

Application of Intuitionist Fuzzy Soft Matrices in Decision Making Problem by Using Medical Diagnosis

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Abstract: Soft set theory is a newly emerging mathematical tool to deal with uncertain problems. In this paper, we proposed the definition of T-product of intuitionistic fuzzy soft matrices with examples. Finally, We extend our approach in application of these matrices in decision making problem, by using medical diagnosis.

Keywords: Soft set, Fuzzy soft set(FSS), Fuzzy soft matrix, T-product of fuzzy soft matrix, Intuitionistic fuzzy soft set(IFSS), Intuitionistic fuzzy soft matrix.

I. Introduction

In 1965, Fuzzy set was introduced by Lotfi.A.Zadeh [15]. An intuitionistic fuzzy was introduced in 1983 by K.Atanassov [1] as an extension of Zadeh's fuzzy set. In the year 1999, Molodtsov [7] introduced soft set theory as a mathematical tool for dealing with the uncertainties. Molodtsov has categorically stated number of applications of this theory in solving practical problems in engineering, medical science, environment, social sciences and management etc. Recently, there are variety of models of medical diagnosis under the general frame work of fuzzy set theory involving fuzzy soft matrices to deal with different complicating aspects of medical diagnosis.

In 2003, Saikia et.al.[9] have extended the method in [4] using intuitionistic fuzzy soft set theory. In 2004, Maji et.al [6] extended soft sets to intuitionistic fuzzy soft sets. In 2010, Chetia and Das [3] have studied Sancher's[11,12] approach of medical diagnosis through IVFSS obtaining an improvement of the same presented in De et.al[4]. In 2010, Cagman et al[2] introduced a new definition of soft set based decision making method which selects the optimum elements from the alternatives.

In 2011, Yong yang et al.[14], have defined T-product of intuitionistic fuzzy soft matrices in continuation of T-product of fuzzy soft matrices. In 2012, Chetia and Das [3] mention IFSM and establish some results on them. In 2013, Rajarajeswary and Dhanalakshmi [8] described intuitionistic fuzzy soft matrices in decision making problems. In 2013, Jalilul and Tapan Kumar Roy[5] introduced some properties on intuitionistic fuzzy soft matrices. In 2013, Saikia, Boruah and Das[10] extended decision making problem of T-Product of intuitionistic fuzzy soft matrices. In 2014, Dr.N.Sarala and S.Rajkumari[13] have introduced intuitionistic fuzzy soft matrices in agriculture.

In this paper, we proposed IFSM and defined T-Product of intuitionistic fuzzy soft matrices with examples. Finally, we extend our approach in decision making problem which played an important role in the fast moving world. For this we have defined decision function and optimum fuzzy set.

II. Preliminaries

In this we section, We recall some basic essential notion of fuzzy soft set theory and T-Product of fuzzy soft matrices.

2.1 Soft Set [7]

Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power Set of U . Let $A \subseteq E$. A pair (F_A, E) is called a soft set over U , where F_A is a mapping given by $F_A : E \rightarrow P(U)$ Such that $F_A(e) = \emptyset$ if $e \notin A$.

Here F_A is called approximate function of the soft set (F_A, E) . The set $F_A(e)$ is called e-approximate value set which consist of related objects of the parameter $e \in E$. In other words, a soft set over U is a parameterized family of subsets of the universe U .

Example 2.1:

Let $U = \{u_1, u_2, u_3, u_4\}$ be a set of four shirts and $E = \{\text{blue}(e_1), \text{green}(e_2), \text{yellow}(e_3)\}$ be a set of parameters. If $A = \{e_1, e_3\} \subseteq E$. Let $F_A(e_1) = \{u_1, u_3, u_4\}$ and $F_A(e_3) = \{u_1, u_2, u_4\}$ then we write the soft set $(F_A, E) = \{(e_1, \{u_1, u_3, u_4\}), (e_3, \{u_1, u_2, u_4\})\}$ over U which describe the "colour of the shirts" Which Mr.X is going to buy.

