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Comparison and Supply Chain Optimization of Poultry Firm Using Mixed Integer and Linear Fractional Program

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Abstract: *In this study, we formulate mixed integer and linear fractional program for manufacturer and retailer system of poultry firm in Bangladesh that is one of the most promising but unstable sectors in Bangladesh. This paper maximizes the profit, minimizes the cost and finally estimates the optimum selling prices of several the firms. In order to validate the model we have made a question survey on some main poultry firms in the district of Mymensingh and Gazipur of Dhaka. We try to figure out the actual situation of the unstable poultry market of Bangladesh. We have made a question survey on some poultry firm. From the survey, it has observed that the selling price of eggs and chicken fluctuate depending on the natural calamities. The formulated mixed integer linear fractional program model maximizes not only the ratio of return on investment but also optimize location, transportation cost, and the investment. In order to comparison, a mixed integer model has been derived for the same constraints. The formulated mixed integer and linear fractional program has solved by branch and bound algorithm-using AMPL. These models determine the sites for manufacturer and the best allocation for both the retailer and manufacturer. It has observed that the profit and selling price have very good relationship with production cost and raw materials cost but no significant relation with fixed cost. This leads to the case where the manufacturer has access to the complete information set that is required to coordinate the supply chain, and, although each buyer has access to the manufacturer's production rate, setup and holding cost information, individual buyers do not have access to the manufacturer's information set that contains information on other buyers.*

Keywords: *Integer program, mixed integer program, Optimization, Poultry firm of Bangladesh.*

I. Introduction

In the global competition market, the importance of SCM is increasing day to day. Maximize the profit and minimize the cost are the main factors that play an important role in supply chain. Further, it is important to make the model optimal for both consumer as well as manufacturer. In this paper, manufacturer-retailer multi-product, multi-facility, and multi-customer location production problem is formulated as a MILFP which maximizes the ratio of return on investment, and at the same time optimizes location, transportation cost, and the investment. MIP model is also derived to determine the sites for manufacturer and the best allocation for both the retailer and manufacturer. Using the suitable transformation of Charnes and Cooper (1962), the formulated MILFP is solved by AMPL. Finally, a numerical example along with the sensitivity of production cost, raw material cost and opening cost is considered to estimate the performance of the models.

Coordination among the members of supply chain is one of the vital issues to overcome the new challenges of the comprehensive enterprise. Without coordination, a supply chain system could not be optimal as a whole since each party always try to enhance his own profits only. That is why to ensure the optimal system and to satisfy customer demands in today's competitive markets; significant information needs to be shared along the supply chain. And a high level of coordination between manufacturer's and retailer's decision making is also required. The concept of Joint Economic Lot Sizing (JELS) has been introduced to filter traditional methods for independent inventory control and to find a more profitable joint production and inventory policy. Qin et al. (2007) have considered volume discounts and franchise fees as coordination mechanism in a system of supply chain with single supplier and single buyer with price sensitive demand. Subsequently, they showed that when demand is price sensitive, channel profits achieved by employing volume discounts and franchise fees is larger than achieved by quantity discounts and franchise fees. Uddin and Sano (2010) developed an MIP based vendor-buyer multiple products-consumers, facility selection problem with a price-sensitive linear demand function. They assumed that by coordinated mechanism among the members of supply chain could achieve the optimal solution and the optimal location for the warehouse. Consequently, Uddin and Sano explained an MIP based a supply chain with a coordination mechanism consisting of a single vendor and buyer is considered. Pourakbar et al. (2007) descried an integrated four-stage supply chain system, incorporating one supplier, multiple producers, multiple distributors and multiple retailers. Then they determined the optimal order quantity of each stage and shortage level of each stage to minimize the cost of the supply chain. Wu and Yen (2009) have provided some patch works to enhance the volubility of the integrated single-vendor single-buyer inventory model. Uddin, Islam and Kazi (2013) deals with supply chain optimization and facility selection problem**.** They describe supply chain consisting single vendor and buyer with deterministic demand function. Jokar and Sajadieh (2009) have described a vendor–buyer integrated production inventory model considering Joint Economic Lot Sizing (JELS) policy with price sensitive linear demand of the customer. Qi et al. (2004) described supply chain coordination with demand disruption. In their model, the market demand function is assumed to be a linear function of the retail price, Q = D-kp, where D is the maximum market demand, p is the retail price, k is a coefficient of price sensitivity, and Q is the real demand under retail price p.

