetic field intensity may suppress or totally prevent the development of internal flow separation in the diverging channel. From Table 1 and Fig. 3 it is also clear that internal flow separation is still possible at low magnetic field intensity if flow Reynolds number is sufficiently high. Hence, in order to prevent the occurrence of internal flow separation in the diverging channel, the imposed external magnetic field intensity on the conducting fluid must be sufficiently high.

VI. Conclusions

We investigated the effects of the externally applied homogeneous magnetic field on the flow for the development of velocity profiles and internal flow separation at a given position in the diverging channel. Our results revealed that early separation occurred with a decrease in magnetic field intensity. We also noticed that the effect of increasing values of the magnetic field intensity is to decrease the magnitude of the axial fluid velocity around the centerline of the channel, but between the centerline and boundary the normal fluid velocity increases by magnetic field. Hence, magnetic field has great influence on fluid velocity components and for the development internal flow separation in the diverging channel.

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