We may represent the soft set in the following form:

U	Blue(e ₁)	Green(e ₂)	Yellow(e ₃)
u ₁	1	0	1
u ₂	0	0	1
u ₃	1	0	0
u ₄	1	0	1

TABLE 2.1.1

2.2 Fuzzy Soft Set [6]

Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the set of all fuzzy sets of U. Let A ⊆ E. A pair (F_A, E) is called a fuzzy soft set (FSS) over U, where F_A is a mapping given by, F_A: E → P(U) Such that F_A(e) = $\tilde{\varphi}$ if e ∉ A, Where $\tilde{\varphi}$ is a null fuzzy set.

Example 2.2:

Consider the example 2.1., here we cannot express with only two real numbers 0 and 1, we can characterized it by a membership function instead of crisp number 0 and 1, which associate with each element a real number in the interval [0,1]. Then

$$(F_A, E) = \{ F_A(e_1) = \{ (u_1, 0.3), (u_2, 0.9), (u_3, 0.4), (u_4, 0.7) \},$$

$F_A(e_3) = \{ (u_1, 0.5), (u_2, 0.6), (u_4, 0.8) \} \}$ is the fuzzy soft set representing the “colour of the shirts” which Mr.X is going to buy.

We may represent the fuzzy soft set in the following form:

U	Blue(e ₁)	Green(e ₂)	Yellow(e ₃)
u ₁	0.3	0.0	0.5
u ₂	0.9	0.0	0.6
u ₃	0.4	0.0	0.0
u ₄	0.7	0.0	0.8

TABLE 2.2.2

2.3 Fuzzy Soft Matrices(FSM) [3]

Let (F_A, E) be a fuzzy soft set over U. Then a subset of U × E is uniquely defined by R_A = { (u, e) : e ∈ A, u ∈ F_A(e) } which is called relation form of (F_A, E). The characteristic function of R_A is written by $\mu_{R_A} : U \times E \rightarrow [0, 1]$, where $\mu_{R_A}(u, e) \in [0, 1]$ is the membership value of u ∈ U for each e ∈ U.

If $[\mu_{ij}] = \mu_{R_A}(u_i, e_j)$, we can define a matrix

$$[\mu_{ij}]_{m \times n} = \begin{bmatrix} \mu_{11} & \mu_{12} & \dots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \dots & \mu_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \mu_{m1} & \mu_{m2} & \dots & \mu_{mn} \end{bmatrix}$$

Which is called an m × n soft matrix of the soft set (F_A, E) over U.

Therefore we can say that a fuzzy soft set (F_A, E) is uniquely characterized by the matrix $[\mu_{ij}]_{m \times n}$ and both concepts are interchangeable.

The set of all m × n fuzzy soft matrices over U will be denoted by FSM_{m × n}.

Example 2.3:

Assume that U = { u₁, u₂, u₃, u₄, u₅ } is a universal set and E = { e₁, e₂, e₃, e₄ } is a set of all parameters. If A ⊆ E = { e₁, e₂, e₄ } and

$$F_A(e_1) = \{ (u_1, 0.3), (u_2, 0.4), (u_3, 0.6), (u_4, 0.7), (u_5, 0.5) \}$$

$$F_A(e_2) = \{ (u_1, 0.2), (u_2, 0.7), (u_3, 0.1), (u_4, 0.8), (u_5, 0.6) \}$$

$$F_A(e_4) = \{ (u_1, 0.1), (u_2, 0.3), (u_3, 0.5), (u_4, 0.4), (u_5, 0.9) \}$$

Then the fuzzy soft set (F_A, E) is a parameterized family { F_A(e₁), F_A(e₂), F_A(e₄) } of all fuzzy set over U.

Hence the fuzzy soft matrix $[\mu_{ij}]$ can be written as

$$[\mu_{ij}] = \begin{bmatrix} 0.3 & 0.2 & 0.0 & 0.1 \\ 0.4 & 0.7 & 0.0 & 0.3 \\ 0.6 & 0.1 & 0.0 & 0.5 \\ 0.7 & 0.8 & 0.0 & 0.4 \\ 0.5 & 0.6 & 0.0 & 0.9 \end{bmatrix}$$

2.4 T-Product of Fuzzy Soft Matrices[10]

Let $\tilde{A}_k = [\mu_{ij}^k] \in \text{FSM}_{m \times n}$, $1 \leq k \leq l$. Then the T-product of fuzzy soft matrices, denoted as $\prod_{k=1}^l \tilde{A}_k = \tilde{A}_1 \times \tilde{A}_2 \times \dots \times \tilde{A}_l$, is defined by $\prod_{k=1}^l \tilde{A}_k = [c_i]_{m \times 1}$, where $c_i = \frac{1}{n} \sum_{j=1}^n \prod_{k=1}^l \mu_{ij}^k$ and $T = * \text{ or } \diamond$ according to the type of problems.

