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Intuitionistic Fuzzy Hx Ring

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Abstract: *In this paper, we define the notion of intuitionistic fuzzy sub HX ring of a HX ring and some of their related properties are investigated. We define the necessity and possibility operators of an intuitionistic fuzzy subset of an intuitionistic fuzzy HX ring and discuss some of its properties. We introduce the concept of an image, pre-image of an intuitionistic fuzzy subset and discuss in detail a series of homomorphic and anti homomorphic properties of an intuitionistic fuzzy set are discussed.*

Keywords: *intuitionistic fuzzy set, fuzzy HX ring, intuitionistic fuzzy sub HX ring, homomorphism and anti homomorphism of an intuitionistic fuzzy HX ring, image and pre-image of an intuitionistic fuzzy set.*

I. Introduction

In 1965, Zadeh [12] introduced the concept of fuzzy subset μ of a set X as a function from X into the closed unit interval [0, 1] and studied their properties. Fuzzy set theory is a useful tool to describe situations in which the data or imprecise or vague and it is applied to logic , set theory, group theory, ring theory, real analysis, measure theory etc. In 1967, Rosenfeld [11] defined the idea of fuzzy subgroups and gave some of its properties. Li Hong Xing [5] introduced the concept of HX group. In 1982 Wang-jin Liu[7] introduced the concept of fuzzy ring and fuzzy ideal. With the successful upgrade of algebraic structure of group many researchers considered the algebraic structure of some other algebraic systems in which ring was considered as first. In 1988, Professor Li Hong Xing [6] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [2,3] gave the structures of HX ring on a class of ring. R.Muthuraj et.al[10] introduced the concept of fuzzy HX ring. In this paper we define a new algebraic structure of an intuitionistic fuzzy sub HX ring of a HX ring and investigate some related properties. We define the necessity and possibility operators of an intuitionistic fuzzy subset of an intuitionistic fuzzy HX ring and discuss some of its properties. Also we introduce the image and pre-image of an intuitionistic fuzzy set in an intuitionistic fuzzy HX ring and discuss some of its properties.

II. Preliminaries

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, R = (R ,+, $\cdot$) is a Ring, e is the additive identity element of R and xy, we mean x.y

2.1 Definition [1]

Let R be a ring. In $2^R - \{\phi\}$, a non-empty set $\vartheta \subset 2^R - \{\phi\}$ with two binary operation ' + ' and '.' is said to be a HX ring on R if ϑ is a ring with respect to the algebraic operation defined by

i. $A + B = \{a + b / a \in A \text{ and } b \in B\}$, which its null element is denoted by Q , and the negative element of A is denoted by – A.
ii. $AB = \{ab / a \in A \text{ and } b \in B\}$,
iii. $A (B + C) = AB + AC$ and $(B + C) A = BA + CA$.

2.2 Definition

Let R be a ring. Let μ be a fuzzy ring defined on R. Let $\vartheta \subset 2^R - \{\phi\}$ be a HX ring. A fuzzy subset λ^μ of ϑ is called a fuzzy HX ring on ϑ or a fuzzy ring induced by μ if the following conditions are satisfied. For all $A,B \in \vartheta$,

i. $\lambda^\mu (A - B) \geq \min \{ \lambda^\mu (A), \lambda^\mu (B) \}$,
ii. $\lambda^\mu (AB) \geq \min \{ \lambda^\mu (A), \lambda^\mu (B) \}$

where $\lambda^\mu (A) = \max\{ \mu(x) / \text{ for all } x \in A \subseteq R \}$.

III. Properties of an intuitionistic fuzzy HX subring

3.1 Definition

Let R be a ring. Let μ be a fuzzy ring on R and a nonempty set $\vartheta \subset 2^R - \{\phi\}$ is a HX ring. An intuitionistic fuzzy subset $\psi = \langle A, \lambda^{\mu}(A), \lambda^{\gamma}(A) \rangle$ of a HX ring ϑ is said to be an intuitionistic fuzzy HX (IFHX) subring of ϑ if the following conditions are satisfied. For all A , B $\in \vartheta$,

(i) $\lambda^{\mu}(A-B) \geq \min\{ \lambda^{\mu}(A), \lambda^{\mu}(B) \}$,

(ii) $\lambda^{\mu}(AB) \geq \min\{ \lambda^{\mu}(A), \lambda^{\mu}(B) \}$,

(iii) $\lambda^{\gamma}(A-B) \leq \max\{ \lambda^{\gamma}(A), \lambda^{\gamma}(B) \}$,

(iv) $\lambda^{\gamma}(AB) \leq \max\{ \lambda^{\gamma}(A), \lambda^{\gamma}(B) \}$

Where $\lambda^{\mu}(A) = \max\{ \mu(x) / x \in A \subseteq R\}$, $\lambda^{\gamma}(A) = \min\{\gamma(x) / x \in A \subseteq R\}$.

3.2 Definition

Let R be a ring. Let μ be a fuzzy ring on R and a nonempty set $\vartheta \subset 2^R - \{\phi\}$ is a HX ring. An intuitionistic fuzzy subset $\psi = \langle A, \lambda^{\mu}(A), \lambda^{\gamma}(A) \rangle$ of a HX ring ϑ is said to be an intuitionistic anti fuzzy HX (IAFHX) subring of ϑ if the following conditions are satisfied. For all A , B $\in \vartheta$,

(i) $\lambda^{\mu}(A-B) \leq \max\{ \lambda^{\mu}(A), \lambda^{\mu}(B) \}$,

(ii) $\lambda^{\mu}(AB) \leq \max\{ \lambda^{\mu}(A), \lambda^{\mu}(B) \}$,

(iii) $\lambda^{\gamma}(A-B) \geq \min\{ \lambda^{\gamma}(A), \lambda^{\gamma}(B) \}$,

(iv) $\lambda^{\gamma}(AB) \geq \min\{ \lambda^{\gamma}(A), \lambda^{\gamma}(B) \}$

Where $\lambda^{\mu}(A) = \min\{ \mu(x) / x \in A \subseteq R \}$, $\lambda^{\gamma}(A) = \max\{\gamma(x) / x \in A \subseteq R\}$.

3.3 Theorem

If ψ_1 and ψ_2 be two intuitionistic fuzzy HX subrings of a HX ring ϑ, then $\psi_1 \cap \psi_2$ is also intuitionistic fuzzy HX subrings of a HX ring ϑ.

Proof

Let $\psi_1 = \{\langle A, \lambda^{\mu}(A), \lambda^{\gamma}(A) \rangle / A \in \vartheta \}$ and $\psi_2 = \{\langle A, \Theta^{\mu}(A), \Theta^{\gamma}(A) \rangle / A \in \vartheta \}$ be two intuitionistic fuzzy HX subrings of a HX ring ϑ.

