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Ultra Upper And Lower Contra Continuous Multifunction

[1]N.Durgadevi, [2]R.Rajrajeswari And [3]P.Thangavelu.
[1, 2] Department of Mathematics, Sri Parasakthi College for Women, Courtallam - 627802, TamilNadu, INDIA.
[3]Department of Mathematics, Karunya University, Coimbatore, TamilNadu, INDIA.

Abstract:*The aim of this paper is to study the ultra contra continuous multifunction.*
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I. Introduction and Preliminaries

"Bitopology" is a new concept introduced by Kelly [2] in the year 1963. He defined the topologies with the help of quasi metrics. Later, instead of taking the topologies defined on quasi metrics, researcher tried the study of bitopological spaces with any two topologies omitting the quasi metrics. Almost all the properties of classical topologies were studied using the pairwise concept, the definition of (1, 2)α - open sets introduced by Thivagar [3], in the year 1991, opened a new era of research in bitopology. He also defined (1, 2)α - continuous function and its weaker and stronger forms between two bitopological spaces. Continuity and multifunction are two basic properties in general topology and set valued analysis. Contra continuous in general topology was introduced and investigated by Dontchev in 1996. That concept was extended to bitopological spaces by Ekici in 2008. By multifunction, we mean a map- ping from a point to a set.

The main purpose of this article is to define and to generalize the ultra- upper and ultra-lower contra continuous multifunction in bitopological spaces. Throughout this paper, X means (X,τ), where X is a non empty set and τ is the topology defined on it. By Y, we mean the bitopological space (Y, σ1, σ2),
where Y is a non empty set with two topologies σ1 and σ2 defined on it.
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Definition1.1. Let A be a subset of a bitopological space (Y, σ1, σ2). Then A is said to be [3] (i) σ1σ2-open if A∈ σ1 ∪ σ2,
(ii) σ1σ2- closed if Ac∈ σ1 ∪ σ2 ,
(iii) (1, 2)α - open or ultra - open if A ⊆ σ1- Int (σ1σ2 – Cl (σ1-Int (A))), where σ1-
Int (A) is the interior of A with respect to the topology σ1and σ1σ2- Cl(A) is the intersection of all σ1σ2 - closed sets containing A. Also A is said to be (1, 2)α - closed if Ac is (1, 2)α - open.
(iv) Int(1, 2)α (A) is the union of all (1, 2)α - open sets contained in A.
(v) Cl(1, 2)α (A) is the intersection of all (1, 2)α - closed sets containing A.
The set of all (1, 2)α - open sets are denoted as (1, 2)αO(X) and if this set forms a topology, then X is called as an ultra space.

Definition1.2. An ultra multifunction [5] Fu: (X,τ) → (Y, σ1, σ2) is a point to a set correspondence and is assumed that Fu(x) = ϕ for all x ∈ X.

Definition1.3. The image set U⊂ X of the multifunction Fu : X → Y is defined [5] by Fu(U) = ∪{Fu(x) / x∈ U}.

Definition1.4. For an ultra multifunction Fu, the upper and lower inverse [5]

of Fu is defined for any set V ⊆ Y, as Fu+(V) = { x∈ X / Fu(x) ⊆ V} and Fu–(V)

= {x∈ X / Fu(x) ∩ V = ϕ}.

Lemma1.5. For any ultra multifunction Fu+(V) ⊆ Fu–(V). This result is proved in [5].

Definition1.6. Let A be a non-empty subset of a space X. Then (1, 2)α - kernal
of A or ultra kernal of A [8] and is denoted by (1, 2)α - Ker(A) and is defined as (1, 2)α - Ker(A) = ∩{G ∈ (1, 2)αO(X) / A ⊆ G}.

Definition1.7. An ultra multifunction Fu: X $\rightarrow$ Y is said to be ultra upper continuous [resp. ultra lower continuous] [5] if for any (1, 2)α - open set V of Y, there exists an open set G of X containing x such that Fu(G) $\subset$ V [resp.
Fu(x) $\cap$ V = ϕ for all x $\in$ G].

Definition1.8. An ultra multifunction Fu : X$\rightarrow$ Y is said to be ultra upper weakly continuous [resp. ultra lower weakly continuous] [9] at the point x$\in$ X, if for any (1, 2)α - open set V of Y with Fu(x)$\subset$ V, there exists an open set G of X containing x such that Fu(G) $\subset$ Cl(1, 2)α (V) for all x$\in$ G [resp. Fu(x) $\cap$ Cl(1, 2)α (V) = ϕ for all x$\in$ G].