Akter and Uddin (2009) argue that as an important sub sector of livestock production, the poultry industry in Bangladesh plays a vital role in economic growth and simultaneously creates numerous employment opportunities. The poultry industry, as a fundamental part of animal production, is committed to supply the nation which a cheap source of good quality nutritious animal protein in terms of meat and eggs. Islam, Uddin and Alam (2014) analyze challenges and prospects of poultry industry in Bangladesh by using data collection from some important poultry industry. Among these, Charnes and Cooper (1962) described a transformation technique which transforms the LFP into equivalent linear program. This method is quite simple but need to solve two transformed model to obtain the optimal solution.

The reminder of this paper is organized as, Data collection; Model formulation which describes the concept of mixed integer linear fractional programming problem, notations, assumption, prerequisites and finally the MIP model. Solution approach and the results of these models are discussed. Finally, the conclusions and contributions of this study are discussed

II. Data Collection

For this study both primary and secondary data sources were used. This study, primary data was collected by means of a questionnaire survey and interview with the poultry farmers in different poultry firms Mymensingh, Kishorgonj, Gazipur and Manikgonj districts of Bangladesh. Questions were asked to know the production cost and profit margin. In the first time the farmers had given their valuable data to complete the study. A basic problem in any analysis of the poultry sector relates to the lack of reliable and adequate data on use by type of feed and by category of poultry output, and on poultry population, disaggregated by scavenging and commercial birds. The study is largely based on information from secondary sources such as agricultural censuses and the FAO yearbook, different issues of statistical pocket book, Bangladesh bureau of statistics. Such information is supplemented by primary data generated from field surveys. In particular, the data on the current poultry production system, generated by the field surveys, have been used. Also, some data were collected from a rapid market survey including a few key-informant interviews. The following information collected from main poultry manufacturer and retailer in Mymensingh, Kishorgonj, Gazipur and Manikgonj districts of Bangladesh.

Figure-1: Shows the study area.

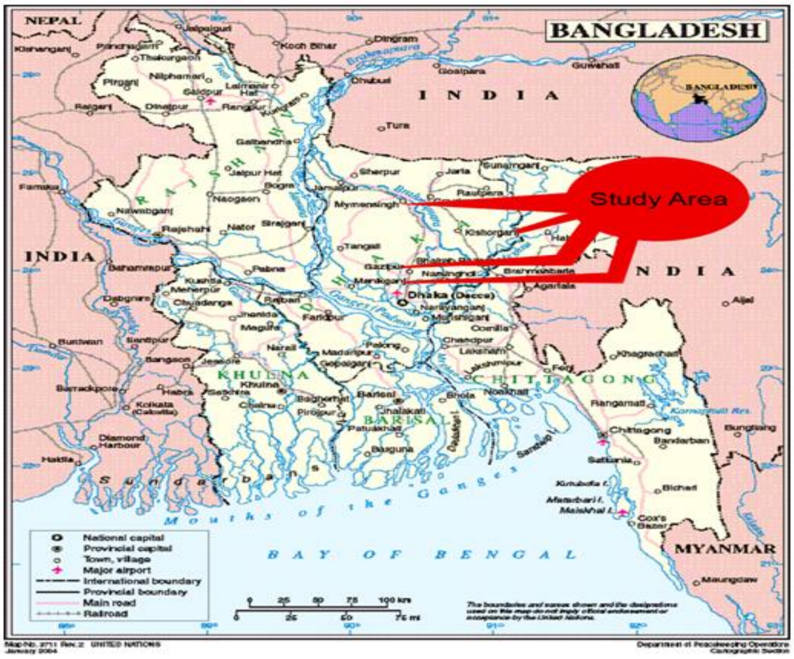


Figure-1: Data collection area.

