Effect of Radiation on Heat Transfer over an Exponentially Stretching Surface with MHD

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Abstract: *The effect of radiation on magnetohydrodynamic (MHD) boundary layer flow of a viscous fluid over an exponentially stretching sheet was examined. The governing partial differential equation are non-dimensionalised and transformed into a system of nonlinear ordinary differential similarity equations, in a single independent variable η and using RungeKutta Gill method with shooting technique. The velocity and temperature distributions are discussed numerically and presented through graphs. It was found that the local heat transfer rate at the surface decreases with increasing values of the magnetic and radiation parameters.*
Keywords: *Boundary layer, heat transfer, MHD, radiation, stretching sheet.*

I. Introduction

The boundary layer flow on a continuous stretching sheet has attracted considerable attention during the last few decades due to its numerous applications in industrial manufacturing processes such as hot rolling, wire drawing, glass-fiber and paper production, drawing of plastic films, metal and polymer extrusion and metal spinning. Both the kinematics of stretching and the simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products (Magyari & Keller 1999). Magyari et al (2001) reported analytical and computational solutions when the surface moves with rapidly decreasing velocity using self similarity method.

Magnetohydrodynamics flows have applications in meteorology, solar physics, cosmic fluid dynamics, astrophysics, geophysics and in the motion of earth's core. In addition from the technological point of view, MHD free convection flows have significant applications in the field of stellar and planetary magneto spheres, aeronautical plasma flows, chemical engineering and electronics.

With general power function form for stretching velocity of the wall, the same problem has been analyzed with surface suction injection by Ali and Al-Yousef (2002). Magyari and Keller (1999) investigated the steady boundary layers on an exponentially stretching continuous surface with an exponential temperature distribution. Partha et al. (2005) studied the effect of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface. Recently, Sajid and Hayat (2008) extended this problem by investigating the radiation effects on the flow over an exponentially stretching sheet, and solved the problem analytically using the homotopy analysis method. The numerical solution for the same problem was then given by Bidin and Nazar (2009). The study of magnetohydrodynamic has important applications, and may be used to deal with problems such as cooling of nuclear reactors by liquid sodium and induction flow meter, which depends on the potential difference in the fluid in the direction perpendicular to the motion and to the magnetic field Ganesan & Palani (2004). At high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperatures and knowledge of radiation heat transfer becomes very important for the design of pertinent equipment (Seddeek 2002).

The aim of the present paper is to study the effect of radiation on heat transfer over an exponentially stretching surface with MHD which has not been considered before.

II. Problem Formulation

A steady two dimensional flow of an incompressible viscous and electrically conducting fluid caused by a stretching sheet, which is placed in a quiescent ambient fluid of uniform temperature T_∞, as shown in Fig.1. A variable magnetic field B(x) is applied normal to the sheet and that the induced magnetic field is neglected, which is justified for MHD flow at small magnetic Reynolds number. Under the usual boundary layer approximations, the flow and heat transfer with the radiation effects are governed by the following equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2 u}{\rho} \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \tag{3}$$
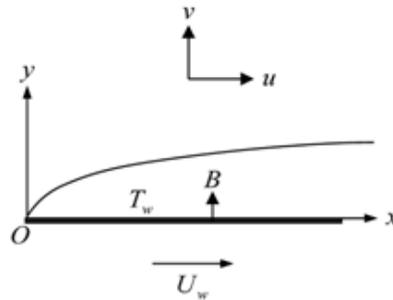

Fig. 1. Physical model and Coordinate system

where u and v are the velocities in the x and y- directions, respectively, ρ is the fluid density, ν the kinematics viscosity, k the thermal conductivity, c_p the specific heat, T the fluid temperature in the boundary layer and q_r is the radiative heat flux. The boundary conditions are given by

$$u = U_w = U_0 e^{x/L}, \quad v = 0$$
$$T = T_w = T_\infty + T_0 e^{x/2L} \quad at \quad y = 0 \tag{4}$$
$$u \to 0, T \to T_\infty \quad as \quad y \to \infty$$

where U_0 is the reference velocity, T_0 the reference temperature and L is the reference length. Most of the effort in understanding fluid radiation is devoted to the derivation of reasonable simplifications (Aboeldahab & El Gendy 2002).