To Prove that $\psi_1 \cap \psi_2$ is also an intuitionistic fuzzy HX subring of a HX ring ϑ.

For any A,B $\in \vartheta$,we have

(i)
$$\begin{aligned}(\lambda^{\mu} \cap \Theta^{\mu})(A-B) &= \min\{ \lambda^{\mu}(A-B), \Theta^{\mu}(A-B)\} \\ &\geq \min\{ \min\{ \lambda^{\mu}(A), \lambda^{\mu}(B)\}, \min\{ \Theta^{\mu}(A), \Theta^{\mu}(B)\} \\ &= \min\{ \min\{ \lambda^{\mu}(A), \Theta^{\mu}(A) \}, \min\{ \lambda^{\mu}(B), \Theta^{\mu}(B)\} \\ &= \min\{(\lambda^{\mu} \cap \Theta^{\mu})(A), (\lambda^{\mu} \cap \Theta^{\mu})(B)\}\end{aligned}$$

Hence, $(\lambda^{\mu} \cap \Theta^{\mu})(A-B) \geq \min\{(\lambda^{\mu} \cap \Theta^{\mu})(A), (\lambda^{\mu} \cap \Theta^{\mu})(B)\}$.

(ii)
$$\begin{aligned}(\lambda^{\mu} \cap \Theta^{\mu})(AB) &= \min\{ \lambda^{\mu}(AB), \Theta^{\mu}(AB)\} \\ &\geq \min\{ \min\{ \lambda^{\mu}(A), \lambda^{\mu}(B)\}, \min\{ \Theta^{\mu}(A), \Theta^{\mu}(B)\} \\ &= \min\{ \min\{ \lambda^{\mu}(A), \Theta^{\mu}(A) \}, \min\{ \lambda^{\mu}(B), \Theta^{\mu}(B)\} \\ &= \min\{(\lambda^{\mu} \cap \Theta^{\mu})(A), (\lambda^{\mu} \cap \Theta^{\mu})(B)\}\end{aligned}$$

Hence, $(\lambda^{\mu} \cap \Theta^{\mu})(AB) \geq \min\{(\lambda^{\mu} \cap \Theta^{\mu})(A), (\lambda^{\mu} \cap \Theta^{\mu})(B)\}$.

(iii)
$$\begin{aligned}(\lambda^{\gamma} \cap \Theta^{\gamma})(A-B) &= \max\{ \lambda^{\gamma}(A-B), \Theta^{\gamma}(A-B)\} \\ &\leq \max\{ \max\{ \lambda^{\gamma}(A), \lambda^{\gamma}(B)\}, \max\{ \Theta^{\gamma}(A), \Theta^{\gamma}(B)\} \\ &= \max\{ \max\{ \lambda^{\gamma}(A), \Theta^{\gamma}(A) \}, \max\{ \lambda^{\gamma}(B), \Theta^{\gamma}(B)\} \\ &= \max\{(\lambda^{\gamma} \cap \Theta^{\gamma})(A), (\lambda^{\gamma} \cap \Theta^{\gamma})(B)\}\end{aligned}$$

Hence, $(\lambda^{\gamma} \cap \Theta^{\gamma})(A-B) \leq \max\{(\lambda^{\gamma} \cap \Theta^{\gamma})(A), (\lambda^{\gamma} \cap \Theta^{\gamma})(B)\}$.

(iv)
$$\begin{aligned}(\lambda^{\gamma} \cap \Theta^{\gamma})(AB) &= \max\{ \lambda^{\gamma}(AB), \Theta^{\gamma}(AB)\} \\ &\leq \max\{ \max\{ \lambda^{\gamma}(A), \lambda^{\gamma}(B)\}, \max\{ \Theta^{\gamma}(A), \Theta^{\gamma}(B)\} \\ &= \max\{ \max\{ \lambda^{\gamma}(A), \Theta^{\gamma}(A) \}, \max\{ \lambda^{\gamma}(B), \Theta^{\gamma}(B)\} \\ &= \max\{(\lambda^{\gamma} \cap \Theta^{\gamma})(A), (\lambda^{\gamma} \cap \Theta^{\gamma})(B)\}\end{aligned}$$

Hence , $(\lambda^{\gamma} \cap \Theta^{\gamma})(AB) \leq \max\{(\lambda^{\gamma} \cap \Theta^{\gamma})(A), (\lambda^{\gamma} \cap \Theta^{\gamma})(B)\}$.

Therefore the intersection of any two IFHX subrings is also an IFHX subring of ϑ.

3.4 Theorem

Let ψ be an intuitionistic fuzzy HX subring of a HX ring ϑ if and only if ψ^c is an intuitionistic anti fuzzy HX subring of a HX ring ϑ.

Proof

Let $\psi = \{\langle A, \lambda^{\mu}(A), \lambda^{\gamma}(A) \rangle / A\in\vartheta\}$ be a intuitionistic fuzzy HX subring of ϑ.
To prove that ψ^c is an intuitionistic anti fuzzy HX subring of ϑ.
For any $A,B \in\vartheta$,we have

(i) $\lambda^{\mu}(A-B) \geq \min\{ \lambda^{\mu}(A), \lambda^{\mu}(B) \}$
$\Leftrightarrow\ 1-(\lambda^{\mu})^c(A-B) \geq \min\{ 1-(\lambda^{\mu})^c(A), 1-(\lambda^{\mu})^c(B)\}$
$\Leftrightarrow\ (\lambda^{\mu})^c(A-B) \leq 1-\min\{ 1-(\lambda^{\mu})^c(A), 1-(\lambda^{\mu})^c(B)\}$
$\Leftrightarrow\ (\lambda^{\mu})^c(A-B) \leq \max\{ (\lambda^{\mu})^c(A), (\lambda^{\mu})^c(B)\}$

(ii) $\lambda^{\mu}(AB) \geq \min\{ \lambda^{\mu}(A), \lambda^{\mu}(B) \}$
$\Leftrightarrow\ 1-(\lambda^{\mu})^c(AB) \geq \min\{ 1-(\lambda^{\mu})^c(A), 1-(\lambda^{\mu})^c(B)\}$
$\Leftrightarrow\ (\lambda^{\mu})^c(AB) \leq 1-\min\{ 1-(\lambda^{\mu})^c(A), 1-(\lambda^{\mu})^c(B)\}$
$\Leftrightarrow\ (\lambda^{\mu})^c(AB) \leq \max\{ (\lambda^{\mu})^c(A), (\lambda^{\mu})^c(B)\}$

(iii) $\lambda^{\gamma}(A-B) \leq \max\{ \lambda^{\gamma}(A), \lambda^{\gamma}(B) \}$
$\Leftrightarrow\ 1-(\lambda^{\gamma})^c(A-B) \leq \max\{ 1-(\lambda^{\gamma})^c(A), 1-(\lambda^{\gamma})^c(B)\}$
$\Leftrightarrow\ (\lambda^{\gamma})^c(A-B) \geq 1-\max\{ 1-(\lambda^{\gamma})^c(A), 1-(\lambda^{\gamma})^c(B)\}$
$\Leftrightarrow\ (\lambda^{\gamma})^c(A-B) \geq \min\{ (\lambda^{\gamma})^c(A), (\lambda^{\gamma})^c(B) \}$.

