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New Exact Solutions of Some Nonlinear Partial Differential Equations via the Hyperbolic-sine Function Method

M.F. El-Sabbagh, R. Zait and R.M. Abdelazeem
Mathematics Department, Faculty of science, Minia university, Egypt.

Abstract: *In this paper, we establish exact solutions for some nonlinear partial differential equations. The hyperbolic-sine method [16] is used to construct periodic and solitary wave solutions for some soliton equations and systems such as the generalized Klien-Gordon, the general improved Kadomtsev-Petviashvili (KP), and the Zakharov-Kuznetsov (ZK) with power law nonlinearity equations, the generalized coupled Drinfeld –sokolov – wilso, and the generalized coupled Hirota-Satsuma Kdv systems.*
Keywords: *Nonlinear PDEs and systems, Exact Solutions, Nonlinear Waves and The hyperbolic-sine function method.*

I. Introduction

The study of numerical methods for the solution of nonlinear partial differential equations has enjoyed an intense period of activity over the last 40 years from both theoretical and practical points of view. Improvements in numerical techniques, together with the rapid advances in computer technology, have meant that many of the partial differential equations arising from engineering and scientific applications, which were previously intractable, can now, be mroutinely solved. Recently there are many new methods to obtain exact solutions of nonlinear PDEs such as sine-cosine function method [1-5], tanh function method [6-8], $({G'}/{G})$-expansion method [9-13], extended Jacobi elliptic function method [14, 15]. The aim of the present paper is to extend the hyperbolic-sine function method introduced to find new solitary solutions of the some nonlinear partial differential equations such as the generalized Klien-Gordon, the general improved Kadomtsev-Petviashvili (KP), and the Zakharov-Kuznetsov (ZK) with power law nonlinearity equations, the generalized coupled Drinfeld –sokolov –wilso, and the generalized coupled Hirota-Satsuma Kdv systems.

II. Hyperbolic-sine function method [16].

Consider the nonlinear partial differential equation in the form

$$F(u_t, u_x, u^n u_x, u_{xxx}, u_{xxt}, \dots) \tag{1},$$

where $u(x,t)$ is the solution of (1); and u_t, u_x etc, are the partial derivatives of u with respect to t and x, respectively. We assume that equation (1) admits travelling wave solution. We use the traveling wave variable:

$$u(x,t) = f(\xi), \quad \xi = x - ct \tag{2},$$

where c is the speed of the travelling wave. This enables us to use the following:

$$\frac{\partial}{\partial t}(\cdot) = -c\frac{d}{d\xi}(\cdot), \frac{\partial}{\partial x}(\cdot) = \frac{d}{d\xi}(\cdot), \frac{\partial^2}{\partial x^2}(\cdot) = \frac{d^2}{d\xi^2}(\cdot), \dots \tag{3}$$

Using the above transformation the nonlinear partial differential equation (1) is transformed to nonlinear ordinary differential equation:

$$G\left(\frac{df}{d\xi},\ f^n\frac{df}{d\xi}, \frac{d^3f}{d\xi^3}, \dots\right) = 0 \tag{4}$$

By integrating equation (4) with respect to ξ, we obtain:

$$H\left(f, f^{n+1}, \frac{d^2f}{d\xi^2}, \dots\right) = 0 \tag{5}$$

The solution of equation (2) can be expressed as:

$$u(x,t) = f(\xi) = \lambda \sinh^{\beta}(\mu\xi) \quad (6),$$

where λ, β and μ are unknown parameters which will be determined. Then we have:

$$\frac{df}{d\xi} = \lambda\, \beta\, \mu \sinh^{\beta-1}(\mu\xi)\, \operatorname{csch}(\mu\xi)$$

$$\frac{d^2f}{d\xi^2} = \lambda\, \mu^2 \beta(\beta-1)\sinh^{\beta-2}(\mu\xi) + \lambda\, \mu^2\beta(\beta-1)\sinh^{\beta}(\mu\xi) + \lambda\, \mu^2 \beta \sinh^{\beta}(\mu\xi) \quad (7)$$

We substitute (6) and (7) in (4) to obtain an equation in different powers of sine-hyperbolic functions. Now equating the coefficients of the same powers of sine-hyperbolic functions we obtain a system of algebraic equations in the parameters λ, β and μ. This system can be solved to obtain the values of λ, β and μ. The exact analytical solution of nonlinear partial differential equation (2) is then obtained by substituting the values of the parameters in equation (6).