For an optically thick gas, the gas self-absorption rises and the situation become difficult. However, the problem can be simplified by using the Rosseland approximation which simplifies the radiative heat flux as:

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \tag{5}$$

where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. This approximation is valid at points optically far from the boundary surface, and it is good only for intensive absorption, which is for an optically thick boundary layer (Bataller 2008). It is assumed that the temperature differences within the flow such that the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms gives:

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \tag{6}$$

Using Eqs. (5) and (6), Eq. (3) reduces to:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3\rho c_p k^*} \frac{\partial^2 T}{\partial y^2} \tag{7}$$

To obtain similarity solutions, it is assumed that the magnetic field B(x) is of the form
$$B = B_0 e^{x/2L} \tag{8}$$
where B_0 is the constant magnetic field.
The continuity equation (1) is satisfied by introducing a stream function ψ such that

$$u = \frac{\partial \psi}{\partial y} \quad and \quad v = -\frac{\partial \psi}{\partial x} \tag{9}$$

The momentum and energy equations can be transformed into the corresponding ordinary differential equations by the following transformation (Sajid & Hayat 2008):

$$u = U_0 e^{x/L} f'(\eta), \quad v = -\left(\frac{\nu U_0}{2L}\right)^{1/2} e^{x/2L} \left(f(\eta) + \eta f'(\eta)\right) \tag{10}$$

$$T = T_\infty + T_0 e^{x/2L} \theta(\eta), \quad \eta = \left(\frac{U_0}{2\nu L}\right)^{1/2} e^{x/2L} y$$

where η is the similarity variable, f(η) is the dimensionless stream function, θ(η) is the dimensionless temperature and primes denote differentiation with respect to η. The transformed ordinary differential equations are:

$$f''' + f f'' - 2f'^2 - Mf' = 0 \qquad (11)$$

$$\left(1 + \frac{4}{3}K\right)\theta'' + \Pr(f\theta' - f'\theta) = 0 \qquad (12)$$

in which $M = \dfrac{2\sigma B_0^2 L}{\rho U_0}$ is the magnetic parameter, $K = \dfrac{4\sigma^* T_\infty^3}{k^* k}$ the radiation parameter and $\Pr = \dfrac{\rho v c_p}{k}$ is the Prandtl number. The transformed boundary conditions are:

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1 \quad at \ \eta = 0$$
$$f'(\infty) \to 0, \ \theta(\infty) \to 0 \qquad as \ \eta \to \infty \qquad (13)$$

The skin-friction coefficient and the local Nusselt number which represents the wall shear stress and the heat transfer rate at the surface, respectively are important physical parameters for this type of boundary layer flow.

III. Solution Of The Problem

The set of coupled non-linear governing boundary layer equations (11) and (12) together with the boundary conditions (13) are solved numerically by using Runge-Kutta Gill method along with the shooting technique. First of all, higher order non-linear differential equations (11) and (12) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique. The resultant initial value problem is solved by employing Runge-Kutta Gill method. The step size Δη = 0.05 is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. From the process of numerical computation, the local Nusselt number, which is proportional to $-\theta'(0)$ is also sorted out and their numerical values are presented in a tabular form.
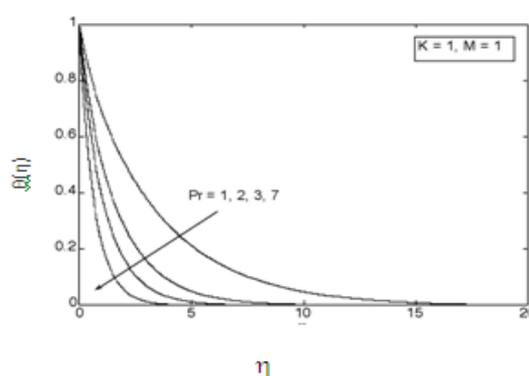
IV. Figures and Tables