2. Ultra upper (lower) contra continuous multifunction

In this section, we introduced two new forms of contra continuous multifunction in bitopological spaces and studied their properties.

Definition2.1. An ultra multifunction Fu: X $\rightarrow$ Y is said to be ultra upper contra continuous (u.u.c.c) at x$\in$ X if for each (1, 2)α - closed set A such that x$\in$ Fu+(V) there exists an open set G of X containing x such that Fu(G) $\subset$ V.

Definition2.2. An ultra multifunction Fu: X $\rightarrow$ Y is said to be ultra lower contra continuous (u.l.c.c) at x$\in$ X if for each (1, 2)α - closed set V containing Fu(x) and Fu(x) $\cap$ V = ϕ there exists an open set G containing x such that Fu(y) $\cap$ V = ϕ for all y$\in$ G.

F is said to be ultra upper(lower) contra continuous on X if F has this property at each point of X.

Example2.3. Let X = {1, 2, 3} with the topology τ = {ϕ, X, {1}} and Y = {a, b, c} with two topologies σ1 = {ϕ, Y, {a}} and σ2 = {ϕ, Y, {a}, {b, c}}. Define an ultra multifunction Fu: X $\rightarrow$ Y as Fu(1) = {b}, Fu(2) = {a, c}, Fu(3) = {a, b}. Here, Fu is both ultra upper contra continuous and ultra lower contra continuous.

Remark2.4. Ultra upper contra continuity implies ultra lower contra conti-nuity.

Proof: Let Fu: X$\rightarrow$ Y be an ultra upper contra continuous multifunction and x$\in$ X. Then, let V be any (1, 2) α - open set in Y such that x$\in$ Fu+ (V). Then by definition, there exists an open set G of X containing x such that Fu+(G)$\subset$ V implies Fu (y) $\cap$ V = ϕ for all y$\in$ G.
But the converse is not true as shown in the following example.

Example2.5. In Example 2.3, define an ultra multifunction Fu: X $\rightarrow$ Y as Fu(1)={a}, Fu(2)={b, c}, Fu(3) ={b}.Here, Fu is ultra lower contra continuous but it is not ultra upper contra continuous.

Theorem2.6. For an ultra multifunction Fu: X $\rightarrow$ Y , the following are equivalent.
(i) Fu is ultra upper contra continuous.
(ii) Fu+(A) is open in X for any (1, 2)α - closed set A $\subset$ Y.
(iii) Fu−(V) is closed in X for any (1, 2)α - open set V $\subset$ Y.
(iv) for each x$\in$ X and each (1,2)α - closed set A containing Fu(x), there exists an open set G containing x such that Fu(y)$\subset$ A for all y $\in$ G.

Proof: (i) $\Leftrightarrow$ (ii) Let A be any (1, 2)α-closed set in Y and x$\in$ Fu+(A). Since Fu is u.u.c.c there exists an open set G containing x such that G$\subset$ Fu+(A).Thus Fu+(A) is open.
(ii) $\Leftrightarrow$ (iii) This follows from the fact that Fu+(Y / V) = X / Fu−(V) for every subset V of Y is open. This implies that Fu−(V) is closed.
(iii) $\Leftrightarrow$ (iv) Let x$\in$ X and A be any (1, 2)α - closed set containing Fu(x). Since A is (1, 2)α - closed Y/A is (1, 2)α - open. This implies that Fu−(Y /A) is closed in X. Then Fu−(Y /A) = X / Fu+(A) is closed in X. Therefore Fu+(A) is open in X. So, there exists an open set G of X such that G$\subset$ Fu+(A) and Fu(G) $\subseteq$ A $\Rightarrow$ Fu(y)$\subseteq$ A for all y$\in$ G.
(iv)$\Leftrightarrow$ (i) It is obvious.

Theorem2.7. For an ultra multifunction Fu: X $\rightarrow$ Y , the following are equivalent.