Example 2.4:

Assume that $\tilde{A}_1, \tilde{A}_2, \tilde{A}_3 \in \text{FSM}_{4 \times 2}$ are given as follows:

$$\tilde{A}_1 = \begin{bmatrix} 0.7 & 0.2 \\ 0.8 & 0.1 \\ 0.3 & 0.6 \\ 0.9 & 0.1 \end{bmatrix}, \tilde{A}_2 = \begin{bmatrix} 0.1 & 0.3 \\ 0.6 & 0.7 \\ 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}, \tilde{A}_3 = \begin{bmatrix} 0.5 & 0.8 \\ 0.3 & 0.4 \\ 0.2 & 0.3 \\ 0.6 & 0.5 \end{bmatrix}$$

Then the * product is

$$\begin{aligned} \prod_{k=1}^3 \tilde{A}_k &= \tilde{A}_1 \times \tilde{A}_2 \times \tilde{A}_3 \\ &= \begin{bmatrix} 0.7 * 0.1 * 0.5 + 0.2 * 0.3 * 0.8 \\ 0.8 * 0.6 * 0.3 + 0.1 * 0.7 * 0.4 \\ 0.3 * 0.8 * 0.2 + 0.6 * 0.3 * 0.3 \\ 0.9 * 0.2 * 0.6 + 0.1 * 0.7 * 0.5 \end{bmatrix} \\ &= \begin{bmatrix} 0.083 \\ 0.172 \\ 0.102 \\ 0.143 \end{bmatrix} \end{aligned}$$

2.5 Intuitionistic Fuzzy Soft Set (IFSS) [6]

Let U be an initial universe, E be the set of parameters and $A \subseteq E$. A pair (\tilde{F}_A, E) is called an intuitionistic fuzzy soft set (IFSS) over U, where \tilde{F}_A is a mapping given by $\tilde{F}_A: E \rightarrow I^U$, where I^U denotes the collection of all intuitionistic fuzzy subsets of U.

Example 2.5:

Suppose that $U = \{u_1, u_2, u_3, u_4\}$ be a set of four shirts and $E = \{\text{Blue}(e_1), \text{Green}(e_2), \text{Yellow}(e_3)\}$ be a set of parameters. If $A = \{e_1, e_3\} \in E$.

Let $\tilde{F}_A(e_1) = \{(u_1, 0.9, 0.1), (u_2, 0.3, 0.4), (u_3, 0.6, 0.2), (u_4, 0.7, 0.3)\}$

$\tilde{F}_A(e_3) = \{(u_1, 0.4, 0.5), (u_2, 0.8, 0.1), (u_3, 0.3, 0.7), (u_4, 0.2, 0.5)\}$

then we write intuitionistic fuzzy soft set is

$(\tilde{F}_A, E) = \{ \tilde{F}_A(e_1) = \{(u_1, 0.9, 0.1), (u_2, 0.3, 0.4), (u_3, 0.6, 0.2), (u_4, 0.7, 0.3)\}$

$\tilde{F}_A(e_3) = \{(u_1, 0.4, 0.5), (u_2, 0.8, 0.1), (u_3, 0.3, 0.7), (u_4, 0.2, 0.5)\} \}$

We would represent this intuitionistic fuzzy soft set in matrix form as

$$\begin{bmatrix} (0.9, 0.1) & (0.0, 0.0) & (0.4, 0.5) \\ (0.3, 0.4) & (0.0, 0.0) & (0.8, 0.1) \\ (0.6, 0.2) & (0.0, 0.0) & (0.3, 0.7) \\ (0.7, 0.3) & (0.0, 0.0) & (0.2, 0.5) \end{bmatrix}$$

