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Area and perimeter relation of Square and rectangle (Relation All Mathematics)

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Abstract: *We are know that the properties of square and rectangle .In this paper we are discuss about relation between square and rectangle with the proof .In our real life and educational life, the geometrical figure like rectangle ,square,… etc. have so much importance that we cannot avoid them . We are trying to give a new concept to the world .I am sure that this concept will be helpful in agricultural, engineering, mathematical branches etc. Inside this research, square and rectangle relation is explained with the help of formula. Square-rectangle relation is explained in two parts i.e.* ***Area relation*** *and* ***perimeter relation****, rectangle can be narrowed in Segment & the rectangle can be of zero area also.*

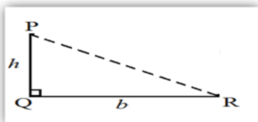
Keywords: *Area, Perimeter, Relation , Seg-rectangle, B- Sidemeasurement*

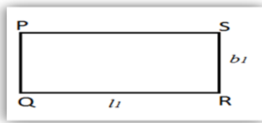
I. Introduction

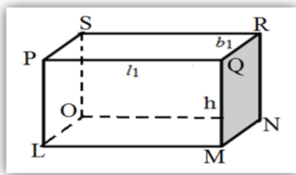
Inside the paper cleared that relation between square and rectangle in two parts. i.e. i) Basic theorem of area relation of square and rectangle, ii) Basic theorem of perimeter relation of square-rectangle. Sidemeasurement is a explained new concept which is very important related to this paper and next papers . Seg-rectangle theorem is proof that perimeter of rectangle is kept constant and opposite sides length increased till width become zero ,then become Segment is rectangle. and that rectangle known as Seg-rectangle. Seg-rectangle is a new concept related to rectangle.

II. Basic concept of Square-rectangle relation

2.1. Side-measurement(B) :-If sides of any geometrical figure are in right angle with each other , then those sides or considering one of the parallel and equal sides after adding them, the addition is the side-measurement .side-measurement indicated as letter 'B'







Side-measurement of right angled triangle - B (ΔPQR) = b+h
In **Δ**PQR ,sides PQ and QR are right angle, performed to each other .
Side-measurement of rectangle-B(□PQRS)= l_1+ b_1
In □PQRS, opposite sides PQ and RS are similar to each other and m<Q = 90^o .here side PQ and QR are right angle performed to each other.
Side-measurement of cuboid–E_B (□PQRS) = l_1+ b_1+ h_1
In E(□PQRS),opposite sides are parallel to each other and QM are right angle performed to each other. Side-measurement of cuboid written as = E_B(□PQRS)

2.2)Important points of square-rectangle relation
I) For explanation of square and rectangle relation following variables are used
. i) Area – A
ii) Perimeter – P
iii) Side-measurement – B
II) For explanation of square and rectangle relation following letters are used
i) Area of square ABCD – A (□ABCD)
ii) Perimeter of square ABCD – P (□ABCD)
iii) Side-measurement of square ABCD – B (□ABCD)
iv) Area of rectangle PQRS – A (□PQRS)
v) Perimeter of rectangle PQRS – P (□PQRS)
vi) Side-measurement of rectangle PQRS – B (□PQRS)

2.3) Relation formula of square-rectangle :

i) Relation area formula of square-rectangle(K) -

P (□ABCD) = P(□PQRS) …[Reference Fig- I]

$2l = l_1 + b_1$

When perimeter of square and rectangle is same at that time difference between area of both sides are maintained with the help of 'relation area formula of square-rectangle' and both sides area of square and rectangle become equal.

Relation area formula of square-rectangle indicated with letter 'K'

$$K = \left[\frac{(l_1 + b_1)}{2} - b_1\right]^2$$

Proof - In □ABCD and □PQRS, …[Reference Fig- I]

P(□ABCD)=P(□PQRS)

but, A(□ABCD)>A(□PQRS)

Now , □PQRS is set in □ABCD as indicated in Fig - I

then we are found ,when perimeter of □ABCD and □PQRS are same then difference between area of both are □PMSN.so that □PMSN is called relation area of square-rectangle. And that indicated with letter 'K'. Now with the help of relation area of square-rectangle we are create Relation area formula of square-rectangle.