Fu is ultra lower contra continuous.
(ii) Fu−(A) is open for any (1, 2)α-closed set A ⊂ Y.
(iii) Fu+(V) is closed in X for any (1, 2)α-open set V□ ⊂ Y.
(iv) for each x□∈ X and each (1, 2)α - closed set A containing Fu(x)□ ∩ V = ϕ ,
there exists an open set G containing x such that Fu(y)□ ∩ A = ϕ for all y□∈ G.
Proof: Similar to Theorem 2.6

Lemma2.8. For any two subsets A, B of a bitopological space (X,τ1, τ2).
The following properties hold: [8]
(i) x□∈ (1, 2)α -Ker(A) if A ∩ B = ϕ for any (1, 2)α - closed set B containing x .
(ii) If A∈ (1, 2)α O(X), then A □≡ (1, 2)α -Ker(A).

Theorem2.9. Let Fu: X □→ Y be an ultra multifunction. If Cl(Fu−(A))□ ⊂□ Fu−((1, 2)αKer(A)) for every (1, 2)α-open subset A of Y, then Fu is ultra upper contra continuous.

Proof: Suppose that Cl(Fu−(A))□⊂ Fu−((1, 2)α Ker(A)) for every subset A of Y. As A□∈ (1, 2)αO(Y). By Lemma 2.8, Cl(Fu−(A))□⊂ Fu−((1, 2)αKer(A)) = Fu−(A). Thus, Cl(Fu−(A)) = Fu−(A) and hence Fu−(A) is closed in X. Consequently, by the above Theorem 2.6, Fu is u.u.c.c.

Theorem2.10. Suppose one of the following properties hold for an ultra multifunction Fu : X□→ Y .
(i)Fu(Cl(A))□⊂ (1, 2)α -Ker(Fu(A)) for every subset A of X.
(ii)Cl(Fu+(V))□⊂ (Fu+((1, 2)α-Ker(V)) for every subset V of Y.
Then Fu is ultra lower contra continuous.

Proof: Given A□⊆ X and Fu(Cl(A))□⊂ (1, 2)α -Ker(Fu(A)). Now, let V□⊆ Y and Fu(Cl(Fu+(V))□⊂ (1, 2)α -Ker(Fu(V)). Hence, Fu(Cl(Fu+(V)))□⊂□□(1, 2)α -Ker(Fu(Fu+(V))) ⇒ Cl(Fu+(V))□⊂ Fu+((1, 2)α-Ker(V)) for every subset V of Y. Now, (1, 2)α -Ker (V) is a (1, 2)α - open set say A. This implies that (1, 2)α -Ker (V)□ □≡ A. Hence, by the definition of Kernel we get, Cl(Fu+(A)□⊆□ Fu+(A). Thus, Fu+(A) is a closed set in X.
By the Theorem 2.6, Fu is ultra lower contra continuous.

Theorem2.11. Let Fu: X□→ Y and Gu: Y□→ Z be ultra multifunction. If Fu is ultra upper continuous and Gu is ultra upper contra continuous, then GuoFu: X□→ Z is ultra upper contra continuous.

Proof: Let A□⊂ Z be a (1, 2)α - closed set. We have, (GuoFu) +(A) = Fu+(Gu + (A)).
Since, Gu is ultra upper contra continuous, Gu+ (A) is an (1, 2)α - open set. Since Fu is ultra upper continuous, Fu+ (Gu+ (A)) is an (1, 2)α-open set. Thus, GuoFu is an ultra upper contra continuous multifunction.

Theorem2.12. Let Fu: X□→ Y and Gu: Y□→ Z be an ultra multifunction. If Fu is ultra lower continuous and Gu is ultra lower contra continuous, then GuoFu: X□→ Z is ultra lower contra continuous.

Proof: Let A□⊂ Z be a (1, 2)α - closed set. We have, (GuoFu)−(A) = Fu−(G−(A)).
Since, Gu is ultra lower contra continuous, Gu−(A) is an (1, 2)α- open set. Since Fu is ultra lower continuous, Fu−(Gu−(A)) is an (1, 2)α-open set. Thus, GuoFu is ultra lower contra continuous multifunction.

Theorem2.13. Let Fu: X□→ Y be an ultra multifunction and let A□⊂ X. If Fu is an ultra lower (upper) contra continuous multifunction then restriction multifunction Fu □ □ A: A→ Y is ultra lower (upper) contra continuous.