2.6 Intuitionistic Fuzzy Soft Matrices (IFSM) [3]

Let U be an initial universe, E be the set of parameters and $A \subseteq E$. Let (\tilde{F}_A, E) be an intuitionistic fuzzy soft set (IFSS) over U. Then a subset of $U \times E$ is uniquely defined by

$$R_A = \{(u, e) : e \in A, u \in \tilde{F}_A(e)\}$$

Which is called relation form of (\tilde{F}_A, E) . The membership and non-membership functions are written by $\mu_{R_A} : U \times E \rightarrow [0,1]$ and $\gamma_{R_A} : U \times E \rightarrow [0,1]$ where $\mu_{R_A} : (u,e) \in [0,1]$ and $\gamma_{R_A} : (u,e) \in [0,1]$ are the membership value and non-membership value of $u \in U$ for each $e \in E$.

If $(\mu_{ij}, \gamma_{ij}) = (\mu_{R_A}(u_i, e_j), \gamma_{R_A}(u_i, e_j))$ we can define a matrix

$$[(\mu_{ij}, \gamma_{ij})]_{m \times n} = \begin{bmatrix} (\mu_{11}, \gamma_{11}) & (\mu_{12}, \gamma_{12}) & \cdots & (\mu_{1n}, \gamma_{1n}) \\ (\mu_{21}, \gamma_{21}) & (\mu_{22}, \gamma_{22}) & \cdots & (\mu_{2n}, \gamma_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (\mu_{m1}, \gamma_{m1}) & (\mu_{m2}, \gamma_{m2}) & \cdots & (\mu_{mn}, \gamma_{mn}) \end{bmatrix}$$

Which is called an $m \times n$ IFSM of the IFSS (\tilde{F}_A, E) over U . Therefore, we can say that IFSS (\tilde{F}_A, E) is uniquely characterized by the matrix $[(\mu_{ij}, \gamma_{ij})]_{m \times n}$ and both concepts are interchangeable. The set of all $m \times n$ IFS matrices will be denoted by $\text{IFSM}_{m \times n}$.

Example 2.6 :

Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{e_1, e_2, e_3, e_4\}$ is a set of parameters. If $A = \{e_1, e_2, e_4\} \subseteq E$ and

$$\tilde{F}_A(e_1) = \{(u_1, 0.6, 0.4), (u_2, 0.9, 0.1), (u_3, 0.2, 0.7), (u_4, 0.3, 0.5)\}$$

$$\tilde{F}_A(e_2) = \{(u_1, 1.0, 0.0), (u_2, 0.3, 0.7), (u_3, 0.2, 0.6), (u_4, 0.5, 0.4)\}$$

$$\tilde{F}_A(e_4) = \{(u_1, 0.8, 0.1), (u_2, 0.4, 0.6), (u_3, 0.7, 0.2), (u_4, 0.3, 0.4)\}$$

Then the IFS set (\tilde{F}_A, E) is a parameterized family $\{\tilde{F}_A(e_1), \tilde{F}_A(e_2), \tilde{F}_A(e_4)\}$ of all IFS sets over U .

Hence IFSM $[(\mu_{ij}, \gamma_{ij})]$ can be written as

$$[(\mu_{ij}, \gamma_{ij})] = \begin{bmatrix} (0.6, 0.4) & (1.0, 0.0) & (0.0, 0.0) & (0.8, 0.1) \\ (0.9, 0.1) & (0.3, 0.7) & (0.0, 0.0) & (0.4, 0.6) \\ (0.2, 0.7) & (0.2, 0.6) & (0.0, 0.0) & (0.7, 0.2) \\ (0.3, 0.5) & (0.5, 0.4) & (0.0, 0.0) & (0.3, 0.4) \end{bmatrix}$$

2.7 Union of Intuitionistic Fuzzy Soft Matrices[5]

If $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})]$, $\tilde{B} = [(\mu_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{B}})] \in \text{IFSM}_{m \times n}$. Then union \tilde{A} and \tilde{B} is denoted by $\tilde{A} \cup \tilde{B}$ is defined as $\tilde{A} \cup \tilde{B} = [\max\{\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}\}, \min\{\gamma_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{B}}\}]$ for all i and j .