K = A(□PMSN)

$= L^2$

$= (L \times L)$

$=[(l-b_1) \times (l-b_1)]$

$= (l - b_1)^2$

$$= \left[\frac{(l_1 + b_1)}{2} - b_1\right]^2$$

$$K = \left[\frac{(l_1 + b_1)}{2} - b_1\right]^2$$ …Here , $K = \left[\frac{(l_1 + b_1)}{2} - b_1\right]^2 = A(\square PMSN) = L^2$

Here Relation area formula of square-rectangle indicated with letter 'K'

Relation area formula of square and rectangle is used by the purpose of both sides of square – rectangle is become equal. When perimeter of are same.

ii) Relation perimeter formula of square-rectangle(V) -

A (□ABCD) = A(□PQRS) … [Reference Fig-(III)]

$l^2 = l_1 \times b_1$

When area of square and rectangle is same at that time difference between perimeter of both are maintained with the help of 'relation perimeter formula of square-rectangle' and both side of perimeter of square and rectangle become equal .

$$\mathbf{V} = \frac{1}{2}\left[\frac{(n^2+1)}{n}\right]$$

Proof - In □ABCD and □PQRS, … [Reference Fig-(III)]

A(□ABCD) = A(□PQRS)

but, P(□ABCD) < P(□PQRS)

Now , □PQRS is set in □ABCD as indicated in Fig- III

then we are found ,when area of □ABCD and □PQRS are same then difference between perimeter of square and rectangle is ratio of $\left[\frac{l+m}{2l}\right]$ .i.e. Relation perimeter of square-rectangle and that indicated with letter 'V' .Relation perimeter of square-rectangle is defined as ratio of sidemeasurement of rectangle to the sidemeasurement of square.

$$V = \left[\frac{l+m}{2l}\right]$$

$$= \frac{1}{2}\left[\frac{l+m}{l}\right]$$

But, $l + m$ = B(□PQRS)

$$V = \frac{1}{2}\left[\frac{B(\square PQRS)}{l}\right]$$

$$V = \frac{1}{2}\left[\frac{l1 + b1}{l}\right]$$

$$V = \frac{1}{2}\left[\frac{l1}{l} + \frac{b1}{l}\right]$$ …. $\mathbf{n} = \frac{l_1}{l} = \frac{l}{b_1}$

$$V = \frac{1}{2}\left[n + \frac{1}{n}\right]$$

$$V = \frac{1}{2}\left[\frac{(n^2+1)}{n}\right]$$ … Here , $V = \frac{1}{2}\left[\frac{(n^2+1)}{n}\right] = \frac{1}{2}\left[\frac{B(\square PQRS)}{l}\right] = \left[\frac{l+m}{2l}\right]$

Here Relation perimeter formula of square-rectangle indicated with letter 'V'
Relation perimeter formula of square and rectangle is used by the purpose of both sides of square – rectangle is become equal. When area of square and rectangle is same.

iii) Ratio of perimeter of square-rectangle (n) –

Ratio of perimeter of square-rectangle is defined as when area of square and rectangle is same then Ratio of length of square-rectangle is equal to the Ratio of breadth of square-rectangle .

Here , Ratio of perimeter of square-rectangle indicated as , $\mathbf{n} = \frac{l_1}{l} = \frac{l}{b_1}$

iii-1) Ratio of length of square-rectangle (n)-

Ratio of length of square-rectangle is defined as when area of square and rectangle is same then ratio of length of rectangle to the side of the square .

here , Ratio of length of square-rectangle indicated as, $\mathbf{n} = \frac{l_1}{l}$

$$V = \frac{1}{2}\left[\frac{(n^2+1)}{n}\right] = \frac{1}{2}\left[\frac{(l_1^2+l^2)}{l.l_1}\right] = \frac{1}{2}\ \frac{(l_1^2+l_1.b_1l)}{l_1.\sqrt{l_1.b_1}l}$$

iii-2) Ratio of breadth of square-rectangle (n)