This investigation is conducted in Mymensingh, Kishorgonj, Gazipur and Manikgonj districts. Information collected from some poultry farmers and marketing actors involved in the sub sector organization in the study areas. Hence, the investigation is limited spatially as well as temporally to make the study more representatives in terms of wider range of commodity, area, and time horizon. Next comes the honesty of the interviewees of the farms whose opinion and answers will affect my study tremendously. There is tendency of not to disclose the actual information and figures in order to maintain the secrecy of the business. Another limitation of this study is farmer's or company's production and marketing strategies and practices is very confidential for any firm, for obvious reasons. So they don't disclose all the information which may make this report more authentic. And some major players of the industry denied disclosing the information and some interviewers failed to answer the questions. I have over come all kind of constraints to collect actual data from farmers, seller, consumer etc. by my personal capacity. Most of the farmers do not have any kind of training about the poultry business. They depend mainly on raw material suppliers. The dominated parties do not share the update situation of poultry business with small scale farmers. The price of poultry raw materials has increased sharply in the international market. Very naturally, the production costs have also hiked up. The farmers are not getting actual price, they are counting huge losses, as the production cost is high and selling price is low. Due to Avian Influenza, the industry suffered losses of around 70 ($.) (As per Breeder's Association of Bangladesh). This was a huge loss for the producers and they did not get any sort of financial help to mitigate it. Saidur Rahman Babu, General Secretary of BAB said that 60 percent of poultry farms and 70 percent hatcheries and breeding farms were closed due to bird flu attack in 2007-2008 periods.

The banks interest rate in this sector is very high which is on an average 12-14% per annum and real effective interest rate is around 18-20% per annum. Moreover, lot of hidden charges and costs are associated in this sector to avail the loan from the banking sector. Actually bank interest rate should be 10%.Moreover, NGOs and also Grameen bank should play more active role to lower the interest rate for poultry sector under their social business program so that rural people can be motivated.

The price of poultry raw materials has increased sharply in the international market. Very naturally, the production costs have also hiked up. So the price of egg and chicken in the Bangladesh's local market has already gone up by nearly 40 per cent over the last one and a half years, which affecting the low-income people. The middle men or whole-seller do not participate in the purchase of products at any predetermined price, both production and price risks are fully borne by the farmers. The farmers are not getting actual price. Therefore the actual producers won't get any benefit of the high price as they are oppressed by the middle men who suck the profit. Since the last 8-10 months they are counting huge losses, as the production cost is high and selling price is low. Moreover, the end users i.e. customer have to pay higher price.