III. T-Product Of Intuitionistic Fuzzy Soft Matrices

In this section, we analyse the work and advocated the definition in two types of T-product of intuitionistic fuzzy soft matrices and some properties.

Definition3.1:

Let $\tilde{A}_k = [(\mu_{ij}^k, \gamma_{ij}^k)] \in \text{IFSM}_{m \times n}$, for $1 \leq k \leq l$. Then T-product X_1 of \tilde{A}_k 's for all $1 \leq k \leq l$, denoted by $\prod_{k=1}^l \tilde{A}_k$, is defined as $\prod_{k=1}^l \tilde{A}_k = [(\mu_{ij}^1, \gamma_{ij}^1) X_1 (\mu_{ij}^2, \gamma_{ij}^2) X_1 \cdots X_1 (\mu_{ij}^l, \gamma_{ij}^l)] = [(\mu_{i1}^{\prime\prime}, \gamma_{i1}^{\prime\prime})]_{m \times 1}$, Where $\mu_{i1}^{\prime\prime} = \frac{1}{n} \sum_{j=1}^n [\max\{\mu_{ij}^1, \mu_{ij}^2, \dots, \mu_{ij}^l\}]$ and $\gamma_{i1}^{\prime\prime} = \frac{1}{n} \sum_{j=1}^n [\min\{\gamma_{ij}^1, \gamma_{ij}^2, \dots, \gamma_{ij}^l\}]$; $i = 1, 2, \dots, m$.

Definition3.2:

Let $\tilde{A}_k = [(\mu_{ij}^k, \gamma_{ij}^k)] \in \text{IFSM}_{m \times n}$, for $1 \leq k \leq l$. Then T-product X_2 of \tilde{A}_k 's for all $1 \leq k \leq l$, denoted by $\prod_{k=1}^l \tilde{A}_k$, is defined as $\prod_{k=1}^l \tilde{A}_k = [(\mu_{ij}^1, \gamma_{ij}^1) X_2 (\mu_{ij}^2, \gamma_{ij}^2) X_2 \cdots X_2 (\mu_{ij}^l, \gamma_{ij}^l)] = [(\mu_{i1}^{\prime\prime\prime}, \gamma_{i1}^{\prime\prime\prime})]_{m \times 1}$, Where $\mu_{i1}^{\prime\prime\prime} = \frac{1}{n} \sum_{j=1}^n [\min\{\mu_{ij}^1, \mu_{ij}^2, \dots, \mu_{ij}^l\}]$ and $\gamma_{i1}^{\prime\prime\prime} = \frac{1}{n} \sum_{j=1}^n [\max\{\gamma_{ij}^1, \gamma_{ij}^2, \dots, \gamma_{ij}^l\}]$; $i = 1, 2, \dots, m$. To illustrate these products, let we denote the following example.

Example 3.3:

Assume that $\tilde{A}_1, \tilde{A}_2, \tilde{A}_3 \in \text{IFSM}_{3 \times 2}$ are given as follows:

$$\tilde{A}_1 = \begin{bmatrix} (0.4, 0.1) & (0.5, 0.3) \\ (0.2, 0.7) & (0.6, 0.2) \\ (0.3, 0.4) & (0.1, 0.8) \end{bmatrix}, \tilde{A}_2 = \begin{bmatrix} (0.8, 0.2) & (0.6, 0.4) \\ (0.7, 0.3) & (0.5, 0.2) \\ (0.4, 0.5) & (0.3, 0.1) \end{bmatrix}, \tilde{A}_3 = \begin{bmatrix} (0.7, 0.3) & (0.4, 0.3) \\ (0.5, 0.4) & (0.8, 0.1) \\ (0.6, 0.2) & (0.2, 0.7) \end{bmatrix}$$