Ratio of breadth of square-rectangle is defined as when area of square and rectangle is same then ratio of side of square to the breadth of rectangle.

here , Ratio of breadth of square-rectangle indicated as, $\mathbf{n} = \frac{l}{b_1}$

$$V = \frac{1}{2}\left[\frac{(n^2+1)}{n}\right] = \frac{1}{2}\left[\frac{(b_1^2+l^2)}{l.b_1}\right] = \frac{1}{2}\ \frac{(b_1^2+l_1.b_1l)}{b_1.\sqrt{l_1.b_1}l}$$

Theorem -1: Basic theorem of area relation of square and rectangle

Perimeter of square and rectangle is same then area of square is more than area of rectangle, at that time area of square is equal to sum of the, area of rectangle and Relation area formula of square-rectangle(K) .

Given -In ,□ABCD and □ PQRS ,

P(□ABCD) = P(□PQRS)

$4l = 2(l_1+b_1)$, $l_1 > l$

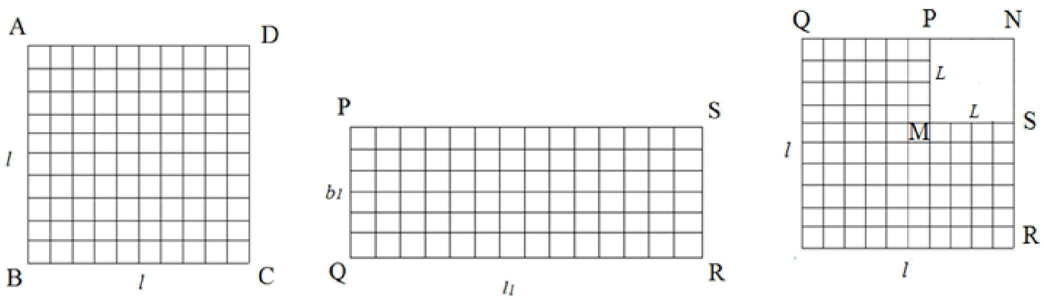


Figure I : Area relation of square and rectangle

To prove - A(□ABCD) = A(□PQRS) + $\left[\frac{(l_1+b_1)}{2} - b_1\right]^2$

Proof - In □ABCD ,

AB =BC=CD=DA= l … (Given **)**

A(□ABCD) = l^2 …(i)

In □ PQRS]

PQ=RS=b_1 and PS=QR=l_1 …(Given)

A(□PQRS) = l_1 x b_1 ...(ii)

Now related to square and rectangle we are know that,

A(□ABCD) > A(□PQRS) …(iii)

Here two sides of equation no.(iii) is not same, so add value of ' K' in RHS. So equation become,

A(□ABCD) = A(□PQRS) + A(□PMSN)

A(□ABCD) = A(□PQRS) + K … K= A(□PMSN) = $L^2 = \left[\frac{(l_1+b_1)}{2} - b_1\right]^2$

A(□ABCD) = A(□PQRS) + $\left[\frac{(l_1+b_1)}{2} - b_1\right]^2$

A(□ABCD) = A(□PQRS) + $\left[\frac{(l_1+b_1)-2b_1}{2}\right]^2$

$A(\square ABCD) = A(\square PQRS) + \left[\frac{(l_1 - b_1)}{2}\right]^2$

Hence Basic theorem of area relation of square and rectangle is proved.

Theorem-2 : Theorem of area relation of two rectangles.

If perimeter of two rectangle is same then rectangle whose length is smaller, its area also is more than another rectangle.