III. Applications

In order to illustrate the effectiveness of the proposed method examples of mathematical interest are chosen as follows:

3.1. The generalized Klien-Gordon equation

In this section we introduce solitary exact solution for a generalized Klien-Gordon equation which is as follows:

$$u_{tt} - u_{xx} - u + u^p = 0 \quad (8),$$

where p is a positive integer. From equations (3), we have:

$$c^2 \frac{d^2f(\xi)}{d\xi^2} - \frac{d^2f(\xi)}{d\xi^2} - f(\xi) - (f(\xi))^p = 0 \quad (9)$$

From equations (6) and (7) we have:

$$(c^2-1)\left[\lambda\, \mu^2\beta(\beta-1)\sinh^{\beta-2}(\mu\xi) + \lambda\, \mu^2\beta(\beta-1)\sinh^{\beta}(\mu\xi) + \lambda\, \mu^2\beta\sinh^{\beta}(\mu\xi)\right] - \lambda \sinh^{\beta}(\mu\xi) - \lambda^{p}\sinh^{p\beta}(\mu\xi) = 0 \quad (10)$$

By balancing the exponents of each pair of sinsh we have:

$$(c^2-1)\lambda\, \mu^2\beta(\beta-1) + (c^2-1)\lambda\, \mu^2\beta - \lambda = 0, \quad \beta - \frac{2}{1-p} = 0$$

$$(c^2-1)\lambda\, \mu^2\beta(\beta-1) - \lambda^{p} = 0 \quad (11)$$

Using MATHEMATICA package software for solving the system equation (11) we obtain:

$$\beta = \frac{2}{1-p}, \qquad \mu = \frac{\sqrt{1-2p+p^2}}{2\sqrt{c^2-1}}, \qquad \lambda = 2^{\frac{1}{1-p}}\left(\frac{1}{1+p}\right)^{\frac{1}{1-p}} \quad (12)$$

Thus we obtain a new exact solution of the general Klien-Gordon equation in the form:

$$u(x,t) = 2^{\frac{1}{1-p}}\left(\frac{1}{1+p}\right)^{\frac{1}{1-p}} \operatorname{sinsh}^{\frac{2}{1-p}}\left(\left(\frac{1-p}{2\sqrt{c^2-1}}\right)(x-ct)\right) \quad (13)$$

As special case if p=3 we get the Klien-Gordon equation [17] in the following form:

$$u_{tt} - u_{xx} - u + u^3 = 0 \quad (14),$$

and thus its exact soliton solution is

$$u(x,t) = 8\sinh^{-1}\left(\frac{-(x-ct)}{\sqrt{c^2-1}}\right), |c| > 1 \quad (15)$$

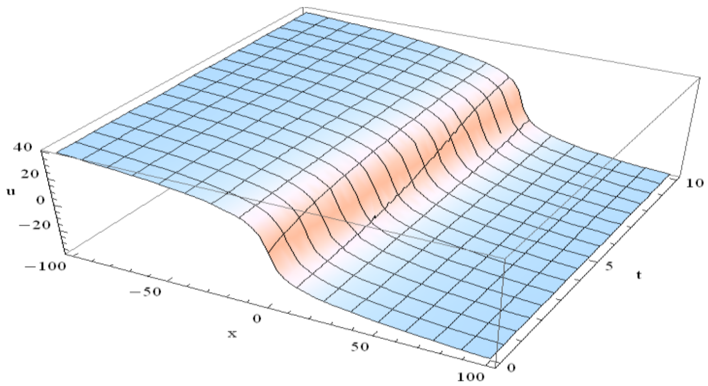

Figure 1. Traveling wave solution of Eq. (8) for solution (15), c = 2, P = 3.