Poultry industry facing crisis for political violence. Bangladesh Poultry Industries Coordination Committee holds a press conference at the National Press Club on 11 December, 2013, Wednesday. The country's poultry industry has suffered losses of more than 50 core ($.) in last three months because of escalating political turmoil, said sector leaders. Around 30% of farms were also closed during the period, they claimed at a press conference in Dhaka. Bangladesh Poultry Industries Coordination Committee (BPICC), a combined body of six major poultry associations in the country, organized it at the National Press Club. The industry leaders urged the ruling party and the opposition to find an immediate solution to the current political crisis so that the businesses could be run in a peaceful environment. Moshiur Rahman, convenor of BPICC, said: "We are unable to bear losses caused by the on-going political unrest. Saidur Rahman Babu, secretary of Breeders Association of Bangladesh (BAB), said: "Farmers had to sell day-old chicks at a rate of 0.17-0.22 ($.) per chick while the production cost was 0.41 ($.)." "Broiler chickens were sold at 0.82-0.94 ($.) per kilogram at farm level against the production cost of 1.29-1.41 ($.)," he added. Kazi Zahedul Hasan, managing director of Kazi Farms Limited and senior vice-president of BAB, said continuous hartals and blockades have almost stalled the supply of eggs."There are thousands of eggs lying in our farms as supply becomes impossible due to hartals and blockades." Not only has that but the supply of poultry feeds to the farms also been disrupted as political unrest continues, Kazi Zahedul Hasan added. He urged the parties concerned to keep vehicles carrying eggs, chicks, chickens and feeds out of the enforcement of hartals and blockades. "We want security for our professions and investments." Taher Ahmed Siddiqui, president of the Egg Producers Association, said the weekly production of eggs is around 10 core and 50 lakh eggs. He said they had suffered losses of 3.2 core ($.) for failure to sell 30% of their total production and 4.3 core ($.) for selling at rates less than production costs. Rafiqul Haque, secretary of the World's Poultry Science Association- Bangladesh Branch (WPSA-BB), said the industry has suffered losses of 11.5 core ($.) from commercial broiler poultry farming. He said: "The industry suffered a loss of around 94 cores ($.) from 2007 to 2012 [for bird flu]. But the present disaster [from political programmes] has caused a loss of 47 core ($.) within only three months." Moshiur Rahman, convener of the BPICC, said: "Some 15,000 tones of broiler chicken and around 3.15 core eggs are left unsold every week. The breeders are forced to destroy around 30 lakh one-day-old chicks every week because of scarcity of feed and transport facilities." Moreover, the poultry sector leaders said they had to pay higher as transportation cost because of the blockades while many vehicles carrying chicks and feed had been vandalised or torched. As a symbolic protest, the farmers gave away a number of chicks and smashed around 5,000 eggs on the street. They warned that next time they would bring lakhs of chicks and release those in front of the residences of the prime minister and the opposition leader.

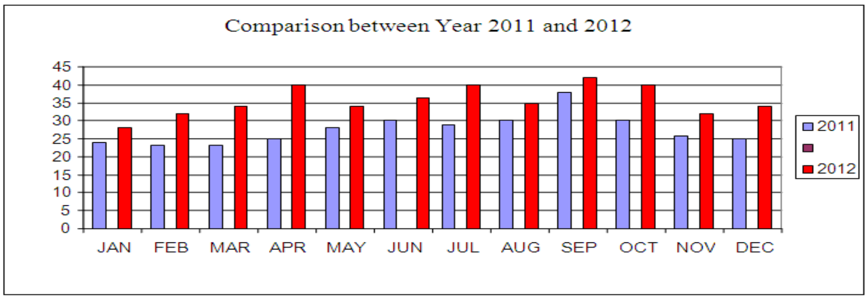
In an innovative way to draw the attention of politicians, poultry farmers 27 December, 2013, Friday held a protest rally in front of the Jatiya Press Club and brought in 25,000 chicks just a day old. With the sector brought to its knees by hartals and blockades, the protestors were giving away chicks to people passing by saying they could not feed the chicks anymore

Figure-2: Farmers protest against hartals and blockades.

The poultry sector is an example how political unrest in a country can bring its businesses to their knees. The situation is so grave that poultry farmers do not worry about losses now, they are only struggling to save the business from destruction. Around 5,000 farmers participated in a protest rally in the capital on that day and brought nearly 25,000 day-old chicks with them. The farmers were so frustrated and helpless that they were offering the chicks to passersby, for free. They cannot even keep their farms shut as production of eggs and chicken involves a natural cycle. “Please take the chicks and feed them. We can't see them dying," a farmer was telling a passerby at the rally in front of the National Press Club. As the supply chain was severely disrupted due to the current political violence, the farmers are unable to sell eggs and chicken or store those.
They cannot even buy feed due to a cash crunch.