Proof: Let B□⊂ Y be a (1, 2)α - closed set and x□∈ A and let x□∈□ (Fu□□A)−(B). Since Fu is lower contra continuous multifunction, then there exists an open set U in X containing x such that U□⊂ (Fu □□A)−(B). This implies that x□∈ U□∩ A which is open in A and also U□∩ A□⊂ (Fu □□A)−(B). Thus, Fu □□A is lower contra continuous.

II. Some stronger and weaker forms of contra multifunction

In this section, we introduced two more new contra continuous multifunctions, named as ultra upper (lower) clopen continuous, which is stronger than ultra upper (lower) contra continuous and ultra upper (lower) weakly continuous multifunction which is weaker than ultra upper (lower) contra continuous respectively. Some of their properties are studied and each function is sustained with suitable examples.

Definition3.1. An ultra multifunction Fu : X□→ Y is called ultra upper (lower) clopen continuous if for each x□∈ X and each (1,2)α - open set V such that x□∈ Fu+(V) [resp. x□∈ Fu−V] there exists a clopen set G containing x such that G□ ⊂ Fu+(V) [resp. G□⊂ Fu−(V)].

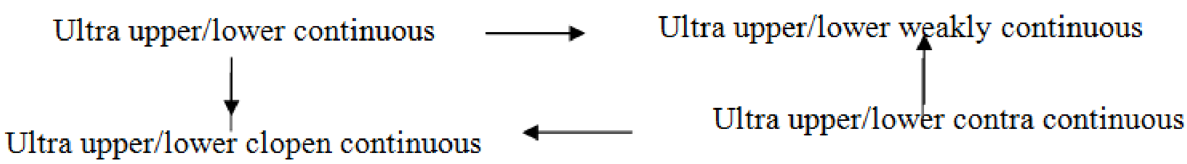
Example3.2. Let X =□{1, 2, 3} with the topology τ =□{ϕ, X,□ {1},□ {2, 3}} and Y =□{a, b, c} with two topologies σ1 =□ {ϕ, Y□} and σ2 =□ {ϕ, Y,□ {c}}. Define a multifunction Fu: X□→ Y as Fu(1) =□ {a}, Fu(2) =□ {b, c}, Fu(3) =□ {c}. Then, Fu is both ultra upper clopen continuous and ultra lower clopen continuous.

Theorem3.3. If Fu: X□→ Y is a multifunction and is ultra upper (lower) contra continuous, then Fu is ultra upper (lower) weakly continuous.

Proof: Let Fu be an ultra upper contra continuous. Also, let x□∈ X and V be any (1, 2)α - open set of V containing Fu(x). Then, Cl(1, 2)α (V) is a (1, 2)α - closed set containing Fu(x). Since, Fu is ultra upper contra continuous, by Theorem 2.6, there exists an open set G containing x such that G ⊂ Fu+(Cl(1, 2)α (V)). Hence, Fu is ultra upper weakly continuous.

The proof is similar for ultra lower contra continuous.

Remark3.4. The following holds for an ultra multifunction Fu: X□ → Y



None of these implications is reversible as shown in the following examples.

Example3.5. Let X =□ {1, 2, 3} with the topology τ =□ {ϕ, X,□ {1},□ {1, 2 }} and Y =□ {a, b, c, d } with two topologies σ1 = {ϕ, Y,□ {a},□ {a, c }, {c, d }, {a, c, d }} and σ2 = {ϕ, Y, {b}}. Define an ultra multifunction Fu: X□→ Y as Fu (1) = □{a}, Fu(2) =□{a, c}, Fu(3) =□ {a, c, d}.Then, it is both ultra upper continuous and ultra lower continuous and also it is an ultra upper weakly continuous and ultra lower weakly continuous but it is not both ultra upper contra continuous and ultra lower contra continuous .

Example3.6. Let X =□ {1, 2, 3} with the topology τ =□ {ϕ, X, □ {1, 2}} and Y = {a, b, c, d} with two topologies σ1 =□ {ϕ, Y, □ {a}, □ {d}, {a, d}} and σ2 =□ {ϕ, Y, □ {c}}. Define an ultra multifunction Fu: X□→ Y as Fu(1) =□ {a}, Fu(2) =□ {b, c}, Fu(3) = □{a, c, d}.Then, Fu is ultra upper weakly continuous but it is not ultra upper continuous.