(iv) $\lambda^{\gamma}(AB) \leq \max\{ \lambda^{\gamma}(A), \lambda^{\gamma}(B) \}$
$\Leftrightarrow\ 1-(\lambda^{\gamma})^c(AB) \leq \max\{ 1-(\lambda^{\gamma})^c(A), 1-(\lambda^{\gamma})^c(B)\}$
$\Leftrightarrow\ (\lambda^{\gamma})^c(AB) \geq 1-\max\{ 1-(\lambda^{\gamma})^c(A), 1-(\lambda^{\gamma})^c(B)\}$
$\Leftrightarrow\ (\lambda^{\gamma})^c(AB) \geq \min\{ (\lambda^{\gamma})^c(A), (\lambda^{\gamma})^c(B) \}$.

Hence $\psi^c = \{\langle A, (\lambda^{\mu})^c(A), (\lambda^{\gamma})^c(A) \rangle / A\in\vartheta\}$ is an intuitionistic anti fuzzy HX subring of ϑ.

3.5 Definition

Let $\psi = \{\langle A, \lambda^{\mu}(A), \lambda^{\gamma}(A) \rangle$ / for all $A\in\vartheta\}$ be an intuitionistic fuzzy subset of a HX ring ϑ . We define the following "necessity" and "possibility" operations :

$\Box\, \psi = \{ \langle A, \lambda^{\mu}(A), 1-\lambda^{\mu}(A) \rangle / A\in\vartheta\}$.
$\Diamond\, \psi = \{ \langle A, 1-\lambda^{\gamma}(A), \lambda^{\gamma}(A) \rangle / A\in\vartheta\}$

3.6 Theorem

If ψ is an intuitionistic fuzzy HX subring of a HX ring ϑ then $\Box\psi$ is an intuitionistic fuzzy HX subring of a HX ring ϑ.

Proof

Let $\Box\psi = \{\langle A, \lambda^{\mu}(A), (\lambda^{\mu})^c(A) \rangle / A\in\vartheta\}$
To prove that $\Box\psi$ is an intuitionistic fuzzy HX subring of ϑ.
Let $\psi = \{\langle A, \lambda^{\mu}(A), \lambda^{\gamma}(A) \rangle / A\in\vartheta\}$ be a intuitionistic fuzzy HX subring of ϑ. We have

(i) $\lambda^{\mu}(A-B) \geq \min\{ \lambda^{\mu}(A), \lambda^{\mu}(B)\}$,
(ii) $\lambda^{\mu}(AB) \geq \min\{ \lambda^{\mu}(A), \lambda^{\mu}(B)\}$,
(iii) $\lambda^{\gamma}(A-B) \leq \max\{ \lambda^{\gamma}(A), \lambda^{\gamma}(B)\}$,
(iv) $\lambda^{\gamma}(AB) \leq \max\{ \lambda^{\gamma}(A), \lambda^{\gamma}(B)\}$.

Now

(i) $(\lambda^{\mu})^c(A-B) = 1-(\lambda^{\mu})(A-B)$
$\leq 1-\min\{ \lambda^{\mu}(A), \lambda^{\mu}(B) \}$
$= 1-\min\{ 1-(\lambda^{\mu})^c(A), 1-(\lambda^{\mu})^c(B)\}$
$= \max\{ (\lambda^{\mu})^c(A), (\lambda^{\mu})^c(B)\}$

(ii) $(\lambda^{\mu})^c(AB) = 1-(\lambda^{\mu})(AB)$
$\leq 1-\min\{ \lambda^{\mu}(A), \lambda^{\mu}(B) \}$
$= 1-\min\{ 1-(\lambda^{\mu})^c(A), 1-(\lambda^{\mu})^c(B)\}$
$= \max\{ (\lambda^{\mu})^c(A), (\lambda^{\mu})^c(B)\}$

Hence ,

$\lambda^{\mu}(A-B) \geq \min\{ \lambda^{\mu}(A), \lambda^{\mu}(B)\}$, $\lambda^{\mu}(AB) \geq \min\{ \lambda^{\mu}(A), \lambda^{\mu}(B)\}$,

$(\lambda^{\mu})^{c}(A-B) \le \max\{(\lambda^{\mu})^{c}(A), (\lambda^{\mu})^{c}(B)\}$ and $(\lambda^{\mu})^{c}(AB) \le \max\{(\lambda^{\mu})^{c}(A), (\lambda^{\mu})^{c}(B)\}$.
Therefore □ψ = $\{\langle A, \lambda^{\mu}(A), (\lambda^{\mu})^{c}(A)\rangle / A\in\vartheta\}$ is a intuitionistic fuzzy HX subring of ϑ.

3.7 Theorem

If ψ is an intuitionistic fuzzy HX subring of a HX ring ϑ then $\Diamond\psi$ is an intuitionistic fuzzy HX subring of a HX ring ϑ.

Proof

Let $\Diamond\psi = \{\langle A, (\lambda^{\gamma})^{c}(A), \lambda^{\gamma}(A)\rangle / A\in\vartheta\}$
To prove that $\Diamond\psi$ is an intuitionistic fuzzy HX subring of ϑ.
Let $\psi = \{\langle A, \lambda^{\mu}(A), \lambda^{\gamma}(A)\rangle / A\in\vartheta\}$ be a intuitionistic fuzzy HX subring of ϑ. We have

(i) $\lambda^{\mu}(A-B) \ge \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$,
(ii) $\lambda^{\mu}(AB) \ge \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$,
(iii) $\lambda^{\gamma}(A-B) \le \max\{\lambda^{\gamma}(A), \lambda^{\gamma}(B)\}$,
(iv) $\lambda^{\gamma}(AB) \le \max\{\lambda^{\gamma}(A), \lambda^{\gamma}(B)\}$.