Table: 1 Parameters of the MILFP and MIP model

Parameters				Locations of the vendor			
	1	2	3	4	5	6	7
Raw Materials(units)	(0.5, 30)	(0.2 , 20)	(0.5 , 30)	(0.1 , 30)	(0.1 , 20)	(0.3 , 20)	(0 .4,30)
TransCostRaw (input)	(0.1, 0.2)	(0.2, 0.2)	(0.1, 0.5)	(0.1, 0.3)	(0.1, 0.4)	(0.2, 0.3)	(0.1, 0.4)
Production cost	(2.0, 120)	(2.2, 125)	(2.1, 130)	(2.3, 120)	(2.2, 130)	(2.3, 120)	(2.1, 125)
Holding cost	(0.1, 1.2)	(0.3, 1.2)	(0.3, 1.4)	(0.5, 1.4)	(0.2, 1.3)	(0.3, 1.5)	(0.4, 1.3)
Shipping cost	(0.1 1.3)	(0.2,1.7)	(0.3,1.6)	(0.2, 1.5)	(0.2, 1.5)	(0.3, 1.4)	(0.4, 1.2)
Capacity(in hund.) units	(700,750)	(750,800)	(900,700)	(700,850)	(850,700)	(900,600)	(800,750)
Travel time units	(10, 8)	(10, 10)	(12, 10)	(15, 12)	(10,10)	(11, 12)	(12, 10)
Required Delivery time	(5, 7)	(10, 10)	(12, 8)	(15, 20)	(10, 10)	(12, 12)	(14, 10)
Obligatory Delivery time	(5, 7)	(10, 10)	(12, 8)	(15, 20)	(10, 10)	(12, 10)	(10, 12)
Trans. cost (/unit time)	(0.1, 1.7)	(0.2, 1.4)	(0.3, 1.5)	(0.2, 1.2)	(0.1, 1.5)	(0.2, 1.6)	(0.3, 1.5)
DelaydefiningFun	(0, 0)	(1, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 1)	(0, 0)
Amountrawneed (in hund.)	(130,120)	(120,180)	(150,200)	(100,100)	(100,100)	(130,1200)	(150,140)



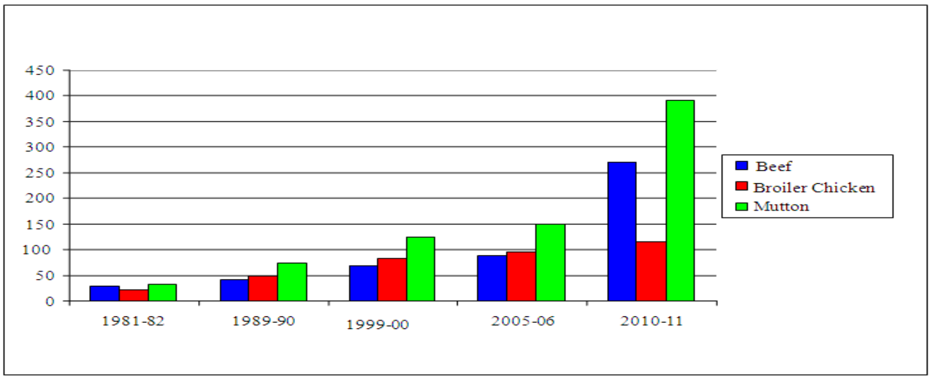
(Source: Field survey)

Figure: 3 Price comparison of egg between year 2011 and 2012.

We can see from fig.3, the prices of egg are in up and down from January to December that is the egg markets are unstable in the whole year.

According to the Food and Agriculture Organization, each person should take 56 kilogram of meat and 365 eggs every year. But in Bangladesh, per head intake of meat is only 11.27kg and egg 30 per year. As a result, people suffer from malnutrition. Dobson and Quarder (2005) commented that the country's pervasive poverty may limit the number of people who can afford to consume chicken as suggested by the simple relationship between per capita GDP and chicken consumption. If population growth continues at this rate, protein deficiency will rise.

Over the past two and half decades, price of meat and egg have increased four times. However, huge supply of poultry meat and egg helped keep the price at a relatively low level. During 1999-2000 and 2010-11, the price of broiler chicken had been stable at around 1.4 ($.) per kg, while the price of beef and mutton increased steadily which resulted in low relative price of chicken, which shows in Fig.4:



Source: Different Issues of Statistical Pocket Book, Bangladesh Bureau of Statistics.

Figure: 4 Trend the Prices of various meat in Bangladesh

III. Model Formulation

This work focuses on developing a MILFP and MIP programs to optimize the capacitated facility location and buyer allocation decisions, and production quantities at these locations to satisfy customer demands.