Then the T-product X_1 of intuitionistic fuzzy soft matrices \tilde{A}_1, \tilde{A}_2 and \tilde{A}_3 is given by

$$\prod_{k=1}^3 \tilde{A}_k = \tilde{A}_1 X_1 \tilde{A}_2 X_1 \tilde{A}_3$$

$$\begin{aligned}
 &= \frac{1}{2} \begin{bmatrix} \max\{0.4,0.8,0.7\} + \max\{0.5,0.6,0.4\}, \min\{0.1,0.2,0.3\} + \min\{0.3,0.4,0.3\} \\ \max\{0.2,0.7,0.5\} + \max\{0.6,0.5,0.8\}, \min\{0.7,0.3,0.4\} + \min\{0.2,0.2,0.1\} \\ \max\{0.3,0.4,0.6\} + \max\{0.1,0.3,0.2\}, \min\{0.4,0.5,0.2\} + \min\{0.8,0.1,0.7\} \end{bmatrix} \\
 &= \frac{1}{2} \begin{bmatrix} (0.8 + 0.6, 0.1 + 0.3) \\ (0.7 + 0.8, 0.3 + 0.1) \\ (0.6 + 0.3, 0.4 + 0.1) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} (1.4, 0.4) \\ (1.5, 0.4) \\ (0.9, 0.5) \end{bmatrix} = \begin{bmatrix} (0.70, 0.20) \\ (0.75, 0.20) \\ (0.45, 0.25) \end{bmatrix}
 \end{aligned}$$

Similarly T-product X_2 of intuitionistic fuzzy soft matrices \tilde{A}_1, \tilde{A}_2 and \tilde{A}_3 is given by

$$\begin{aligned}
 \prod_{k=1}^3 \tilde{A}_k &= \tilde{A}_1 X_2 \tilde{A}_2 X_2 \tilde{A}_3 \\
 &= \frac{1}{2} \begin{bmatrix} \min\{0.4,0.8,0.7\} + \min\{0.5,0.6,0.4\}, \max\{0.1,0.2,0.3\} + \max\{0.3,0.4,0.3\} \\ \min\{0.2,0.7,0.5\} + \min\{0.6,0.5,0.8\}, \max\{0.7,0.3,0.4\} + \max\{0.2,0.2,0.1\} \\ \min\{0.3,0.4,0.6\} + \min\{0.1,0.3,0.2\}, \max\{0.4,0.5,0.2\} + \max\{0.8,0.1,0.7\} \end{bmatrix} \\
 &= \frac{1}{2} \begin{bmatrix} (0.4 + 0.4, 0.3 + 0.4) \\ (0.2 + 0.5, 0.7 + 0.2) \\ (0.3 + 0.1, 0.5 + 0.8) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} (0.8, 0.7) \\ (0.7, 0.9) \\ (0.4, 1.3) \end{bmatrix} = \begin{bmatrix} (0.40, 0.35) \\ (0.35, 0.45) \\ (0.20, 0.65) \end{bmatrix}
 \end{aligned}$$

Proportion 3.4:

Let $\tilde{A}, \tilde{B}, \tilde{C} \in \text{IFSM}_{m \times n}$. Then (i) $\tilde{A} \otimes \tilde{B} = \tilde{B} \otimes \tilde{A}$, (ii) $(\tilde{A} \otimes \tilde{B}) \otimes \tilde{C} = \tilde{A} \otimes (\tilde{B} \otimes \tilde{C})$, where \otimes denotes the T-product X_1 or X_2 of IFSM.

Proof:

(i) Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})]$ and $\tilde{B} = [(\mu_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{B}})]$
 Then $\tilde{A} \otimes \tilde{B} = \sum_{i=1, j=1}^{m, n} [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})] \otimes [(\mu_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{B}})]$
 $= \sum_{i=1, j=1}^{m, n} [(\mu_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{B}})] \otimes [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})] = \tilde{B} \otimes \tilde{A}$.

(ii) Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})]$, $\tilde{B} = [(\mu_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{B}})]$ and $\tilde{C} = [(\mu_{ij}^{\tilde{C}}, \gamma_{ij}^{\tilde{C}})]$
 Then $(\tilde{A} \otimes \tilde{B}) \otimes \tilde{C} = (\sum_{i=1, j=1}^{m, n} [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})] \otimes [(\mu_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{B}})]) \otimes [(\mu_{ij}^{\tilde{C}}, \gamma_{ij}^{\tilde{C}})]$
 $= [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})] \otimes (\sum_{i=1, j=1}^{m, n} [(\mu_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{B}})] \otimes [(\mu_{ij}^{\tilde{C}}, \gamma_{ij}^{\tilde{C}})])$
 $= \tilde{A} \otimes (\tilde{B} \otimes \tilde{C})$

Hence the result is as such proved.

