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An Analysis of Multi Server Retrail Queue with Vacation Time in Embedded Markov Processes

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Abstract: *In this paper we investigate of multi-server retrial queue system .Server provides two stages of homogeneous service in succession. The customer has to complete first service from first server then to the second server of the service. After completion of the second service, the second server takes Bernoulli vacation. On arriving customer on finding a free for the first server enters into service immediately, and goes for the second server; otherwise the customer enters into an orbit of infinite service. An orbiting customer competes for the service by sending signals at random times until a free server is captured. Using the above concept we obtain steady state behaviour of multiserver.*

Keywords: *M/M/2 Retrial, Steady state equation, Bernoulli vacation*

I. Introduction

In this paper, an M/M/2 queueing model with server vacation is taken .The server providing service one by one ,provides the second server is an fixed size k(≥ 1).The customers are queued up for the first service ,which is essential for all customer s .The second server give an optional service is demand for some the customer whereas the others leave the system after the first server provide the service . Considerable attention has been paid to the analysis of queueing systems retrial [3] queues.

II. Model Description

Assume that the customers arrive at the system in accordance with a Poisson process with rate λ.If arriving customer finds the server idle ,the customer enters the service immediately and go for the second server .If the server found to be blocked ,the arriving customer enters a retrial queue . The customer at the head of the retail queue attempts to reach the server in a retrial time distributed with general distribution function A(x), density function a(x). and Laplace –Stieltjes transform $A^{*}(x)$.The service times are independent ,identically distributed with common distribution function B(X),density function b(x) and Laplace –Stieltjes transform $B^{*}(x)$.The vacation period of the server has ,each service completion epoch ,the server may go for vacation. Here one of the service dependent upon the another service for the customer .Here any customer who has not yet completed service in unit-I will be called a I-customer .If have said α because of free from the server, otherwise customer wait to get the server in the probability 1- α.Customer go for the service in unit-II get a service β otherwise wait and get the service 1- β. Anyone who has completed service in I but not yet in II will be referred to as a II-customer.

The finite capacity of unit-II is expressed by the restriction. There can never be more the k+1 II-customers in the system. Whenever the number of II-Customers reaches K+1 we say that the system blocks.
The service mechanism in II may be different depending on whether Unit-I is blocked or not. The description below covers a large number of different operating procedures for unit-II.Some rather practical examples could be the following:

- If blocking is penalized we may wish to study the effect and cost of accelerated service in unit-I
- If the occurrence rather than the duration of blocking is generalized we may require that several customers be processed in II before the system becomes unblocked and service in I is resumed.
- If the time between the beginning of a blocked period and the next departure of an II-customer is too long, the blocked customer in I may require extra service thus extending the blocked period.

An instance of this might be an ingot which has to be reheated if the time between its service completion in I and its access to II is too long, so that its temperature drops below an allowable level.

III. Analysis Of The System

(A). Service In Ii When The System Is Not Blocked:

Let T_1 and T_2 be two successive epochs of arrival in unit-II and let the system be unblocked at time T_1+0.The system is then necessarily unblocked during the entire interval (T_1 , T_2) .we will then assume that in the interval (T_1 , T_2)the departure process from II is a Markovian death process with possibly state-dependent death rates . Specifically , let $T_1<t<t+dt<T_2$ and let there be j II-customers in the system at time t , then the event that a customer's leaves in (t , t +dt) depends only on j and has probability σ_j dt with $\sigma_0=0$ and $\sigma_j>0$ for j=1,2,…………..k.