3.2. The general improved Kadomtsev-Petviashvili (KP) equation

Consider the following nonlinear partial differential equation (known as the general improved Kadomtsev-Petviashvili (KP) equation) [18-22]

$$(u_t + \varepsilon u^p u_x + u_{xxx})_x + \alpha u_{yy} = 0 \quad (16),$$

where ε, α are arbitrary nonzero constants. In this case we use the following transformation:

$\xi = x + y - ct, \qquad u = u(\xi),$

Thus

$$\frac{d}{d\xi}\left(-c\frac{df(\xi)}{d\xi} + \varepsilon f(\xi)^p \frac{df(\xi)}{d\xi} + \frac{d^3 f(\xi)}{d\xi^3}\right) + \alpha \frac{d^2 f(\xi)}{d\xi^2} = 0 \quad (17)$$

By integration twice we have:

$$(\alpha - c)f(\xi) + \frac{\varepsilon}{p+1}\left(f(\xi)\right)^{p+1} + \frac{d^2 f(\xi)}{d\xi^2} = 0 \quad (18)$$

From equations (6) and (7) we have:

$$(\alpha - c)\lambda \sinh^\beta(\mu\xi) + \frac{\varepsilon}{p+1}\lambda^{p+1}\sinh^{(p+1)\beta}(\mu\xi) + \lambda\mu^2\beta(\beta-1)\mathrm{sinhs}^{\beta-2}(\mu\xi) +$$
$$\lambda\mu^2\beta(\beta-1)\sinh^\beta(\mu\xi) + \lambda\mu^2\beta\sinh^\beta(\mu\xi) = 0 \quad (19)$$

By balancing the exponents of each pair of sinsh we have:

$$\beta + \frac{2}{p} = 0, \qquad \frac{\varepsilon}{p+1}\lambda^{p+1} + \lambda\mu^2\beta(\beta-1) = 0,$$

$$(\alpha - c)\lambda + \lambda\mu^2\beta(\beta-1) + \lambda\mu^2\beta = 0 \quad (20)$$

Using MATHEMATICA package software for solving the system of equations (20) we obtain:

$$\beta = -\frac{2}{p}, \quad \mu = \frac{p}{2}\sqrt{c-}\,\alpha, \lambda = 2^{\frac{-1}{p}}\left(\frac{(\alpha-c)(p^2+3p+2)}{\varepsilon}\right)^{\frac{1}{p}} \quad (21)$$

Thus we now have new exact solution of the general improved Kadomtsev-petviashvili equation is given by:

$$u(x,y,t) = 2^{\frac{-1}{p}}\left(\frac{(\alpha-c)(p^2+3p+2)}{\varepsilon}\right)^{\frac{1}{p}}\sinh^{-\frac{2}{p}}\left(\frac{p}{2}\sqrt{c-}\,\alpha(x+y-ct)\right), c > \alpha, \ p \neq -1, p \neq -2 \quad (22)$$

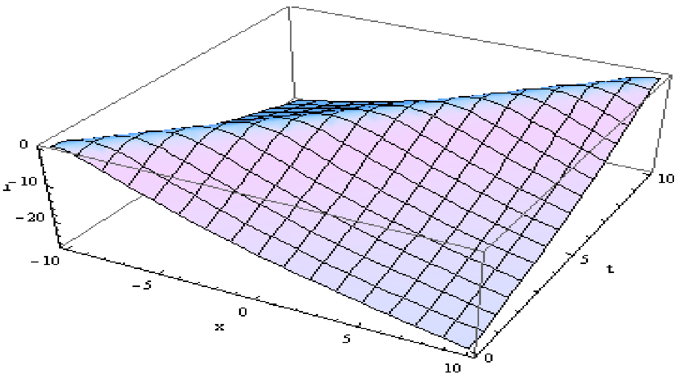

Figure 2. Traveling wave solution of Eq. (16) for solution (22), $\alpha = 1, P = 1, c = 2, y = 10, \varepsilon = 1$.