Mathematically the LFP problem can be represented as: $Z = \dfrac{C^T x + \alpha}{D^T x + \beta}$

Subject to

$$x \in X = \{x \in R^n : Ax = B, x \geq 0\}$$

Where,

x is the set of decision variables of n×1

A is the constraint matrix of order m×n

C and D is the contribution coefficient vector of order n×1

B is the constant or resource vector of order m×1

α, ß are scalar, which determines some constant profit and cost respectively

n and m are the number of variables and constraints respectively.

In this work, we used the Charnes and Coorper transformation technique. Charnes and Cooper (1962) considered the LFP problem defined above and assumed that

1) The feasible region X is non–empty and bounded,
2) C x+ α and D x+ ß do not vanish simultaneously in X

Introducing the variable transformation y =t x, where $t \geq 0$, Charnes and Cooper proved that LFP problem is reduced to either of the following two equivalent linear programs.

(EQP) Maximize, $Z1 = Cy + \alpha t$

Subject to

$$Ay + Bt = 0$$
$$Dy + \beta t = 1$$
$$y, t \geq 0$$

And

(EQN) Maximize, $Z2 = -Cy - \alpha t$

Subject to

$$Ay - Bt = 0$$
$$Dy + \beta t = -1$$
$$y, t \geq 0$$

Notations and Assumptions

Notation for the multiproduct multicustomer and multi-facility vendor-buyer system

Index and Parameters

i	Index for product, for all i=1, 2,……………….., m.
j	Index for retailer, for all j=1, 2,………… .., n.
l	Index for location of the manufacturer, for all l=1, 2,………… .., L.
s_{ij}	The price of ith product to j^{th} retailer ($/unit).
α_l	The fixed cost for opening the manufacturer at location l ($).
ß	Any positive scalar.
s^1_i	The price of unit raw materials for i^{th} product at l^{th} manufacturer ($/unit).
a^l_i	The amount of raw materials need to produce ith product at l^{th} manufacturer ($/unit). .
t^1_i	Unit transportation cost of raw materials for ith product at l^{th} manufacturer ($/unit).
p^l_{ij}	The production cost of i^{th} product to j^{th} buyer at l^{th} manufacturer ($/unit).
$h^1 ij$	Unit holding cost of i^{th} product from l^{th} manufacturer to retailer j for some given unit of time ($/unit-time).
ss^l_{ij}	The shipment cost of ith product from l^{th} manufacturer to j^{th} retailer ($/unit).
d_{ij}	The total demands of i^{th} product by j^{th} retailer (unit).
w^l_i	The capacity for ith product at l^{th} manufacturer (unit).
t^l_j	The required time for delivery of products from l^{th} manufacturer to retailer j (unit).
t^{*l}_j	The time within which should be delivery from l^{th} manufacturer to retailer (unit).
p	Penalty cost for delay in delivery for one unit of demand in one unit of time ($/unit).
s^{*l}_j	The transportation cost per unit product from l^{th} manufacturer to retailer j ($/unit).

c1i, b1i are any constant.

Penalty defining function

The function could be defined as

$$g^l{}_j = \begin{cases} 1, if\ \ t^l{}_j > t^{*l}{}_j, \\ 0, else \end{cases}$$

, where t^l_j is the required time for delivery of products from l^{th} vendor to buyer j and t^{*l}_j is the time within which should be delivery from l^{th} vendor to buyer.

Decision Variables

$$y^l{}_j = \begin{cases} 1, if\ \ customer\ \ j\ \ is\, assaign\ to\ \ manufacturer\ l\ , \\ 0, else \end{cases}$$

$$x_l = \begin{cases} 1, if\ \ location\ l\ is\ used, \\ 0, else \end{cases}$$

Q^l_{ij}= the production quantity of product i for retailer j at l^{th} manufacturer (unit).