(B). Service In Ii When The System Is Blocked:

Let T be the any epoch in which the number of II-customers in the system reaches k+1.T is necessarily the time of a service completion in unit-I and we refer to the corresponding customer as the blocking customer. The system remains blocked until a later time T' when the blocking customer is released into unit II.We assume that the duration T'-T of the blocked time is stochastically independent of the arrival process, the service process in I and conditionally independent of the service process in unit II before time T.
The probability that T'-T is at most x and that the number of II-customer at T'+0 is equal to j will be denoted by $H_j(x)$ with

$$\sum_{j=1}^{k} H_j(x) = \bar{H}(x) \tag{3.1}$$

$\bar{H}(x)$ is an honest probability distribution with finite mean $\tilde{\alpha}$. We set $H_0(x) \equiv 0$.

We assume that at T'+0 the service mechanism in unit II becomes again as described in (i) above. The reader should note that two consecutive blocked intervals must always be separated by an interval of time during which the system is not blocked. In order to see this, we observe that at the end of a blocked period the blocking customer is released into unit II. There is therefore no customer in unit I who has completed service. This automatically makes the system unblocked.

(C). The Simple Death (Departure) Process:

Let $F_\sigma(.)$ denote the negative exponential distribution of mean σ^{-1}.Suppose that we have a simple Markovian death process with I individuals at time t=0,0<i≤k.If at any time t there are v individuals then the probability that a death, here a departure from II occurs in (t,t+dt) is given by $\sigma_v dt + o(dt)$. We denote by $p_{ij}(t)$ the conditional probability that there are j individuals at time t , given that there are where I at t=0.The probability $p_{ij}(t)$ are given by

$$p_{ij}(t) = F_{\sigma i} * F_{\sigma i-1} * F_{\sigma i-2} * \dots\dots\dots\dots\dots\dots . F_{\sigma 1},$$

$$p_{ij} = 0, j > i \geq 0,$$

$$p_{ij}(t) = F_{\sigma i} * \dots\dots\dots\dots\dots\dots * F_{\sigma i+1} - F_{\sigma i} * \dots\dots\dots\dots\dots\dots . F_{\sigma i}, i > j > 0,$$

$$p_{ii}(t) = 1 - F_{\sigma i}(t) = e^{-\sigma_i t}, 1 \leq i \leq k, \tag{3.2}$$

These expressions are elementary and follow from the independence of the times between successive deaths. This it is a consequence of the Markov assumption.

A (k+1) –state Markov renewal process, related to the service process in unit-II
As long as there is a steady supply of customers in unit I, it is possible to describe the behavior of the system in terms of a finite Markov renewal process. This will be made precise later, but at this state it is worthwhile to make the following heuristic consideration .Consider any instant of time in which there are
$i \geq j$Customers in I, one of which just begin service. If we disregard for the time being any new arrivals to I, then let $\tau_0, \tau_1, \tau_2, \tau_3 \dots\dots\dots\dots \tau_{i-1}$ be the epochs in which these I customers begin service in unit I and let τ_1 be the time of the i[Th] service .Let ζ_n ,n=0,1,2,3i be the number of II-customers at
$\tau_n + 0$,n=0,1,2,……………i.It then follows readily from the assumptions on the system that the random variables $\zeta_n, \tau_0 = 0, \zeta_1, \tau_1 - \tau_0 \dots\dots\dots\dots\dots\dots\dots\dots\dots . \zeta_i, \tau_i - \tau_{i-1}, \dots$may be regarded as the first i+1 states and sojourn times in a Markov renewal processes with k+1 states 1,2,3,…………………..k+1.The transition probability

$$R_{r,v}(x) = P\{\zeta_{n+1} = v, \tau_{n+1} - \tau_n \leq x | \zeta_n = r\} \tag{3.3}$$

And

$$R_{r,v}(x) = \int_0^x p_{r\gamma-1}(u)\, dH(u), 0 \leq v - 1 \leq r \leq k \tag{3.4}$$

$$R_{r,v}(x) = \sum_{v=\gamma-1}^{k} \int_0^x dH_v(x-u) \int_0^u p_{rv-1}(w)\, dH(w), 1 \leq \gamma \leq k+1.$$

This formula corresponds to the case where there is a blocked period between the two successive service completions in unit I.