Now

(i)
$$\begin{aligned}(\lambda^{\gamma})^{c}(A-B) &= 1-(\lambda^{\gamma})(A-B)\\ &\ge 1-\max\{\lambda^{\gamma}(A), \lambda^{\gamma}(B)\}\\ &= 1-\max\{1-(\lambda^{\gamma})^{c}(A), 1-(\lambda^{\gamma})^{c}(B)\}\\ &= \min\{(\lambda^{\gamma})^{c}(A), (\lambda^{\gamma})^{c}(B)\}\end{aligned}$$

(ii)
$$\begin{aligned}(\lambda^{\gamma})^{c}(AB) &= 1-(\lambda^{\gamma})(AB)\\ &\ge 1-\max\{\lambda^{\gamma}(A), \lambda^{\gamma}(B)\}\\ &= 1-\max\{1-(\lambda^{\gamma})^{c}(A), 1-(\lambda^{\gamma})^{c}(B)\}\\ &= \min\{(\lambda^{\gamma})^{c}(A), (\lambda^{\gamma})^{c}(B)\}\end{aligned}$$

Hence, $\lambda^{\gamma}(A-B) \le \max\{\lambda^{\gamma}(A), \lambda^{\gamma}(B)\}$, $\lambda^{\gamma}(AB) \le \max\{\lambda^{\gamma}(A), \lambda^{\gamma}(B)\}$.
$(\lambda^{\gamma})^{c}(A-B) \ge \min\{(\lambda^{\gamma})^{c}(A), (\lambda^{\gamma})^{c}(B)\}$ and $(\lambda^{\gamma})^{c}(AB) \ge \min\{(\lambda^{\gamma})^{c}(A), (\lambda^{\gamma})^{c}(B)\}$.
Therefore, $\Diamond\psi = \{\langle A, (\lambda^{\gamma})^{c}(A), \lambda^{\gamma}(A)\rangle / A\in\vartheta\}$ is a intuitionistic fuzzy HX subring of ϑ.

3.8 Theorem

An IFS $\psi = \{\langle A, \lambda^{\mu}(A), \lambda^{\gamma}(A)\rangle / A\in\vartheta\}$ is an intuitionistic fuzzy HX subring of a HX ring ϑ if and only if the fuzzy subsets $\lambda^{\mu}(A)$, $(\lambda^{\gamma})^{c}(A)$ are fuzzy HX subring of a HX ring ϑ.

Proof

Let $\psi = \{\langle A, \lambda^{\mu}(A), \lambda^{\gamma}(A)\rangle / A\in\vartheta\}$ be a intuitionistic fuzzy HX subring of ϑ. We have

(i) $\lambda^{\mu}(A-B) \ge \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$,
(ii) $\lambda^{\mu}(AB) \ge \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$,
(iii) $\lambda^{\gamma}(A-B) \le \max\{\lambda^{\gamma}(A), \lambda^{\gamma}(B)\}$,
(iv) $\lambda^{\gamma}(AB) \le \max\{\lambda^{\gamma}(A), \lambda^{\gamma}(B)\}$.

Clearly, $\lambda^{\mu}(A)$ is a fuzzy HX subring of ϑ by (i) and (ii) .
Now we have to show $(\lambda^{\gamma})^{c}$ is a fuzzy HX subring of ϑ.

(i)
$$\begin{aligned}(\lambda^{\gamma})^{c}(A-B) &= 1-(\lambda^{\gamma})(A-B)\\ &\ge 1-\max\{\lambda^{\gamma}(A), \lambda^{\gamma}(B)\}\\ &= 1-\max\{1-(\lambda^{\gamma})^{c}(A), 1-(\lambda^{\gamma})^{c}(B)\}\\ &= \min\{(\lambda^{\gamma})^{c}(A), (\lambda^{\gamma})^{c}(B)\}\\ (\lambda^{\gamma})^{c}(A-B) &\ge \min\{(\lambda^{\gamma})^{c}(A), (\lambda^{\gamma})^{c}(B)\}\end{aligned}$$

(ii)
$$\begin{aligned}(\lambda^{\gamma})^{c}(AB) &= 1-(\lambda^{\gamma})(AB)\\ &\ge 1-\max\{\lambda^{\gamma}(A), \lambda^{\gamma}(B)\}\\ &= 1-\max\{1-(\lambda^{\gamma})^{c}(A), 1-(\lambda^{\gamma})^{c}(B)\}\\ &= \min\{(\lambda^{\gamma})^{c}(A), (\lambda^{\gamma})^{c}(B)\}\\ (\lambda^{\gamma})^{c}(AB) &\ge \min\{(\lambda^{\gamma})^{c}(A), (\lambda^{\gamma})^{c}(B)\}.\end{aligned}$$

Thus $(\lambda^{\gamma})^{c}$ is a fuzzy HX subring of ϑ.

Conversely,

$\lambda^{\mu}(A)$ and $(\lambda^{\gamma})^{c}(A)$ are fuzzy HX subring of a HX ring ϑ

To prove that $\psi = \{\langle A, \lambda^{\mu}(A), \lambda^{\gamma}(A)\rangle / A\in\vartheta\}$ be a intuitionistic fuzzy HX subring of ϑ.

Now we know that

$$(\lambda^{\gamma})^{c}(A-B) \geq \min\{(\lambda^{\gamma})^{c}(A), (\lambda^{\gamma})^{c}(B)\}$$
$$1-(\lambda^{\gamma})(A-B) \geq \min\{1-(\lambda^{\gamma})(A), 1-(\lambda^{\gamma})(B)\}.$$
$$1-(\lambda^{\gamma})(A-B) = 1-\max\{\lambda^{\gamma}(A), \lambda^{\gamma}(B)\} \quad \text{implies}$$
$$(\lambda^{\gamma})(A-B) \leq \max\{\lambda^{\gamma}(A), \lambda^{\gamma}(B)\}$$

Also

$$(\lambda^{\gamma})^{c}(AB) \geq \min\{(\lambda^{\gamma})^{c}(A), (\lambda^{\gamma})^{c}(B)\}$$
$$1-(\lambda^{\gamma})(AB) \geq \min\{1-(\lambda^{\gamma})(A), 1-(\lambda^{\gamma})(B)\}.$$
$$1-(\lambda^{\gamma})(AB) = 1-\max\{\lambda^{\gamma}(A), \lambda^{\gamma}(B)\} \quad \text{implies}$$
$$(\lambda^{\gamma})(AB) \leq \max\{\lambda^{\gamma}(A), \lambda^{\gamma}(B)\}$$

Already we have

$$\lambda^{\mu}(A-B) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}, \quad \lambda^{\mu}(AB) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}.$$

Hence $\psi = \{\langle A, \lambda^{\mu}(A), \lambda^{\gamma}(A)\rangle / A \in \vartheta\}$ be a intuitionistic fuzzy HX subring of ϑ.

3.9 Theorem

An IFS $\psi = \{\langle A, \lambda^{\mu}(A), \lambda^{\gamma}(A)\rangle / A \in \vartheta\}$ is an intuitionistic fuzzy HX subring of a HX ring ϑ if and only if the fuzzy subsets $(\lambda^{\mu})^{c}$ and (λ^{γ}) are anti-fuzzy HX subring of a HX ring ϑ.