Proportion 3.5:

Let $\tilde{A}, \tilde{B}, \tilde{C} \in \text{IFSM}_{m \times n}$. Then (i) $\tilde{A} \tilde{\cup} (\tilde{B} \otimes \tilde{C}) = (\tilde{A} \tilde{\cup} \tilde{B}) \otimes (\tilde{A} \tilde{\cup} \tilde{C})$,
 (ii) $\tilde{A} \otimes (\tilde{B} \tilde{\cup} \tilde{C}) = (\tilde{A} \otimes \tilde{B}) \tilde{\cup} (\tilde{A} \otimes \tilde{C})$, where \otimes denotes the T-product X_1 or X_2 of IFSM.

Proof:

(i) Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})]$, $\tilde{B} = [(\mu_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{B}})]$ and $\tilde{C} = [(\mu_{ij}^{\tilde{C}}, \gamma_{ij}^{\tilde{C}})]$
 Then $\tilde{A} \tilde{\cup} (\tilde{B} \otimes \tilde{C}) = [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})] \tilde{\cup} ([(\mu_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{B}})] \otimes [(\mu_{ij}^{\tilde{C}}, \gamma_{ij}^{\tilde{C}})])$
 $= ([(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})] \tilde{\cup} [(\mu_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{B}})]) \otimes ([(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})] \tilde{\cup} [(\mu_{ij}^{\tilde{C}}, \gamma_{ij}^{\tilde{C}})])$
 $= (\max\{\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}\}, \min\{\gamma_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{B}}\}) \otimes (\max\{\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{C}}\}, \min\{\gamma_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{C}}\})$
 $= (\tilde{A} \tilde{\cup} \tilde{B}) \otimes (\tilde{A} \tilde{\cup} \tilde{C})$.

Similarly, we can prove(ii) also.

IV. Application Of Ifsm In Medical Diagnosis

In this section, we are submitting the problem which is based on T-product X_1 of IFSM.

4.1 Intuitionistic Fuzzy Soft Matrices in Medical Diagnosis:

In this field, we put forward four kinds of diabetic patients went to a hospital for checking blood sugar contents in their body and there by three type of diabetes symptoms are ranked according to the glucose level.

Let us assume that S is the set of symptoms of diabetic disease and u_1, u_2, u_3 and u_4 are the set of patients. We should call this matrix as the symptoms of diabetic disease matrix.

Let N number of patients went to check up their blood sugar in a hospital and jointly from the m number of objects which have n number of features, ie., parameter(E). Here the decision maker has his own choice of parameters belonging to the parameter set E. In this decision making problem it is assumed that the parameter presented in intuitionistic fuzzy soft set format and then it is changed in the form of intuitionistic fuzzy soft matrix. We have to find out the object out of these m objects.

Now, we apply T-product of IFSM in decision making problem. For this, we first define maximum score set and optimum decision set.

Definition 4.2:

Let $\tilde{A}_k \in \text{IFSM}_{m \times n}$, $k = 1, 2, \dots, l$. The T-product (say X_1) is $\prod_{k=1}^l \tilde{A}_k = [(\mu_{i1}^{\tilde{A}_k}, \gamma_{i1}^{\tilde{A}_k})]_{m \times 1}$. Then the set $O_s = \{j: c_j = \max\{\mu_{i1}^{\tilde{A}_k} - \gamma_{i1}^{\tilde{A}_k}\}, i = 1, 2, \dots, m\}$ is called the maximum score set and $O_d = \{u_j: u_j \in U \text{ and } j \in O_s\}$ is called the optimum decision set of U.

V. Solution Procedure

The algorithm for the solution is given below.

5.1 ALGORITHM:

Input : Intuitionistic fuzzy soft sets with m objects, each of which has n parameters.

Output: An optimum set.

Step1: Choose the set of parameters.

Step2: Construct the intuitionistic fuzzy soft matrices for each of the parameters.

Step3: Compute T-product of the intuitionistic fuzzy soft matrices.

Step4: Find the maximum score set.

Step5: Find the optimum decision set.