Given – In □ABCD, □PQRS and □ LMNO,

$P(\square ABCD) = P(\square PQRS) = P(\square LMNO)$

$2l = l_1+b_1 = l_2+b_2$ … $l_1<l_2$

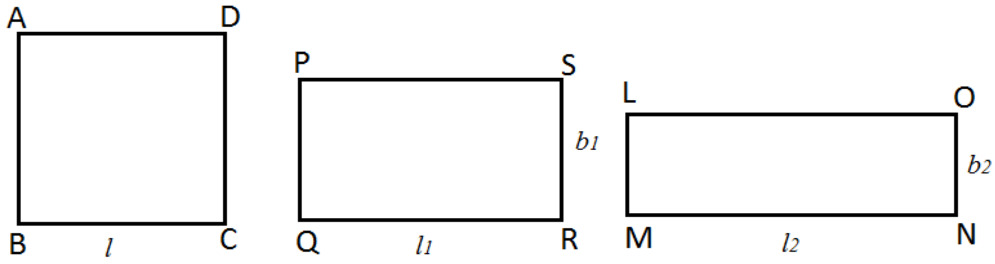


Figure II : Area relation of two rectangles

To prove – **$A(\square PQRS) = A(\square LMNO) + (b_1-b_2).[(l_1+b_1) – (b_1+b_2)]$**

Proof - In □ABCD and □PQRS,

$A(\square ABCD) = A(\square PQRS) + \left[\frac{(l_1+b_1)}{2} - b_1\right]^2$ …(i)

... (Basic theorem of area relation of square and rectangle)

In □ABCD and □ LMNO,

$A(\square ABCD) = A(\square LMNO) + \left[\frac{(l_2+b_2)}{2} - b_2\right]^2$ …(ii)

... (Basic theorem of area relation of square and rectangle)

$A(\square PQRS) + \left[\frac{(l_1+b_1)}{2} - b_1\right]^2 = A(\square LMNO) + \left[\frac{(l_2+b_2)}{2} - b_2\right]^2$ …From equation no.(i) and (ii)

$A(\square PQRS) = A(\square LMNO) + \left[\frac{(l_2+b_2)}{2} - b_2\right]^2 - \left[\frac{(l_1+b_1)}{2} - b_1\right]^2$

$= A(\square LMNO) + \left[\frac{(l_2+b_2)}{2} - b_2 + \frac{(l_1+b_1)}{2} - b_1\right] x \left[\frac{(l_2+b_2)}{2} - b_2 - \frac{(l_1+b_1)}{2} + b_1\right]$

…$(a^2-b^2) = (a+b).(a-b)$, $l_1+b_1 = l_2+b_2$ …(Given)

$A(\square PQRS) = A(\square LMNO) + (b_1-b_2).[(l_1+b_1) – (b_1+b_2)]$

.Hence , Theorem of area relation of two rectangles is proved.

Theorem-3: Basic theorem of perimeter relation of square-rectangle

Area of square and rectangle is same then perimeter of rectangle is more than perimeter of square , at that time perimeter of rectangle is equal to product of the, perimeter of square and Relation perimeter formula of square-rectangle(V).

Given - In □ABCD and □PQRS,

$A(\square ABCD) = A(\square PQRS)$ … (Fig-III)

$l^2 = l_1 \times b_1$

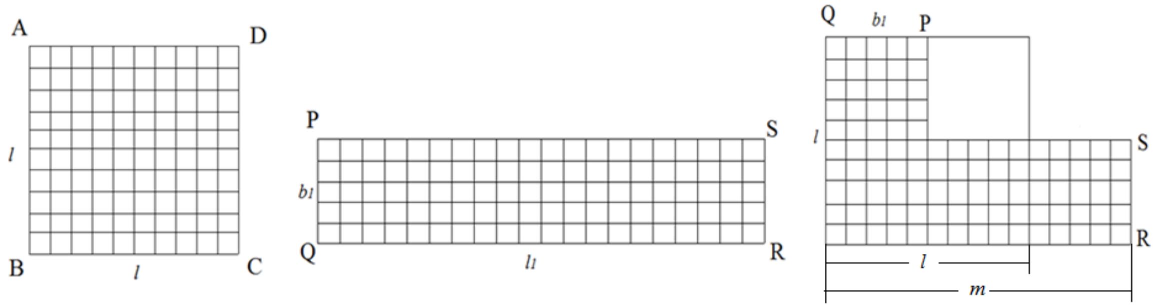


Figure III : Perimeter relation of square-rectangle

To prove - **P(□ PQRS)= P(□ABCD) x** $\frac{1}{2}\left[\frac{(n^2+1)}{n}\right]$

Proof –

In □ABCD , P(□ABCD) = 4 l …(i)