Proof

Let $\psi = \{\langle A, \lambda^{\mu}(A), \lambda^{\gamma}(A)\rangle / A \in \vartheta\}$ be a intuitionistic fuzzy HX subring of ϑ. We have

(i) $\lambda^{\mu}(A-B) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$,

(ii) $\lambda^{\mu}(AB) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$,

(iii) $\lambda^{\gamma}(A-B) \leq \max\{\lambda^{\gamma}(A), \lambda^{\gamma}(B)\}$,

(iv) $\lambda^{\gamma}(AB) \leq \max\{\lambda^{\gamma}(A), \lambda^{\gamma}(B)\}$.

From (iii) and (iv) it is clear that (λ^{γ}) is an anti-fuzzy HX subring of ϑ.

Now ,

$$\begin{aligned}
(\lambda^{\mu})^{c}(A-B) &= 1-(\lambda^{\mu})(A-B) \\
&\leq 1-\min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\} \\
&= 1-\min\{1-(\lambda^{\mu})^{c}(A), 1-(\lambda^{\mu})^{c}(B)\} \\
&= \max\{(\lambda^{\mu})^{c}(A), (\lambda^{\mu})^{c}(B)\} \\
(\lambda^{\mu})^{c}(A-B) &\leq \max\{(\lambda^{\mu})^{c}(A), (\lambda^{\mu})^{c}(B)\}.
\end{aligned}$$

Also

$$\begin{aligned}
(\lambda^{\mu})^{c}(AB) &= 1-(\lambda^{\mu})(AB) \\
&\leq 1-\min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\} \\
&= 1-\min\{1-(\lambda^{\mu})^{c}(A), 1-(\lambda^{\mu})^{c}(B)\} \\
&= \max\{(\lambda^{\mu})^{c}(A), (\lambda^{\mu})^{c}(B)\} \\
(\lambda^{\mu})^{c}(AB) &\leq \max\{(\lambda^{\mu})^{c}(A), (\lambda^{\mu})^{c}(B)\}.
\end{aligned}$$

Hence, $(\lambda^{\mu})^{c}$ and (λ^{γ}) are anti-fuzzy HX subring of a HX ring ϑ.

Conversely,

$(\lambda^{\mu})^{c}$ and (λ^{γ}) are anti-fuzzy HX subring of a HX ring ϑ

To prove that $\psi = \{\langle A, \lambda^{\mu}(A), \lambda^{\gamma}(A)\rangle / A \in \vartheta\}$ be a intuitionistic fuzzy HX subring of ϑ.

It is clear that $\lambda^{\gamma}(A-B) \leq \max\{\lambda^{\gamma}(A), \lambda^{\gamma}(B)\}$ and

$\lambda^{\gamma}(AB) \leq \max\{\lambda^{\gamma}(A), \lambda^{\gamma}(B)\}$.

Now

$$(\lambda^{\mu})^{c}(A-B) \leq \max\{(\lambda^{\mu})^{c}(A), (\lambda^{\mu})^{c}(B)\}$$
$$1-(\lambda^{\mu})(A-B) = \max\{1-(\lambda^{\mu})(A), 1-(\lambda^{\mu})(B)\}.$$
$$1-(\lambda^{\mu})(A-B) = 1-\min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\} \quad \text{implies}$$
$$(\lambda^{\mu})(A-B) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}.$$

Also

$$(\lambda^{\mu})^{c}(AB) \leq \max\{(\lambda^{\mu})^{c}(A), (\lambda^{\mu})^{c}(B)\}$$
$$1-(\lambda^{\mu})(AB) = \max\{1-(\lambda^{\mu})(A), 1-(\lambda^{\mu})(B)\}.$$
$$1-(\lambda^{\mu})(AB) = 1-\min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\} \quad \text{implies}$$
$$(\lambda^{\mu})(AB) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}.$$

Thus $\lambda^{\mu}(A-B) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$, $\lambda^{\mu}(AB) \geq \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$,

$\lambda^{\gamma}(A-B) \leq \max\{\lambda^{\gamma}(A), \lambda^{\gamma}(B)\}$ and $\lambda^{\gamma}(AB) \leq \max\{\lambda^{\gamma}(A), \lambda^{\gamma}(B)\}$.

Hence $\psi = \{\langle A, \lambda^{\mu}(A), \lambda^{\gamma}(A)\rangle / A \in \vartheta\}$ be a intuitionistic fuzzy HX subring of ϑ.

3.10 Definition

Let ϑ_1 and ϑ_2 be any two HX rings. Then the function f : $\vartheta_1 \to \vartheta_2$ is said to be a homomorphism if it satisfies the following axioms:

i) f(A+B) = f(A) + f(B) and

ii) f(AB) = f(A) f(B), for all A,B $\in \vartheta_1$.

3.11 Definition

Let ϑ_1 and ϑ_2 be any two HX rings. Then the function f : $\vartheta_1 \to \vartheta_2$ is said to be an anti homomorphism if it satisfies the following axioms:

i) f(A+B) = f(B) + f(A) and

ii) f(AB) = f(B) f(A), for all A,B $\in \vartheta_1$.

3.12 Definition

Let R_1 and R_2 be any two rings. Let $\vartheta_1 \subset 2^{R1}$-{$\phi$} and $\vartheta_2 \subset 2^{R2}$-{$\phi$} be any two HX rings. Let A = { (x, $\mu_A(x)$, $\gamma_A(x)$) / x$\in R_1$ } and B = { (y, $\alpha_B(y)$, $\beta_B(y)$) / y$\in R_2$ } be any two intuitionistic fuzzy sets on R_1 and R_2 respectively. Let C = { (U, $\lambda_C^{\mu}(U)$, $\theta_C^{\gamma}(U)$) / U$\in \vartheta_1$ } and D = { (V, $\eta_D^{\alpha}(V)$, $\psi_D^{\beta}(V)$) / V$\in \vartheta_2$ } any two intuitionistic fuzzy sets in ϑ_1 and ϑ_2 resp. Let f be a function from ϑ_1 into ϑ_2 then the image of C on ϑ_1 under f is defined as

$$\eta_D^{\alpha}(V) = \begin{cases} \max\{\lambda_C^{\mu}(U) : U \in f^{-1}(V)\}, f^{-1}(V) \neq \phi \\ 0 \qquad ,otherwise \end{cases} \quad \text{and} \quad \psi_D^{\beta}(V) = \begin{cases} \min\{\theta_C^{\gamma}(U) : U \in f^{-1}(V)\}, f^{-1}(V) \neq \phi \\ 1 \qquad ,otherwise \end{cases}$$

Where $\eta_D^{\alpha} = f(\lambda_C^{\mu})$ also Pre-image of D on ϑ_2 under f is defined as $\left(f^{-1}(\eta_D^{\alpha})\right)(U) = \eta_D^{\alpha}(f(U))$, $\left(f^{-1}(\psi_D^{\beta})\right)(U) = \psi_D^{\beta}(f(U))$.