VI. Case Study

Let $U = \{u_1, u_2, u_3, u_4\}$ be the set of four persons related with diabetic disease. The four persons have been went to a hospital for medical treatment and all the four persons were identified by diabetic and in this case all the person were given proper medical treatment and all of them have been cured, but particularly one person was cured in an extraordinary manner by getting maximum score point.

Step1: Let us consider $E = \{e_1, e_2, e_3, e_4\}$ as the set of parameter for identifying the person affected by diabetic.

e_1 is the patient suffers from frequent need to urinate,

e_2 is the patient suffers from irritation and itching,

e_3 is the patient suffers from excessive thirst and hunger,

and e_4 is the patient suffers from blurred vision.

Step2: Let \tilde{A} , \tilde{B} and \tilde{C} be intuitionistic fuzzy soft matrices on the basis of glucose contents in the blood. Let it may be classified as Type 1 namely found in childhood, Type 2 namely mostly found in 40 aged people and Type 3 found during pregnancy and these are all identified as diabetic kind of diseases.

$$\tilde{A} = \begin{bmatrix} (0.5,0.2) & (0.7,0.3) & (0.4,0.2) & (0.6,0.4) \\ (0.6,0.3) & (0.5,0.4) & (0.5,0.5) & (0.3,0.2) \\ (0.4,0.2) & (0.4,0.4) & (0.3,0.6) & (0.8,0.1) \\ (0.5,0.4) & (0.3,0.5) & (0.7,0.1) & (0.4,0.5) \end{bmatrix}, \tilde{B} = \begin{bmatrix} (0.4,0.3) & (0.3,0.2) & (0.6,0.3) & (0.4,0.6) \\ (0.8,0.1) & (0.1,0.7) & (0.5,0.3) & (0.6,0.2) \\ (0.5,0.4) & (0.4,0.6) & (0.3,0.7) & (0.9,0.1) \\ (0.3,0.2) & (0.6,0.3) & (0.5,0.4) & (0.2,0.6) \end{bmatrix}$$

$$\tilde{C} = \begin{bmatrix} (0.2,0.7) & (0.3,0.5) & (0.7,0.2) & (0.3,0.6) \\ (0.6,0.4) & (0.5,0.4) & (0.6,0.2) & (0.3,0.2) \\ (0.5,0.3) & (0.7,0.1) & (0.4,0.2) & (0.1,0.8) \\ (0.2,0.6) & (0.6,0.3) & (0.4,0.6) & (0.3,0.5) \end{bmatrix}$$

Step3: Now, we compute T-product X_1 between the three matrices. i.e.,

$$\tilde{A}X_1\tilde{B}X_1\tilde{C} = \frac{1}{4} \begin{bmatrix} (0.5 + 0.7 + 0.7 + 0.6, 0.2 + 0.2 + 0.2 + 0.4) \\ (0.8 + 0.5 + 0.6 + 0.6, 0.1 + 0.4 + 0.2 + 0.1) \\ (0.5 + 0.7 + 0.4 + 0.9, 0.2 + 0.1 + 0.2 + 0.1) \\ (0.5 + 0.6 + 0.7 + 0.4, 0.2 + 0.3 + 0.1 + 0.5) \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} (2.5,1.0) \\ (2.5,0.8) \\ (2.5,0.6) \\ (2.2,1.1) \end{bmatrix} = \begin{bmatrix} (0.63,0.25) \\ (0.63,0.20) \\ (0.63,0.15) \\ (0.55,0.28) \end{bmatrix}$$

Step4: The maximum score set

$$O_s = \{c_1=0.63-0.25, c_2=0.63-0.20, c_3=0.63-0.15, c_4=0.55-0.28\}$$

$$= \{c_1=0.38, c_2=0.43, c_3=0.48, c_4=0.27\}$$

Step5: The maximum score $c_3 = 0.48$, scored by u_3 and the patient was highly cured and the decision was in favour of patient u_3 who is observing as maximum curable person.

VII. Conclusion

In this paper, we proposed the theory of intuitionistic fuzzy soft matrices in the field of medical diagnosis, which will be applied intuitionistic fuzzy soft matrices in decision making problems and defined two different types of T-product of intuitionistic fuzzy soft matrices. We utilized one of them through a new solution procedure to solve real life decision problem which will involve more number of decision maker.

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