In □PQRS , P(□PQRS) = $2(l_1 + b_1)$ …(ii)

Now related to square and rectangle we are know that,

P(□PQRS) > P(□ABCD) …(iii)

Here two sides of equation no.(iii) is not same, so multiply value of ' V' in RHS. So equation become,

P(□PQRS)= P(□ABCD) . V

…V = $\frac{B(□PQRS)}{l} = \frac{1}{2}\left[\frac{(n^2+1)}{n}\right]$

… Relation perimeter formula of square-rectangle – V and Ratio of perimeter of square-rectangle - n

P(□PQRS)= P(□ ABCD) x $\left[\frac{(n^2+1)}{n}\right]$

Hence , Basic theorem of perimeter relation of square-rectangle is proved.

Theorem-4 : Theorem of perimeter relation between two rectangles .

If area of two rectangle is same then rectangle whose length is more, its perimeter also is more than another rectangle.

Given - In □ABCD, □PQRS and □ LMNO,

A(□ABCD)= A(□PQRS) = A(□LMNO)

Here, $l_1 \times b_1 = l_2 \times b_2$ …$l_1 < l_2$

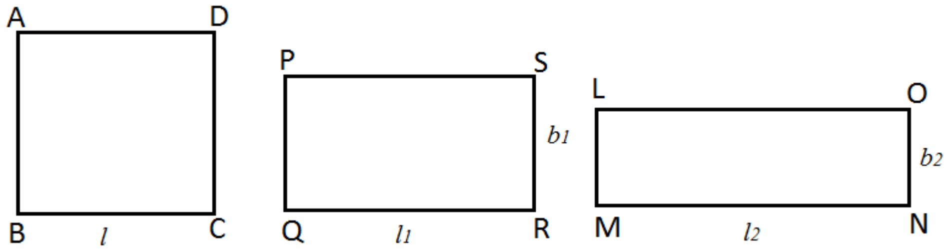


Figure IV : Perimeter relation between two rectangles

To prove - **P(□PQRS) = P(□LMNO) x** $\left[\frac{n_2}{n_1} \cdot \frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right]$

Proof - In □ABCD and □PQRS ,

$\left[\frac{n_1}{(n_1{}^2+1)}\right]$ P(□ABCD)=P(□PQRS)x2

… (Basic theorem of perimeter relation of square-rectangle) …(i)

In □ABCD and □PQRS,

P(□ABCD) = P(□LMNO) x 2 . $\left[\frac{n_2}{(n_2{}^2+1)}\right]$

… (Basic theorem of perimeter relation of square-rectangle) …(ii)

P(□PQRS) x 2 $\left[\frac{n_1}{(n_1{}^2+1)}\right]$ = P(□LMNO) x 2 . $\left[\frac{n_2}{(n_2{}^2+1)}\right]$ … From equation no. (i) and (ii)

P(□PQRS) = P(□LMNO) x $\left[\frac{(n_1{}^2+1)}{n_1}\right] \cdot \left[\frac{n_2}{(n_2{}^2+1)}\right]$

P(□PQRS) = P(□LMNO) x $\left[\frac{n_2}{n_1} \cdot \frac{(n_1{}^2+1)}{(n_2{}^2+1)}\right]$

Hence , Theorem of perimeter relation between two rectangles is proved.

Theorem-5 : Seg-rectangle theorem

If perimeter of rectangle is kept constant and opposite sides length increased till width become zero ,then become Segment is Seg-rectangle.