3.13 Theorem

Let R_1 and R_2 be any two rings. Let A = { (x, $\mu_A(x)$, $\gamma_A(x)$) / x$\in R_1$ } and B = { (y, $\alpha B(y)$, $\beta B(y)$) / y$\in R_2$ } be any two intuitionistic fuzzy sets on R_1 and R_2 respectively. Let C = { (U, $\lambda_C^{\mu}(U)$, $\theta_C^{\gamma}(U)$) / U$\in \vartheta_1$ } and D = { (V, $\eta_D^{\alpha}(V)$, $\psi_D^{\beta}(V)$) / V$\in \vartheta_2$ } be any two intuitionistic fuzzy sets in ϑ_1 and ϑ_2 resp. Let f be a onto homomorphism from ϑ_1 to ϑ_2. If C be the intuitionistic fuzzy HX subring of ϑ_1 then f(C) is a intuitionistic fuzzy HX subring of ϑ_2.

Proof.

Let C be the intuitionistic fuzzy HX subring of ϑ_1 then

$$(i) \quad \lambda_C^{\mu}(U-T) \geq \min\{\lambda_C^{\mu}(U), \lambda_C^{\mu}(T)\}$$

$$(ii) \quad \lambda_C^{\mu}(UT) \geq \min\{\lambda_C^{\mu}(U), \lambda_C^{\mu}(T)\}$$

$$(iii) \quad \theta_C^{\gamma}(U-T) \leq \max\{\lambda_C^{\mu}(U), \lambda_C^{\mu}(T)\}$$

$$(iv) \quad \theta_C^{\gamma}(UT) \leq \max\{\lambda_C^{\mu}(U), \lambda_C^{\mu}(T)\}$$

To Prove that f(C) is an intuitionistic fuzzy HX subring of ϑ_2.

Let V = f(U) , W = f(T) $\in \vartheta_2$, where U,T$\in \vartheta_1$.

Now $\eta_D^{\alpha}\,[f(U)-f(T)] = \eta_D^{\alpha}\,[f(U-T)]$ (f is homomorphism)

$= \lambda_C^{\mu}\,[U-T]$ (f is onto)

$\geq \min\{\lambda_C^{\mu}(U), \lambda_C^{\mu}(T)\}$

$\geq \min\{\eta_D^{\alpha}(f(U)), \eta_D^{\alpha}(f(T))\}$

Hence, $\eta_D^{\alpha}[f(U)-f(T)] \geq \min\{\eta_D^{\alpha}(f(U)), \eta_D^{\alpha}(f(T))\}$

Again $\eta_D^{\alpha}\,[f(U)\,f(T)] = \eta_D^{\alpha}\,[f(UT)]$ (f is homomorphism)

$= \lambda_C^{\mu}\,[UT]$ (f is onto)

$\geq \min\{\lambda_C^{\mu}(U), \lambda_C^{\mu}(T)\}$

$\geq \min\{\eta_D^{\alpha}(f(U)), \eta_D^{\alpha}(f(T))\}$

Hence, $\eta_D^{\alpha}[f(U)\,f(T)] \geq \min\{\eta_D^{\alpha}(f(U)), \eta_D^{\alpha}(f(T))\}$

Now

$$\psi_D^{\beta}[f(U)-f(T)] \;=\; \psi_D^{\beta}[f(U-T)] \quad (\text{ f is homomorphism})$$
$$= \theta_C^{\gamma}[U-T] \quad (\text{ f is onto })$$
$$\leq \max\left\{\theta_C^{\gamma}(U), \theta_C^{\gamma}(T)\right\}$$
$$= \max\left\{\psi_D^{\beta}(f(U)), \psi_D^{\beta}(f(T))\right\}$$

Hence, $\psi_D^{\beta}[f(U)-f(T)] \leq \max\left\{\psi_D^{\beta}(f(U)), \psi_D^{\beta}(f(T))\right\}$

Also

$$\psi_D^{\beta}[f(U)f(T)] \;=\; \psi_D^{\beta}[f(UT)] \quad (\text{ f is homomorphism })$$
$$= \theta_C^{\gamma}(UT) \quad (\text{ f is onto })$$
$$\leq \max\left\{\theta_C^{\gamma}(U), \theta_C^{\gamma}(T)\right\}$$
$$= \max\left\{\psi_D^{\beta}(f(U)), \psi_D^{\beta}(f(T))\right\}$$

Hence, $\psi_D^{\beta}[f(UT)] \leq \max\left\{\psi_D^{\beta}(f(U)), \psi_D^{\beta}(f(T))\right\}$

Thus D = f(C) is an intuitionistic fuzzy HX subring of ϑ_2.

3.13 Theorem

Let R_1 and R_2 be any two rings. Let A = { (x, $\mu_A(x)$, $\gamma_A(x)$) / x$\in R_1$ } and B = { (y, $\alpha_B(y)$, $\beta_B(y)$) / y$\in R_2$ } be any two intuitionistic fuzzy sets on R_1 and R_2 respectively. Let C = { (U, $\lambda_C^{\mu}(U)$, $\theta_C^{\gamma}(U)$) / U$\in \vartheta_1$ } and D = { (V, $\eta_D^{\alpha}(V)$, $\psi_D^{\beta}(V)$) / V$\in \vartheta_2$ } be any two intuitionistic fuzzy sets in ϑ_1 and ϑ_2 resp. Let f be an onto homomorphism from ϑ_1 to ϑ_2. If D be the intuitionistic fuzzy HX subring of ϑ_2 then $f^{-1}(D)$ is a intuitionistic fuzzy HX subring of ϑ_1.

Proof.

Let D be the intuitionistic fuzzy HX subring of ϑ_2 then

$$(i) \quad \eta_D^{\alpha}[V-W] \geq \min\left\{\eta_D^{\alpha}(V), \eta_D^{\alpha}(W)\right\}$$
$$(ii) \quad \eta_D^{\alpha}[VW] \geq \min\left\{\eta_D^{\alpha}(V), \eta_D^{\alpha}(W)\right\}$$
$$(iii) \quad \psi_D^{\beta}[V-W] \leq \max\left\{\psi_D^{\beta}(V), \psi_D^{\beta}(W)\right\}$$
$$(iv) \quad \psi_D^{\beta}[VW] \leq \max\left\{\psi_D^{\beta}(V), \psi_D^{\beta}(W)\right\}$$

To Prove that $f^{-1}(D)$ is a intuitionistic fuzzy HX subring of ϑ_1.

Let U,T$\in \vartheta_1$ and f(U) = V , f(T) =W$\in \vartheta_2$.