3.3. The Zakharov-Kuznetsov (ZK) equation with power law nonlinearity

This ZK appears in many areas of physics, applied Mathematics, and Engineering. In particular, it shows up in the areas of Plasma Physics. The ZK govern the behaviour of weakly nonlinear ion-acoustics waves in a plasma comprising of cold ion and hot isothermal electron in the presence of a uniform magnetic field.

The ZK equation [23-28] is given by

$$u_t + au^n u_x + b\left(u_{xx} + u_{yy}\right)_x = 0 \qquad (23),$$

In equation (23), a and b are nonzero real valued constants. The first term represents the evolution term while the second term is the nonlinear term and finally the third and fourth terms together, in parentheses , are the dispersion terms. The solitons are a result of a delicate balance between dispersion and nonlinearity. The exponent n, which indicates the power law , is a positive real number. The special case where $n = \frac{1}{2}$ gives the modified ZK equation.

We use the traveling wave variable:

$$u(x, y, t) = f(\xi), \qquad \xi = x + y - ct \qquad (24)$$

Then from equations (3), we have:

$$-c\frac{df(\xi)}{d\xi} + a(f(\xi))^n \frac{df(\xi)}{d\xi} + b\frac{d}{d\xi}\left(2\frac{d^2 f(\xi)}{d\xi^2}\right) = 0 \qquad (25)$$

By integration we have:

$$-c\, f(\xi) + \frac{a}{n+1}\left(f(\xi)\right)^{n+1} + 2b\frac{d^2 f(\xi)}{d\xi^2} = 0 \qquad (26)$$

From equations (6) and (7) we have:

$$-c\lambda \sinh^{\beta}(\mu\xi) + \frac{a}{n+1}\lambda^{n+1}\sinh^{(n+1)\beta}(\mu\xi) + 2b\lambda\mu^2\beta(\beta-1)\sinh^{\beta-2}(\mu\xi) + 2b\lambda\mu^2\beta(\beta-1)\sinh^{\beta}(\mu\xi) + 2b\lambda\mu^2\beta\sinh^{\beta}(\mu\xi) = 0 \qquad (27)$$

By balancing the exponents of each pair of sinsh we have:

$$-c\lambda + 2b\lambda\mu^2\beta(\beta-1) + 2b\lambda\mu^2\beta = 0, \quad \beta + \frac{2}{n} = 0, \qquad \frac{a}{n+1}\lambda^{n+1} + 2b\lambda\mu^2\beta(\beta-1) = 0 \qquad (28)$$

Using MATHEMATICA package software for solving the system (28) we obtain:

$$\lambda = 2^{\frac{-1}{n}}\left(\frac{-2c-2cn-cn^2}{a}\right)^{\frac{1}{n}}, \quad \beta = \frac{-2}{n}, \quad \mu = \frac{\sqrt{c}n}{2\sqrt{2b}} \qquad (29)$$

Thus we obtain new exact solution of the ZK equation in the form:

$$u(x, y, t) = 2^{\frac{-1}{n}}\left(\frac{-2c-2cn-cn^2}{a}\right)^{\frac{1}{n}}\sinh^{\frac{-2}{n}}\left(\frac{\sqrt{c}n}{2\sqrt{2b}}\,(x+y-ct)\right), c > 2b, b > 0, a > 0 \qquad (30)$$

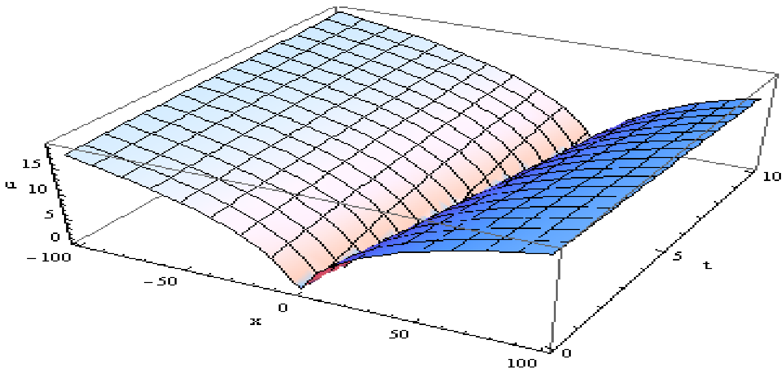

Figure 3. Traveling wave solution of Eq. (23) for solution (30), $n = 1, a = -10, b = 1, c = 3, y = 1$.