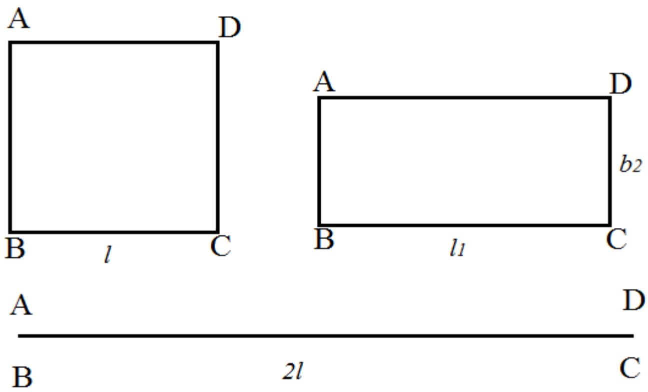


Figure V : Segment AB-CD is seg-rectangle

Given –In □ABCD]

P(□ABCD) = P(Seg AB-CD)

Now become, $l_1 = 2l$ … ($b_1 = 0$)

To prove - **Seg AB-CD is Seg-rectangle**

proof – In □ABCD ,

A(□ABCD) = l^2 … (i)

In Segment AB-CD, b =0

Suppose Segment AB-CD is rectangle

Now , In Seg AB-CD,

A(Seg AB-CD)= O …(ii) ($b_1 = 0$) … Given

A(□ABCD) = A (Seg AB-CD) + $\left[\frac{(l_1+b_1)}{2} - b_1\right]^2$ … (Basic theorem of relation of area of square and rectangle)

$= 2l \times 0 + \left[\frac{(2l+0)}{2} - 0\right]^2$

$= 0 + \left[\frac{2l}{2}\right]^2$

A(□ABCD) $= l^2$

But this equation is satisfied with Basic theorem of area relation of square and rectangle.

Hence , Seg-rectangle theorem is proved.

(Seg AB-CD is Seg-rectangle , so it can be written as □AB-CD

Theorem- 6: Theorem of Seg-rectangles ratio

If opposite sides of two rectangles are increased till their width becomes zero , then their perimeters ratio is equal to their lengths ratio.

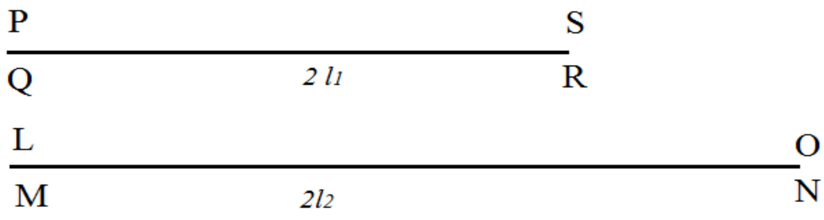


Figure VI : Seg-rectangles ratio

Given –In □ PQRS and □LMNO ,

$b_1 = b_2 = 0$, ($l_1 \times b_1 = l_2 \times b_2 = 0$) and $L_1=2l_1$, $L_2=2l_2$

To prove - **P(□PQRS) : P(□LMNO) = $l_1 : l_2$**

Proof - In □ PQRS and □LMNO ,

P(□ PQRS) = P(□LMNO) x $\left[\frac{n_2}{n_1} \cdot \frac{(n_1^2+1)}{(n_2^2+1)}\right]$ … (Theorem of perimeter relation between two rectangles)

P(□ PQRS) = P(□LMNO) x$\left[\frac{2l_2}{2l_1} \cdot \frac{(2l_1^2+2\,l_1.b_1)}{(2l_2^2+2l_2.b_2)}\right]$ …$l^2 = l_1.b_1$

P(□ PQRS) = P(□LMNO) x$\left[\frac{2l_2}{2l_1} \cdot \frac{4l_1^2}{4l_2^2}\right]$

$= P(\square LMNO) \times \left[\frac{l_2}{l_1} \cdot \frac{l_1^2}{l_2^2}\right]$ … ($b_1 = b_2 = 0$, Given)

$P(\square PQRS) = P(\square LMNO) \times \frac{l_1}{l_2}$

$P(\square PQRS) = P(\square LMNO) \times \frac{l_1}{l_2}$

$\frac{P(\square PQRS)}{P(\square LMNO)} = \frac{l_1}{l_2}$

$P(\square PQRS) : P(\square LMNO) = l_1 : l_2$

Hence , Theorem of Seg-rectangle ratio is proved.

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