$$R_{r,v}(x) = 0, r < \gamma - 1.$$

For future reference, we note that

$\sum_{v=1}^{k+1} R_{r,v}(x) = R_r(x)$

$$= \sum_{v=1}^{r+1} \int_0^u p_{rv-1}(u)\, dH(u),$$

$= H(x), for\ r = 1,2,3, \ldots .k\ and$ .... (3.5)

$= \sum_{v=1}^{k+1} \sum_{v=\gamma-1}^{k} \int_0^x dH_v(x-v) \int_0^u P_{v,\gamma-1}(w)\, dH(w)$

$= H * \tilde{H}(x), r = k+1.$

We note that $R_r(+\infty) = 1$ for r=1, 2, 3,……..k+1

$\int_0^\infty x dR_r(x) = \alpha, r = 1,2, \ldots \ldots \ldots k$

$= \alpha + \tilde{\alpha}, r = k+1$ (3.6)

The stochastic matrix $\{R_{r\gamma}(+\infty)\}$ is irreducible. We will denote its stationary probabilities by $\theta_1, \theta_2, \theta_3, \theta_4 \ldots \ldots .\theta_{k+1}$.We note for future use that

$\sum_{v=1}^{k+1} \theta_r \int_0^\infty x dR_r(x) = \alpha + \theta_{k+1}\tilde{\alpha}.$ (3.7)

We define

$R_{rv}^{(i)}(x) = P\{\tau_i \le x, \zeta_i = v | \zeta_0 = r\}, i \ge 1$ (3.8)

And observe that

$R_{rv}(x) = \sum_{v=1}^{k+1} R_{rv}^{(i-1)}(x-v) dR_{rv}(u)$ (3.9)

So that the matrix $R^{(i)}(.) = \{R_{rv}^{(i)}(.)\}$ is the i- fold matrix –convolution of the matrix R(.)={$R_{rv}(.)$} with itself .In terms of Laplace –Stieltjes transform we obtain the following .If$\tilde{R}^{(i)}(s)$is the (k+1)X(K+1) matrix of Laplace –Stieltjes transform of the distribution $R_{rv}^{(i)}(.)$ we have

$R_{rv}^{(i)}(.) = [\tilde{R}(s)]^i, i \ge 1$ (3.10)

We define $\tilde{R}^{(0)}(s) = 1,$ the identified matrix.The successive Busy Cycles in Parallel queues In the queueing system under study, it does not suffice to distinguish simply between busy and idle periods for the unit-I. The possibility that unit-I is simultaneously empty and blocked may lead to quit complicated transitions , which we have to discuss in detail. Let $\varphi_0 = 0$ and let $\varphi_1, \varphi_{2,\ldots\ldots}$be the successive epochs in which unit I become empty: ie φ_n is the instant in which, for the n^th^ time, there is a service completion in unit I such that all remaining customers in the system are II-Customers$(n \ge)$. We will call the interval(φ_0, φ_1) the initial busy cycle and the intervals (φ_1, φ_2), (φ_2, φ_3), (φ_3, φ_4),…… the ordinary busy cycles. Furthermore, let$\zeta_0^* = r$ and let ζ_0^* be the number of II-customers in the system at the epochs$(\varphi_3, +)$ we will call ζ_n^* the state of the circle of the busy cycle $(\varphi_n, \varphi_{n+1})$.It follows readily from the basic assumptions concerning the input and service processes ,that the random variables ζ_n^*,n>=0 from a (k+1) –state Markov chain and that the random variables $\varphi_n - \varphi_{n-1}$,n>=0 .Are the conditionally independent given the Markov chain$\{\zeta_n^*\}$. Thus the busy cycles and their states from a (k+1)-state Markov renewal processes. This will be one of the basic imbedded processes of the queueing system .For n>=1,the transition probability matrix of the Markov renewal processes of buys is cycles is define by