3.4. The generalized coupled Drinfeld –sokolov –wilso system

This system [29-32] is given by

$$u_t - 3vv_x = 0, \qquad v_t - 3v_{xxx} - a(uv)_x = 0 \tag{31}$$

We assume the solution of the system (30) in the form:

$$u(x,t) = f(\xi), \qquad v(x,t) = g(\xi), \qquad \xi = x - ct \tag{32}$$

From equations (2) and (3), we have:

$$-c\frac{df(\xi)}{d\xi} - 3g\frac{dg(\xi)}{d\xi} = 0, \; -c\frac{dg(\xi)}{d\xi} - 3\frac{d^3g(\xi)}{d\xi^3} - a\frac{d}{d\xi}\big(f(\xi)g(\xi)\big) = 0 \tag{33}$$

By integration we have:

$$f(\xi) = -\frac{3}{2c}(g(\xi))^2, \qquad -cg(\xi) - 3\frac{d^2g(\xi)}{d\xi^2} - a\big(f(\xi)g(\xi)\big) = 0 \tag{34}$$

Thus from (33) and (34) we have:

$$-cg(\xi) - 3\frac{d^2g(\xi)}{d\xi^2} - \frac{3a}{2c}(g(\xi))^3 = 0 \qquad (35),$$

We assume via hyperbolic sine method that

$$g(\xi) = \lambda \sinh^{\beta}(\mu\xi) \tag{36}$$

Thus we have:

$$-c\lambda \sinh^{\beta}(\mu\xi) - 3\big[\lambda\mu^2\beta(\beta-1)\sinh^{\beta-2}(\mu\xi) + \lambda\,\mu^2\beta(\beta-1)\sinh^{\beta}(\mu\xi) + \lambda\,\mu^2\beta\sinh^{\beta}(\mu\xi)\big] + \frac{3a}{2c}\lambda^3\beta\sinh^{3\beta}(\mu\xi) = 0. \tag{37}$$

By balancing the exponents of each pair of sinsh we have:

$$-c\lambda - 3\lambda\mu^2\,\beta(\beta-1) - 3\lambda\mu^2\,\beta = 0, \;\; \beta + 1 = 0, \;\; -3\lambda\mu^2\,\beta(\beta-1) + \frac{3a}{2c}\lambda^3 = 0 \tag{38}$$

Using MATHEMATICA package software for solving the system equation (38) we obtain:

$$\mu = \sqrt{\frac{c}{3}}, \qquad \lambda = \frac{2c}{\sqrt{3a}}, \qquad \beta = -1 \tag{39}$$

Thus the exact solution of the generalized coupled Drinfeld –sokolov –wilso system is given as follows:

$$v(x,t) = \frac{2c}{\sqrt{3a}}\sinh^{-1}\left(\sqrt{\frac{c}{3}}\,(x-ct)\right), a > 0, c > 0 \qquad (40)(a),$$

and

$$u(x,t) = \frac{2c}{a}\sinh^{-2}\left(\sqrt{\frac{c}{3}}\,(x-ct)\right), a > 0, c > 0 \qquad (40)(b).$$