From example 2.3, we have, Fu is both u.u.c.c and u.u.w.c but not u.u.c.

Example3.7. Let X =□ {1, 2, 3} with the topology τ = {ϕ, X, {1},{2, 3}} and Y =□ {a, b, c} with two topologies σ1= {ϕ, Y,□ {a}} and σ2 =□ {ϕ, Y,□ {a}, {b, c}}.. Define an ultra multifunction Fu: X□→ Y as Fu(1) =□ {a}, Fu(2) =□{ a, c}, Fu(3) =□ {a, b}. Here, Fu is both ultra upper contra continuous and ultra lower contra continuous but not ultra upper clopen continuous and also not ultra lower clopen continuous.

III. The graph multifunction and Applications

In this section, we derived some properties of ultra upper weakly continuous and ultra lower weakly continuous which can be extended to ultra upper contra continuous and ultra lower contra continuous multifunction.

Definition4.1. The graph of a multifunction [9] $Fu: X \rightarrow Y$ is denoted as $FuG(x)$ and is defined as $FuG(x) = \{x \times Fu(x)\}$ for $x \in X$.

Theorem4.2. Let $Fu: X \rightarrow Y$ be an ultra multifunction. Then FuG is ultra upper weakly continuous in $x \in X$ if and only if Fu is ultra upper weakly continuous in x.

Proof: Let Fu be an ultra upper weakly continuous function in $x \in X$. Let $W \subset X \times Y$ be a $(1, 2)\alpha$ - open set of $X \times Y$ such that $x \times Fu(x) \in W$. By a $(1, 2)\alpha$ - open set W in $X \times Y$, we mean $W = U \times V$ where U is open in X and V is $(1, 2)\alpha$ - open in Y. Now, let U be an open set of X containing x and V be a $(1, 2)\alpha$ - open set of Y such that $Fu(x) \subset V$. As Fu is u.u.w.c, for any $Fu(x) \subset V$, there exists an open set G contained in U and $Fu(x) \subset Cl(1, 2)\alpha(V)$ for all $x \in G$. Hence we have, $Fu(G) \subset Cl(1, 2)\alpha(V)$. Ultimately, $x \times Fu(x) \subset G \times Cl(1, 2)\alpha(V) \subset Cl(G) \times Cl(1, 2)\alpha(V) \subset Cl(X \times Y)(W)$. By $Cl(X \times Y)(W)$, we mean the closure of W in $X \times Y$. Hence FuG is u.u.w.c.

Conversely, let FuG be u.u.w.c at $x \in X$. Let $X \times V$ be a $(1, 2)\alpha$ - open set of $X \times Y$ such that V is an $(1, 2)\alpha$ - open set in Y containing Fu(x) and $x \times Fu(x) \subset X \times V$. Since FuG is u.u.w.c, there exists an open set U of X containing x such that $x \times Fu(x) \subset Cl(X \times Y)(X \times V) \subset X \times Cl(1, 2)\alpha(V)$ for all $x \in G$. Hence if $x \in U$, then $Fu(U) \subset Cl(1, 2)\alpha(V)$. We conclude that $Fu(x) \subset Cl(1, 2)\alpha(V)$ for all $x \in U$. This implies that Fu is u.u.w.c.

Theorem4.3. The ultra multifunction $Fu: (X,\tau) \rightarrow (Y, \sigma1, \sigma2)$ is u.l.w.c in $x \in X$ if and only if the graph of the ultra multifunction FuG is u.l.w.c in x.

Proof: Suppose Fu is a u.l.w.c function in x. Let $W \subset X \times Y$ be a $(1,2)\alpha$ - open set in $X \times Y$ such that $(x \times Fu(x)) \cap W = \phi$. Then there exists an open set U of X and a $(1, 2)\alpha$ - open set V of Y such that $x \in U$ and $Fu(x) \cap V = \phi$ and $U \times V \subset W$. Since Fu is u.l.w.c, there exists an open set G of X containing x and contained in U such that $Fu(x) \cap Cl(1, 2)\alpha(V) = \phi$ for all $x \in G$. Hence $FuG(x) \cap G \times Cl(1, 2)\alpha(V) = [x \times Fu(x)] \cap [G \times Cl(1, 2)\alpha(V)] = \phi$ for all $x \in G$. Also $G \cap Cl(1, 2)\alpha(V) \subset U \cap Cl(1, 2)\alpha(V) \subset Cl(U) \cap Cl(1, 2)\alpha(V) \subset Cl(X \times Y)(W)$. Hence FuG is u.l.w.c.