Now
$$[f^{-1}(\eta_D^{\alpha})](U-T) \;=\; \eta_D^{\alpha}[f(U-T)]$$
$$= \eta_D^{\alpha}[f(U)-f(T)] \quad (\text{ f is homomorphism })$$
$$\geq \min\left\{\eta_D^{\alpha}(f(U)), \eta_D^{\alpha}(f(T))\right\}$$
$$= \min\left\{[f^{-1}(\eta_D^{\alpha})](U), [f^{-1}(\eta_D^{\alpha})](T)\right\}$$

Hence, $[f^{-1}(\eta_D^{\alpha})](U-T) \geq \min\left\{[f^{-1}(\eta_D^{\alpha})](U), [f^{-1}(\eta_D^{\alpha})](T)\right\}$

Also
$$[f^{-1}(\eta_D^{\alpha})](UT) \;=\; \eta_D^{\alpha}[f(UT)]$$
$$= \eta_D^{\alpha}[f(U)f(T)] \quad (\text{ f is homomorphism })$$
$$\geq \min\left\{\eta_D^{\alpha}(f(U)), \eta_D^{\alpha}(f(T))\right\}$$
$$= \min\left\{[f^{-1}(\eta_D^{\alpha})](U), [f^{-1}(\eta_D^{\alpha})](T)\right\}$$

Hence, $[f^{-1}(\eta_D^{\alpha})](UT) \geq \min\left\{[f^{-1}(\eta_D^{\alpha})](U), [f^{-1}(\eta_D^{\alpha})](T)\right\}$

again
$$[f^{-1}(\psi_D^{\beta})](U-T) \;=\; \psi_D^{\beta}[f(U-T)]$$
$$= \psi_D^{\beta}[f(U)-f(T)] \quad (\text{ f is homomorphism })$$
$$\leq \max\left\{\psi_D^{\beta}(f(U), \psi_D^{\beta}(f(T))\right\}$$
$$= \max\left\{[f^{-1}(\psi_D^{\beta})](U), [f^{-1}(\psi_D^{\beta})](T)\right\}$$

Hence, $[f^{-1}(\psi_D^{\beta})](U-T) \leq \max\left\{[f^{-1}(\psi_D^{\beta})](U), [f^{-1}(\psi_D^{\beta})](T)\right\}$

Also

$$[f^{-1}(\psi_D^\beta)](UT) = \psi_D^\beta\,[f(UT)]$$
$$= \psi_D^\beta\,[f(U)f(T)] \quad (\text{ f is homomorphism })$$
$$\le \max\left\{\psi_D^\beta(f(U),\, \psi_D^\beta(f(T))\right\}$$
$$= \max\left\{ [f^{-1}(\psi_D^\beta)](U)\, ,[f^{-1}(\psi_D^\beta)](T) \right\}$$

Hence, $f^{-1}[(\psi_D^\beta)(UT)] \le \max\left\{ f^{-1}(\psi_D^\beta)(U), f^{-1}(\psi_D^\beta)(T)\right\}$

Therefore, $f^{-1}(D)$ is a intuitionistic fuzzy HX subring of ϑ_1.

3.14 Theorem

Let R_1 and R_2 be any two rings. Let A = { (x, $\mu_A(x)$, $\gamma_A(x)$) / x∈ R_1 } and B = { (y, $\alpha_B(y)$, $\beta_B(y)$) / y∈ R_2 } be any two intuitionistic fuzzy sets on R_1 and R_2 respectively.

Let C = { (U, $\lambda_C^\mu(U)$, $\theta_C^\gamma(U)$) / U∈ ϑ_1 } and D = { (V, $\eta_D^\alpha(V)$, $\psi_D^\beta(V)$) / V∈ ϑ_2 } be any two intuitionistic fuzzy sets in ϑ_1 and ϑ_2 resp. Let f be an onto anti-homomorphism from ϑ_1 to ϑ_2. If C be the intuitionistic fuzzy HX subring of ϑ_1 then f(C) is a intuitionistic fuzzy HX subring of ϑ_2.

Proof.

Let C be the intuitionistic fuzzy HX subring of ϑ_1 then

$$(i)\quad \lambda_C^\mu(U-T) \ge \min\left\{ \lambda_C^\mu(U), \lambda_C^\mu(T)\right\}$$
$$(ii)\quad \lambda_C^\mu(UT) \ge \min\left\{ \lambda_C^\mu(U), \lambda_C^\mu(T)\right\}$$
$$(iii)\quad \theta_C^\gamma(U-T) \le \max\left\{ \lambda_C^\mu(U), \lambda_C^\mu(T)\right\}$$
$$(iv)\quad \theta_C^\gamma(UT) \le \max\left\{ \lambda_C^\mu(U), \lambda_C^\mu(T)\right\}$$

To Prove that f(C) is an intuitionistic fuzzy HX subring of ϑ_2.

Let V = f(U) , W = f(T) $\in\vartheta_2$, where U,T$\in\vartheta_1$.

Now
$$\eta_D^\alpha\,[f(U)-f(T)] = \eta_D^\alpha\,[f(T-U)] \quad (\text{ f is an anti homomorphism })$$
$$= \lambda_C^\mu\,[T-U] \quad (\text{ f is onto })$$
$$\ge \min\left\{\lambda_C^\mu(T), \lambda_C^\mu(U)\right\}$$
$$\ge \min\left\{\lambda_C^\mu(U), \lambda_C^\mu(T)\right\}$$
$$\ge \min\left\{\eta_D^\alpha(f(U)), \eta_D^\alpha(f(T))\right\}$$

Hence , $\eta_D^\alpha[f(U)-f(T)] \ge \min\left\{\eta_D^\alpha(f(U)), \eta_D^\alpha(f(T))\right\}$

Again
$$\eta_D^\alpha\,[f(U)\,f(T)] = \eta_D^\alpha\,[f(TU)] \quad (\text{ f is an anti homomorphism })$$
$$= \lambda_C^\mu\,[TU] \quad (\text{ f is onto })$$
$$\ge \min\left\{\lambda_C^\mu(T), \lambda_C^\mu(U)\right\}$$
$$\ge \min\left\{\lambda_C^\mu(U), \lambda_C^\mu(T)\right\}$$
$$\ge \min\left\{\eta_D^\alpha(f(U)), \eta_D^\alpha(f(T))\right\}$$

Hence , $\eta_D^\alpha[f(U)\,f(T)] \ge \min\left\{\eta_D^\alpha(f(U)), \eta_D^\alpha(f(T))\right\}$

Now
$$\psi_D^\beta[f(U)-f(T)] = \psi_D^\beta\,[f(T-U)] \quad (\text{ f is an anti homomorphism})$$
$$= \theta_C^\gamma\,[T-U] \quad (\text{ f is onto })$$
$$\le \max\left\{\theta_C^\gamma(T), \theta_C^\gamma(U)\right\}$$
$$\le \max\left\{\theta_C^\gamma(U), \theta_C^\gamma(T)\right\}$$
$$= \max\left\{\psi_D^\beta(f(U))\, , \psi_D^\beta(f(T))\right\}$$

Hence , $\psi_D^\beta[f(U)-f(T)] \le \max\left\{\psi_D^\beta(f(U))\, , \psi_D^\beta(f(T))\right\}$

Also
$$\psi_D^\beta\,[f(U)f(T)] = \psi_D^\beta\,[f(UT)] \quad (\text{ f is an anti homomorphism })$$
$$= \theta_C^\gamma(UT) \quad (\text{ f is onto })$$
$$\le \max\left\{\theta_C^\gamma(U)\, , \theta_C^\gamma(T)\right\}$$

$$= \max \left\{ \psi_D^{\beta}(f(U)), \psi_D^{\beta}(f(T)) \right\}$$

Hence , $\quad \psi_D^{\beta}[f(UT)] \le \max \left\{ \psi_D^{\beta}(f(U)), \psi_D^{\beta}(f(T)) \right\}$

Thus D = f(C) is an intuitionistic fuzzy HX subring of ϑ_2.