$\aleph_{rv}(x) = P\{\zeta_{n+1}^* = v, \varphi_{n+1} - \varphi_n \le x |, \zeta_n^* = r\}$ (3.11)

$1 \le r \le k+1, 1 \le v \le k+1$. We will represent its Laplace –Stieltjes transform by the matrix $\tilde{\aleph}(s) = \tilde{\aleph}_{rv}(s)$. For the initial busy cycle, the transition probability matrix $\aleph^{(i)}(.)$ is defined by

$\aleph_{rv}^{(1)}(x) = P\{\zeta_1^* = v, \varphi_1 \le x |, \zeta_0^* = r, \xi(0) = I\}$ (3.12)

$1 \le r \le k+1, 1 \le v \le k+1$.Its Laplace-Stieltjes transform will be denoted by

$\tilde{\aleph}^{(1)}(s), \xi(0)$is the number of I-customers at t=0. Let at any time the number of I-customers be I and let one of these customers just enter service (i>=j).The additional time required until for the first time thereafter, there are no more I-customers in the system ,will be called a busy period with I I-customers initially .A busy period with one I-customer initially is simply called a busy period.

IV. Conclusion

In this paper, we have proposed multiserver retrial queue with Bernoulli vacation time. The system of equations for this second server is obtained. We analyzed that multi-server retrial queue system. Server provides two stages of homogeneous service in succession. We have formulated the queueing model by a Steady state conditions. For future study, we plan to take into account the impatience of customers in the queueing model and investigate the ergodic condition. Furthermore, we also pay attention to the derivation of the waiting time distribution.

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References

[1]. Abramowitz, M and Stegun, I.A (1972) Hand book of Mathematical functions. Dover, New York.
[2]. Artalejo, J.R (1993) Explicit formulae for the characteristics of the M/H2/I retrial queue. Operate Res.soc.44,(309-313).
[3]. Artalejo, J.R and Falin, G.I (1994), Stochastic decomposition for retrial queues top2, 329-342.
[4]. Cooper, R.B (1990) .Queuing theory .In Heyman, D.P., Sobel, M.J.eds stochastic Models Amsterdam: North-Holland (Elsevier),pp 469-518.
[5]. Falin, G.I.(1980) An M/M/1 queue with repeated calls in the presence of persistence function: Paper 1606-80.All-Union Institute for scientific and Technical information ,Moscow.
[6]. Falin,G.I.(1984) An Double channel queuing system with repeated calls :Paper 4221-84.All-Union Institute for scientific and Technical information ,Moscow.
[7]. Fayolly, G(1986) A simple telephone exchange with delayed feedbacks. In teletraffic Analysis and computer performance evolution et.O.J.Boxma, J.W.Conen and H.C.Tijns, Elsevier, Amsterdam.
[8]. Fuhrman, S.W .and Cooper, R.B (1985) stochastic decompositi0n in the M/G/1 queue with generalized vacations. Operate .Res 33, 1117-1129.
[9]. Farahmand .K(1990) Single line queue with repeated demands QUESTS A6223-228.
[10]. Fayolle,G(1986) A simple telephone exchange with delayed feedbacks. In teletraffic analysis and computer performance evaluation et.O.J.Boxma, J.W.Conen and H.C.Tijns, Elsevier, Amsterdam.
[11]. Hanschke, T (1987) Explicit formulas for the characteristics of the M/M/2 queue with repeated attempts, J.Appl.Prob 24,486-494.
[12]. Kendal, D.G (1957) Some problems in the theory of dams J.R statst Soc.B12, 207-212.
[13]. Ross, S.M (1993) .introduction to probability models 5th ed,San Diego Ca,Academic press.
[14]. Syski, R(1986) Introduction to congestion theory in telephone system 2nd ed.New York Elsevier.
[15]. N.Tian, G.Zhang, vacation queueing models: Theory and Application, Springer-Verlag, New York,2006.