IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 10, Issue 4 Ver. VI (Jul-Aug. 2014), PP 24-25
www.iosrjournals.org

Method of Construction of Triangular Designs

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Abstract: *This paper describes a new method of construction of triangular design. Consequently, a new series of truly self dual triangular design is obtained.*

I. Introduction

A triangular association scheme is defined as follows:

Consider a set of cardinality n(say), $S = 1, 2, ..., n$. The treatments of association scheme are all possible $\binom{n}{2}$ unordered pairs of elements of S. Two unordered pairs corresponding to two different treatments are first associates, if they have one element is common, otherwise second associates, e.g. (i, j) and (i, j^1) are first associates if $(j \neq j^1)$

The following relations clearly hold :

$$n_1 = 2(n-2),\ n_2 = \frac{(n-2)(n-3)}{2}.$$

$$p_1 = \begin{pmatrix} p_{11}^1 & p_{12}^1 \\ p_{21}^1 & p_{22}^1 \end{pmatrix} = \begin{pmatrix} n-2 & n-3 \\ n-3 & \frac{(n-3)(n-4)}{2} \end{pmatrix}$$

$$p_2 = \begin{pmatrix} p_{11}^2 & p_{12}^2 \\ p_{21}^2 & p_{22}^2 \end{pmatrix} = \begin{pmatrix} 4 & 2n-8 \\ 2n-8 & \frac{(n-4)(n-5)}{2} \end{pmatrix}$$

For the definition of triangular design see, Raghavarao (1988).For the classification and analysis of triangular design one can refer Bose and Shimanoto (1952). Two self dual designs are said to be truly self dual if both are isomorphic to each other.[one can refer Clatworthy (1973).]

II. Construction

In this section we describe the method how truly self dual triangular designs are obtained. We also give two triangular designs which are truly self dual.

Theorem : A series of two associate triangular PBIB designs with parameters

$$v = \frac{n(n-1)}{2},\ b = \frac{n(n-1)}{2},\ r = \frac{(n-2)(n-3)}{2},\ k = \frac{(n-2)(n-3)}{2},$$

$$\lambda_1 = \frac{(n-2)(n-3)}{4},\ \lambda_2 = \frac{(n-2)(n-3)-2(n-1)}{2},\ n_1 = 2(n-2), n_2 = \frac{(n-2)(n-3)}{2}, \tag{1.1}$$

always exists when $n \geq 6$ and is truly self dual.

Proof : Suppose s be the set of n positive integers numbered from 1,2, …, n . we identify 2 – sets of s as treatments and $(n-2)-$sets of s as blocks. A block corresponding to an $(n-2)$ set of s consists of all 2 – sets formed with the $(n-2)$set. Thus we get the design with parameters given in (1.1), which can be easily verified.

Now by introducing the isomorphism
$(i_i\, i_2 \ldots\ldots) \leftrightarrow (i_{2+1} \ldots\ldots i_n)$
where $i_1 \ldots\ldots i_2 \neq i_{2+1} \ldots\ldots i_n$; $i = 1, 2, \ldots\ldots, n,$ between treatments and blocks and noting that if $n < 2\,(n-2)$ the association scheme is with two associate classes. It can also be verified that the design with parameters (1.1) is truly self dual.

As a particular case of (1.1) for n = 7, we get a triangular PBIB design with parameters v = b = 21, r = k = 10, λ_1=5, λ_2 = 4 ; n_1=10, n_2=10 we find that the triangular design T_{95} is Clatworthy (1973) and the triangular design obtained by us with parameters (1.1) for n = 7 are isomorphic. Therefore, two associate class triangular design T_{95} is in fact truly self dual. But T_{95} in Clatworthy (1973) is marked as only self dual.