Table 1. Values of $\theta'(0)$ for different values of K, M and Pr

K	M	Pr	Magyari and Keller (1999)	El-Aziz (2009)	Bidin and Nazar (2009)	Present results
0	0	1	-0.954782	-0.954785	-0.9548	-0.9548
		2			-1.4714	-1.4715
		3	-1.869075	-1.869074	-1.8691	-1.8691
		5	-2.500135	-2.500132		-2.5001
		10	-3.660379	-3.660372		-3.6603
	1	1				-0.8613
1	0				-0.5315	-0.5312
	1					-0.4504
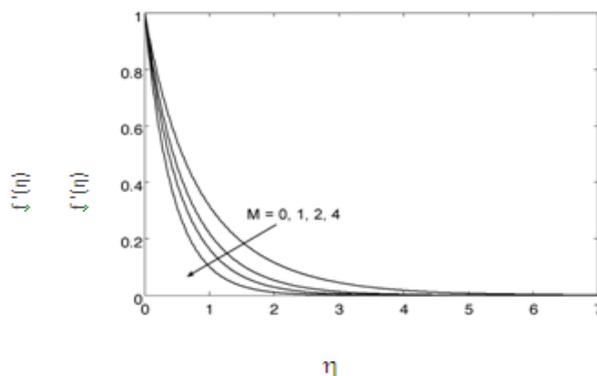
Fig. 2. Velocity profile for different values of M   **Fig. 3.** Temperature profile for different values of Pr
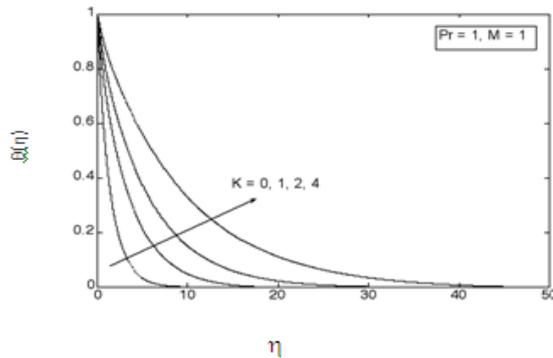
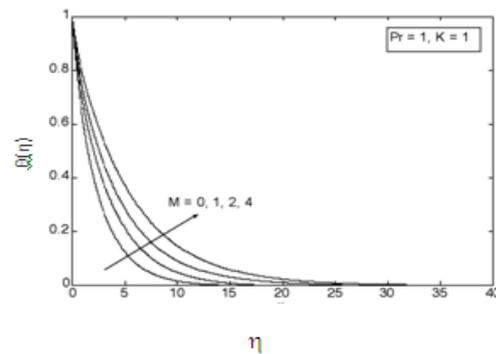**Fig. 4** Temperature profiles for different values of K   **Fig. 5** Temperature profile for different values of M

V.   Results and Discussion

Table 1 shows numerical values of magnetic field parameter M, radiation parameter K and Prandtl number Pr effects on - $\theta'(0)$. Comparison with the existing results from the literature gives a favourable agreement, as presented in Table 1.

The profiles for velocity and temperature are shown in Fig.2-5 respectively with various values of the parameters. The velocity profiles for different values of the magnetic parameter M presented in Fig. 2 show that the rate of transport is considerably reduced with the increase of M. It clearly indicates that the transverse magnetic field opposes the transport phenomena. This is because the variation of M leads to the variation of the Lorentz force due to the magnetic field, and the Lorentz force produces more resistance to the transport phenomena. We note that the Prandtl number Pr and the radiation parameter K have no influence on the flow field, which is clear from Equation (11).The velocity gradient at the surface f "(0) which represents the surface shear stress increases with increasing M. Thus, the magnetic parameter M acts as a controlling parameter to control the surface shear stress.

The temperature profiles for different values of Pr, K and M with other parameters are fixed to unity are presented in Figs. 3, 4 and 5, respectively. Figs. 2 to 5 show that the far field boundary conditions are satisfied asymptotically, thus supporting the accuracy of the numerical results obtained. It is evident from Figs.3 to 5 that the thermal boundary layer thickness increases as M and K increase but opposite trends are observed for increasing values of Pr. This results in decreasing manner of the local Nusselt number -θ′(0), which represents the heat transfer rate at the surface, with increasing M and K but opposite trends are observed for increasing values of Pr. This is because, when Pr increases, the thermal diffusivity decreases and thus the heat is diffused away from the heated surface more slowly and in consequence increase the temperature gradient at the surface

VI.   Conclusion

The effect of radiation on heat transfer over an exponentially stretching surface with MHD was investigated. The numerical results obtained agreed very well with previously reported cases available in the literature. It was found that the surface shear stress increases with the magnetic parameter M, while the heat transfer rate increases with Prandtl number Pr, but decreases with both magnetic parameter M and radiation parameter K.

Acknowledgements

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