Conversely, let FuG be u.l.w.c in $x \in X$ and let $X \times V$ be a $(1, 2)\alpha$ - open set of $X \times Y$ such that $Fu(x) \cap V = \phi$. The set $X \times V$ is open in $X \times Y$ and also $[x \times Fu(x)] \cap (X \times V) = \phi$. Since FuG is u.l.w.c, there exists an open set G of X containing x such that $FuG(x) \cap Cl(X \times Y)(X \times V) = \phi$ for all $x \in X$. Again $Cl(X \times Y)(X \times V) = Cl(X) \times Cl(1, 2)\alpha(V) = X \times Cl(1, 2)\alpha(V)$. It follows that $Fu(x) \cap Cl(1, 2)\alpha(V) = \phi$ for all $x \in G$. Hence Fu is u.l.w.c.

Theorem4.4. Let $Fu: X \rightarrow Y$ be an ultra multifunction. Then FuG is ultra upper (lower) contra continuous in $x \in X$ then Fu is ultra upper (lower) contra continuous in x.

Proof: Since u.u.c.c $\Rightarrow$ u.u.w.c, the conclusion follows from the above Theorem 4.3.

Definition4.5. An ultra multifunction [9] $Fu: X \rightarrow Y$ is said to be ultra punctually connected if Fu(x) is connected for each $x \in X$.

Theorem4.6. If $Fu: X \rightarrow Y$ is an ultra upper (lower) weakly continuous punctually connected surjective multifunction and X is connected, then Y is connected.

Proof: Suppose Fu is u.u.w.c and Y is not ultra connected, then there exists two non-empty $(1, 2)\alpha$ - open sets A and B such that $Y = A \cup B$ also A and B are both $(1, 2)\alpha$-open and $(1, 2)\alpha$ - closed. As Fu is surjective, $X = Fu+(Y) = Fu+(A \cup B)$. Since Fu is ultra punctually connected for any $x \in X$, $Fu+(x) \subset A \cup B$, $Fu+(x) \subset A$ or $Fu+(x) \subset B$ which implies $x \in Fu+(A)$ or $x \in Fu+(B)$. Hence we have, $X = Fu+(A) \cup Fu+(B)$. As Fu+ is surjective, there exists an $x \in X$ such that $Fu+(x) \subset A$ and there exists an x such that $Fu+(x) \subset B$ and so Fu+(A) and Fu+(B) are nonempty. By Theorem 2.3 [9], $Fu+(A) \subset Int(Fu+(Cl(1, 2)\alpha(A)))$. As A is $(1, 2)\alpha$-closed, we get $Fu+(A) \subset Int((Fu+(A)))$. Hence Fu+(A) is both open and closed. Similarly, we can say that Fu+(B) is also open. Hence X is separable, which is a contradiction.

Now, let Fu be u.l.w.c. Assume $Y = A \cup B$. If $x \in Fu+(A)$, then $x \in Fu-(A)$ (by Lemma 1.5) and thus $Fu(x) \cap A = \phi$. Then there exists an open set G of X, such that $Fu(x) \cap Cl(1, 2)\alpha\ (A) = \phi$, for all $x \in G$. But $Cl(1, 2)\alpha\ (A) = A$ and so $Fu(x) \cap A = \phi$, for all $x \in G$. Also Fu is punctually connected, from $Fu+(A) \cap Fu+(B) = \phi$, it comes out that $Fu(x) \subset A$ and so $G \subset Fu+(A)$. Thus we proved that for any $x \in Fu+(A)$, there is an open set G of X containing x such that $G \subset Fu+(A)$. Hence, Fu+(A) is an open set in X. Similarly, Fu+(B) is also an open set in X. Since Fu is surjective, we get that $X = Fu+(Y) = Fu+(A \cup B)$ and so X is separable, which is a contradiction.

Theorem4.7. If Fu: $X \rightarrow Y$ is an ultra upper (lower) contra continuous punctually connected surjective multifunction and X is connected, then Y is connected.

Proof: Since u.u.c.c $\Rightarrow$ u.u.w.c, the conclusion follows from the above
Theorem 3.4.

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