Triangular design T_{95} obtained by us which is truly self dual whose blocks are written in the form of columns:

1	**2**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**2**	**2**	**2**	**2**	**3**	**3**	**3**	**4**	**4**	**5**
3	**3**	**2**	**2**	**2**	**2**	**2**	**3**	**4**	**5**	**6**	**3**	**4**	**5**	**6**	**4**	**5**	**6**	**5**	**6**	**6**
4	**4**	**4**	**3**	**3**	**3**	**8**	**7**	**7**	**7**	**7**	**7**	**7**	**7**	**7**	**8**	**8**	**8**	**9**	**9**	**10**
5	**5**	**5**	**6**	**4**	**4**	**9**	**9**	**8**	**8**	**8**	**8**	**9**	**10**	**11**	**9**	**10**	**11**	**10**	**11**	**11**
6	**6**	**6**	**9**	**6**	**5**	**10**	**10**	**10**	**9**	**9**	**13**	**12**	**12**	**12**	**12**	**12**	**12**	**13**	**13**	**14**
7	**7**	**8**	**13**	**10**	**11**	**11**	**11**	**11**	**11**	**10**	**14**	**14**	**13**	**13**	**13**	**14**	**15**	**14**	**15**	**15**
12	**8**	**12**	**5**	**14**	**15**	**12**	**12**	**13**	**14**	**15**	**15**	**15**	**15**	**14**	**17**	**16**	**16**	**16**	**16**	**17**
13	**9**	**16**	**16**	**17**	**18**	**13**	**16**	**16**	**17**	**18**	**16**	**16**	**17**	**18**	**18**	**18**	**17**	**17**	**18**	**18**
14	**10**	**17**	**19**	**20**	**19**	**14**	**17**	**19**	**20**	**19**	**17**	**19**	**20**	**19**	**19**	**20**	**19**	**19**	**20**	**19**
15	**11**	**18**	**21**	**21**	**20**	**15**	**18**	**21**	**21**	**20**	**18**	**21**	**21**	**20**	**21**	**21**	**20**	**20**	**21**	**21**

Remark : Triangular Design T_{94} of Clatworthy (1973) is marked as self dual
whereas we are reporting truly self dual with parameters

v = b = 21, r = k = 10, λ_1=6, λ_2 = 3 ; n_1=10, n_2=10

The solution of T_{94} obtained by us is presented below in which blocks are written in the form of columns:

1	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**2**	**2**	**2**	**2**	**3**	**7**	**7**	**7**	**7**	**8**	**12**
2	**2**	**2**	**2**	**2**	**2**	**3**	**3**	**3**	**4**	**3**	**3**	**3**	**4**	**4**	**8**	**8**	**8**	**9**	**9**	**13**
3	**3**	**3**	**4**	**4**	**5**	**4**	**4**	**5**	**5**	**4**	**4**	**5**	**5**	**5**	**9**	**9**	**10**	**10**	**10**	**14**
4	**5**	**6**	**5**	**6**	**6**	**5**	**6**	**6**	**6**	**5**	**6**	**6**	**6**	**6**	**10**	**11**	**11**	**11**	**11**	**15**
7	**7**	**7**	**7**	**7**	**7**	**8**	**8**	**8**	**9**	**12**	**12**	**12**	**13**	**16**	**12**	**12**	**12**	**13**	**16**	**16**
8	**8**	**8**	**9**	**9**	**10**	**9**	**9**	**10**	**10**	**13**	**13**	**14**	**14**	**17**	**13**	**13**	**14**	**14**	**17**	**17**
9	**10**	**11**	**10**	**11**	**11**	**10**	**11**	**11**	**11**	**14**	**15**	**15**	**15**	**18**	**14**	**15**	**15**	**15**	**18**	**18**
12	**12**	**12**	**13**	**13**	**14**	**16**	**16**	**17**	**19**	**16**	**16**	**17**	**19**	**19**	**16**	**16**	**17**	**19**	**19**	**19**
13	**14**	**15**	**14**	**15**	**15**	**17**	**18**	**18**	**20**	**17**	**18**	**18**	**20**	**20**	**17**	**18**	**18**	**20**	**20**	**20**
16	**17**	**18**	**19**	**20**	**21**	**19**	**20**	**21**	**21**	**19**	**20**	**21**	**21**	**21**	**19**	**20**	**21**	**21**	**21**	**21**

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