Assumptions
1. Each manufacturing facility is able to produce all of the products.
2. The selling price for a product may vary from retailer to retailer depending on the discussions, order
3. sizes, discounts, historical relationships, etc.
4. The company and retailer have agreed beforehand on the inventory distribution pattern so the
5. shipping plans would be formulated accordingly.

Prerequisites of the Objective Function

Total return: $Z1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L} Q^l{}_{ij} s_{ij}$

Fixed opening cost: $c1 = \sum_{l=1}^{L} x_l \alpha_l$

Input cost: $c2 = \sum_{l=1}^{L}\sum_{i=1}^{m} s^l{}_i a^l{}_i + \sum_{l=1}^{L}\sum_{i=1}^{m} t^l{}_i \; a^l{}_i$

Production cost: $c3 = \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} Q^l{}_{ij} \; p^l{}_{ij}$

Shipment cost: $c4 = \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} Q^l{}_{ij} \; ss^l{}_{ij}$

Inventory holding cost: $c5 = \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} Q^l{}_{ij} \; h^l{}_{ij} / 2$

Penalty cost: $c6 = \sum_{l=}^{L}\sum_{j=1}^{n} p\, d_{ij} y^l{}_j (t^l{}_j - t^{*l}{}_j) g^l{}_j$

Transportation cost: $c7 = \sum_{l=1}^{L}\sum_{j=1}^{n} t^l{}_j \, s^{*l}{}_j$

Linear demand function: D (p) = $c_i^l + b_i^l * s_i^l$

For MILFP Model,

The objective function is: $$Maximize = \frac{Z1}{Z2} \qquad (1.1)$$

Where,
Z1 is the total return
Z2 is the total investments (sum of C1 to C7) explained in previous subsection.
Subject to

$\sum_{l=1}^{L}\sum_{i=1}^{m} Q^l{}_{ij} = \sum_{i=1}^{m} d_{ij} \;, \forall j$	(1.2)
$\sum_{l=1}^{L}\sum_{j}^{n} Q^l{}_{ij} = \sum_{j=1}^{n} d_{ij}, \forall i$	(1.3)
$\sum_{l=1}^{L} Q^l{}_{ij} = d_{ij} \;, \forall i,j$	(1.4)
$\sum_{j=1}^{n} Q^l{}_{ij} \le w^l{}_i \; \forall i,l$	(1.5)
$\sum_{j=1}^{n}\sum_{i=1}^{m} Q^l{}_{ij} \le \beta \, x_l \;, \forall l$	(1.6)

$\sum_{l=1}^{L} y^l{}_j = 1, \forall j$	(1.7)
$c^l_i = b^l_i * s^l_i, \forall i$	(1.8)
$Q^l{}_{ij}, s_{ij}, \alpha_l, d_{ij}, w^l{}_i, ss^l{}_{ij}, h^l{}_{ij}, p^l{}_{ij}, t^l{}_i, t^l{}_j, t*^l{}_j, p, s^{*l}{}_j, a^l{}_i, s^l{}_i \geq 0, x_l, y^l{}_j$ are binary $\forall i,j,l$	(1.9)

For MIP Model,
We formulated the equivalent mixed integer programming problem that estimate the total profit as well as optimal allocation and distribution.
The objective function is: $Maximize = Z1 - Z2$
Subject to
The set of constraints (1.2) to (1.9) described in the previous subsection.

IV. Solution Approach And Result Discussion

In order to solve the formulated MILFP, we need to apply suitable transformation. In this section, we have applied the Charnes and Cooper transformation to solve the formulated MILFP as described in above section.
For any nonnegative r the let the new decision could be redefined as follows:

$$z_l = rx_l, \text{ for } r \geq 0 \text{ and } l = 1,.....,L$$

$$z^l{}_j = ry^l{}_j, \text{ for } r \geq 0 \text{ and } j = 1,....,n, \ l = 1,........,L$$

$$z^l{}_{ij} = rQ^l{}_{ij}, \text{ for } r \geq 0 \text{ and } i = 1,....,m, \ j = 1,....,n, \ l = 1,........,L$$