3.15 Theorem

Let R_1 and R_2 be any two rings. Let A = { (x, $\mu_A(x)$, $\gamma_A(x)$) / x$\in R_1$ } and B = { (y, $\alpha_B(y)$, $\beta_B(y)$) / y$\in R_2$ } be any two intuitionistic fuzzy sets on R_1 and R_2 respectively.
Let C = { (U, $\lambda_C^{\mu}(U)$, $\theta_C^{\gamma}(U)$) / U$\in \vartheta_1$ } and D = { (V, $\eta_D^{\alpha}(V)$, $\psi_D^{\beta}(V)$) / V$\in \vartheta_2$ } be any two intuitionistic fuzzy sets in ϑ_1 and ϑ_2 resp. Let f be an onto anti- homomorphism from ϑ_1 to ϑ_2. If D be the intuitionistic fuzzy HX subring of ϑ_2 then $f^{-1}(D)$ is a intuitionistic fuzzy HX subring of ϑ_1.

Proof.

Let D be the intuitionistic fuzzy HX subring of ϑ_2 then

$$(i) \quad \eta_D^{\alpha}[V-W] \ge \min \left\{ \eta_D^{\alpha}(V), \eta_D^{\alpha}(W) \right\}$$

$$(ii) \quad \eta_D^{\alpha}[VW] \ge \min \left\{ \eta_D^{\alpha}(V), \eta_D^{\alpha}(W) \right\}$$

$$(iii) \quad \psi_D^{\beta}[V-W] \le \max \left\{ \psi_D^{\beta}(V), \psi_D^{\beta}(W) \right\}$$

$$(iv) \quad \psi_D^{\beta}[VW] \le \max \left\{ \psi_D^{\beta}(V), \psi_D^{\beta}(W) \right\}$$

To Prove that $f^{-1}(D)$ is a intuitionistic fuzzy HX subring of ϑ_1.

Let U,T$\in \vartheta_1$ and f(U) = V , f(T) =W$\in \vartheta_2$.

Now, $\quad [f^{-1}(\eta_D^{\alpha})](U-T) = \eta_D^{\alpha}[f(U-T)]$

$$= \eta_D^{\alpha}[f(T)-f(U)] \quad \text{(f is an anti homomorphism)}$$

$$\ge \min \left\{ \eta_D^{\alpha}(f(T)), \eta_D^{\alpha}(f(U)) \right\}$$

$$\ge \min \left\{ \eta_D^{\alpha}(f(U)), \eta_D^{\alpha}(f(T)) \right\}$$

$$= \min \left\{ [f^{-1}(\eta_D^{\alpha})](U), [f^{-1}(\eta_D^{\alpha})](T) \right\}$$

Hence, $\quad [f^{-1}(\eta_D^{\alpha})](U-T) \ge \min \left\{ [f^{-1}(\eta_D^{\alpha})](U), [f^{-1}(\eta_D^{\alpha})](T) \right\}$

Also $\quad [f^{-1}(\eta_D^{\alpha})](UT) = \eta_D^{\alpha}[f(UT)]$

$$= \eta_D^{\alpha}[f(T)f(U)] \quad \text{(f is an anti homomorphism)}$$

$$\ge \min \left\{ \eta_D^{\alpha}(f(T)), \eta_D^{\alpha}(f(U)) \right\}$$

$$\ge \min \left\{ \eta_D^{\alpha}(f(U)), \eta_D^{\alpha}(f(T)) \right\}$$

$$= \min \left\{ [f^{-1}(\eta_D^{\alpha})](U), [f^{-1}(\eta_D^{\alpha})](T) \right\}$$

Hence, $\quad [f^{-1}(\eta_D^{\alpha})](UT) \ge \min \left\{ [f^{-1}(\eta_D^{\alpha})](U), [f^{-1}(\eta_D^{\alpha})](T) \right\}$

again

$$[f^{-1}(\psi_D^{\beta})](U-T) = \psi_D^{\beta}[f(U-T)]$$

$$= \psi_D^{\beta}[f(T)-f(U)] \quad \text{(f is an anti homomorphism)}$$

$$\le \max \left\{ \psi_D^{\beta}(f(T), \psi_D^{\beta}(f(U)) \right\}$$

$$\le \max \left\{ \psi_D^{\beta}(f(U), \psi_D^{\beta}(f(T)) \right\}$$

$$= \max \left\{ [f^{-1}(\psi_D^{\beta})](U), [f^{-1}(\psi_D^{\beta})](T) \right\}$$

Hence, $\quad [f^{-1}(\psi_D^{\beta})](U-T) \le \max \left\{ [f^{-1}(\psi_D^{\beta})](U), [f^{-1}(\psi_D^{\beta})](T) \right\}$

Also

$$[f^{-1}(\psi_D^{\beta})](UT) = \psi_D^{\beta}[f(UT)]$$

$$= \psi_D^{\beta}[f(T)f(U)] \quad \text{(f is an anti homomorphism)}$$

$$\le \max \left\{ \psi_D^{\beta}(f(T), \psi_D^{\beta}(f(U)) \right\}$$

$$\le \max \left\{ \psi_D^{\beta}(f(U), \psi_D^{\beta}(f(T)) \right\}$$

$$= \max \left\{ [f^{-1}(\psi_D^{\beta})](U), [f^{-1}(\psi_D^{\beta})](T) \right\}$$

Hence, $f^{-1}[(\psi_D^\beta)(UT)] \le \max\left\{ f^{-1}(\psi_D^\beta)(U), f^{-1}(\psi_D^\beta)(T) \right\}$

Therefore, $f^{-1}(D)$ is a intuitionistic fuzzy HX subring of ϑ_1.

IV. Conclusion

In this paper we introduce the concept of intuitionistic fuzzy HX ring and discuss the basic results on intuitionistic HX subring. Further investigation may be in intuitionistic fuzzy HX ideals on HX ring. which will give a new horizon in the further study.

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