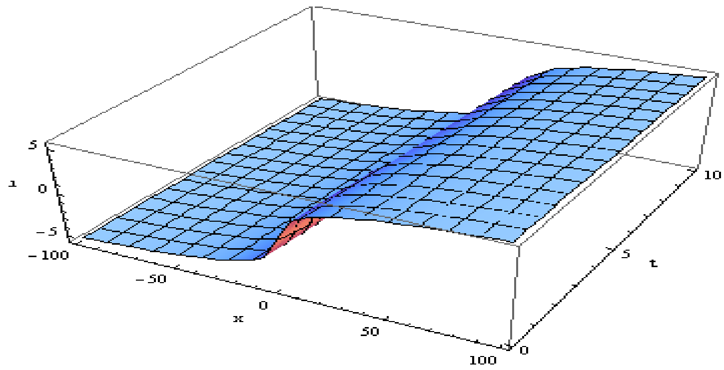

Figure 4. Traveling wave solution of Eq. (31) for solution (40)(a), c = a = 1

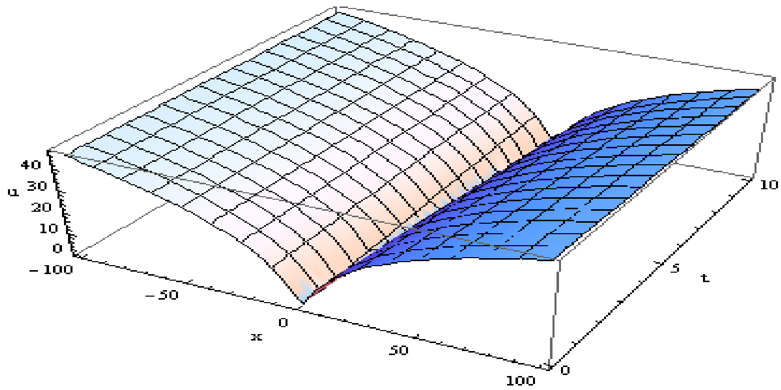

Figure 4. Traveling wave solution of Eq. (31) for solution (40)(b), c = a = 1

3.5. The generalized coupled Hirota-Satsuma Kdv system

The generalized coupled Hirota-Satsuma Kdv system [33, 34] is given as follows:

$$u_t - au_{xxx} - 3uu_x + 6vv_x = 0, \qquad v_t + bv_{xxx} + 3(uv)_x = 0 \tag{41}$$

where a, b are nonzero constants.

To obtain the travelling wave solutions we use the following transformations:

$$u(x,t) = f(\xi), \qquad v(x,t) = g(\xi), \qquad \xi = x - ct \tag{42}$$

From equations (2) and (3), we have:

$$-c\frac{df(\xi)}{d\xi} - a\frac{d^3f(\xi)}{d\xi^3} - 3f(\xi)\frac{df(\xi)}{d\xi} + 6g(\xi)\frac{dg(\xi)}{d\xi} = 0,$$
$$-c\frac{dg(\xi)}{d\xi} + b\frac{d^3g(\xi)}{d\xi^3} + 3\frac{d}{d\xi}\big(f(\xi)g(\xi)\big) = 0 \tag{43}$$

By integration we have

$$-c\,f(\xi) - a\frac{d^2f(\xi)}{d\xi^2} - \frac{3}{2}\big(f(\xi)\big)^2 + 3\big(g(\xi)\big)^2 = 0, -c\,g(\xi) + a\frac{d^2g(\xi)}{d\xi^2} + 3fg = 0 \tag{44}$$

We assume via the hyperbolic-sine method that:

$$f(\xi) = \lambda_1\sinh^{\beta_1}(\mu\xi), \qquad g(\xi) = \lambda_2\sinh^{\beta_2}(\mu\xi) \tag{45}$$

from equation (45), equation (44) becomes in the following form:

$$-c\lambda_1\sinh^{\beta_1}(\mu\xi) - a[\lambda_1\mu^2\beta_1(\beta_1-1)\sinh^{\beta_1-2}(\mu\xi) + \lambda_1\mu^2\beta_1(\beta_1-1)\sinh^{\beta_1}(\mu\xi) + \lambda_1\mu^2\beta_1\sinh^{\beta_1}(\mu\xi)$$
$$-\frac{3}{2}\lambda_1{}^2\sinh^{2\beta_1}(\mu\xi) + 3\lambda_2{}^2\sinh^{2\beta_2}(\mu\xi) = 0,$$
$$-c\lambda_2\sinh^{\beta_2}(\mu\xi) + b[\lambda_2\mu^2\beta_2(\beta_2-1)\sinh^{\beta_2-2}(\mu\xi) + \lambda_2\mu^2\beta_2(\beta_2-1)\sinh^{\beta_2}(\mu\xi) + \lambda_2\mu^2\beta_2\sinh^{\beta_2}(\mu\xi)$$
$$+ 3\lambda_1\lambda_2\sinh^{\beta_1+\beta_2}(\mu\xi) = 0 \tag{46}$$

By balancing the exponents of each pair of sinh we have:

$$-c\lambda_1 - a\lambda_1\mu^2\beta_1(\beta_1-1) - a\lambda_1\mu^2\beta_1 = 0,\ 2\beta_1 = 2\beta_2 = \beta_1 - 2, -a\lambda_1\mu^2\beta_1(\beta_1-1) - \frac{3}{2}\lambda_1{}^2 + 3\lambda_2{}^2 = 0,$$
$$-c\lambda_2 + b\lambda_2\mu^2\beta_2(\beta_2-1) + b\lambda_2\mu^2\beta_2 = 0,\ \beta_2 - 2 = \beta_1 + \beta_2,\ b\lambda_2\mu^2\beta_2(\beta_2-1) + 3\lambda_1\lambda_2 = 0 \tag{47}$$

Using MATHEMATICA package software for solving the system equation we have:

$$\beta_1 = \beta_2 = -2,\ \ b = -a, \quad \lambda_1 = \frac{c}{2}, \lambda_2 = \frac{c}{2}\sqrt{\frac{3}{2}},\ \mu = \frac{1}{2}\sqrt{\frac{c}{a}} \tag{48}$$

Thus the exact solution of the generalized coupled Hirota-Satsuma Kdv system (41) is given as:

$$u(x,t) = \frac{c}{2}\sinh^{-2}\left(\frac{1}{2}\sqrt{\frac{c}{a}}(x-ct)\right), \quad a>0, c>a \qquad (49)(a),$$

and

$$v(x,t) = \frac{c}{2}\sqrt{\frac{3}{2}}\sinh^{-2}\left(\frac{1}{2}\sqrt{\frac{c}{a}}\,(x-ct)\right), \quad a>0, c>a \qquad (49)(b),$$

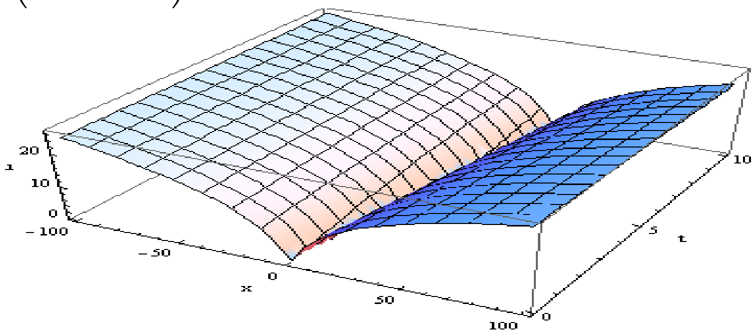

Figure 5. Traveling wave solution of Eq. (41)for solution (49)(a), c = 2, a = 1.

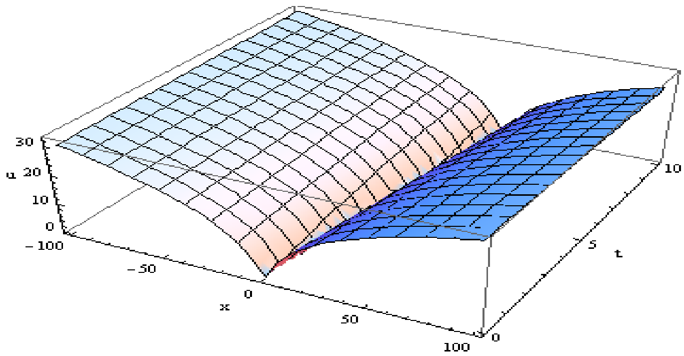

Figure 6. Traveling wave solution of Eq. (41)for solution (49)(b), c = 2, a = 1.

IV. Conculsion

In this paper, the hyperbolic-sine function method has been successfully applied to obtain new solutions of some nonlinear partial differential equations. Thus,the hyperbolic-sine function method can be extended to solve the problems of nonlinear partial differential equations which arising in the theory of solitons and other areas.

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