Since $r \geq 0$, y^l_j and x_l are binary; as a result, z_l and z^l_j are become either zero or r. Further, since, Q^l_{ij} is non negative, consequently, z^l_{ij} are also remaining non-negative. Therefore, MILFP can be reformulated into two equivalent linear problems as follows:

(EQP) Maximize: $\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L} z^l{}_{ij} S_{ij}$

Subject to

$\sum_{l=1}^{L}\sum_{i=1}^{m} z^l{}_{ij} = r\sum_{i=1}^{m} d_{ij}, \forall j$	(1.10)
$\sum_{l=1}^{L}\sum_{j}^{n} z^l{}_{ij} = r\sum_{j=1}^{n} d_{ij}, \forall i$	(1.11)
$\sum_{l=1}^{L} z^l{}_{ij} = r\, d_{ij}, \forall i,j$	(1.12)
$\sum_{j=1}^{n} z^l{}_{ij} \leq r\, w^l{}_i \ \forall i,l$	(1.13)
$\sum_{j=1}^{n}\sum_{i=1}^{m} z^l{}_{ij} \leq 10000\, z_l, \forall l$	(1.14)
$\sum_{l=1}^{L} z^l{}_j = r, \forall j$	(1.15)
$\sum_{l=1}^{L} z_l\alpha_l + \sum_{l=1}^{L}\sum_{i=1}^{m} r\, t^l{}_i\, d_i + \sum_{l=1}^{L}\sum_{i=1}^{m} r\, s^l{}_i a^l{}_i + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z^l{}_{ij}\, p^l{}_{ij} + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z^l{}_{ij}\, ss^l{}_{ij} + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z^l{}_{ij}\, h^l{}_{ij}/2 + \sum_{i=1}^{m}\sum_{l=}^{L}\sum_{j=1}^{n} p\, d_{ij} z^l{}_j (t^l{}_j - t^{*l}{}_j) g^l{}_j + \sum_{l=1}^{L}\sum_{j=1}^{n} r\, t^l{}_j\, s*^l{}_j = 1$	(1.16)

(EQN) Maximize: $\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L} -z^{l}{}_{ij} s_{ij}$

Subject to

$\sum_{l=1}^{L}\sum_{i=1}^{m} z^{l}{}_{ij} = -r\sum_{i=1}^{m} d_{ij} \ ,\forall j$	(1.17)
$\sum_{l=1}^{L}\sum_{j}^{n} z^{l}{}_{ij} = -r\sum_{j=1}^{n} d_{ij}, \forall i$	(1.18)
$\sum_{l=1}^{L} z^{l}{}_{ij} = -r\, d_{ij} \ , \forall i,j$	(1.19)
$\sum_{j=1}^{n} z^{l}{}_{ij} \le -r\, w^{l}{}_{i} \ \forall i,l$	(1.20)
$\sum_{j=1}^{n}\sum_{i=1}^{m} z^{l}{}_{ij} \le -10000\ z_{l} \ ,\forall l$	(1.21)
$\sum_{l=1}^{L} z^{l}{}_{j} = -r, \forall j$	(1.22)
$\sum_{l=1}^{L} z_{l}\alpha_{l} + \sum_{l=1}^{L}\sum_{i=1}^{m} r\ t^{l}{}_{i}\ d_{i} + \sum_{l=1}^{L}\sum_{i=1}^{m} r\ s^{l}{}_{i} a^{l}{}_{i} + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z^{l}{}_{ij}\ p^{l}{}_{ij} + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z^{l}{}_{ij}\ ss^{l}{}_{ij} +$ $\sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z^{l}{}_{ij}\ h^{l}{}_{ij}/2 + \sum_{i=1}^{m}\sum_{l=}^{L}\sum_{j=1}^{n} p\, d_{ij} z^{l}{}_{j} (t^{l}{}_{j} - t^{*l}{}_{j}) g^{l}{}_{j} + \sum_{l=1}^{L}\sum_{j=1}^{n} r\, t^{l}{}_{j}\, s^{*l}{}_{j} = -1$	(1.23)
$Q^{l}{}_{ij}\ ,s_{ij}\ ,\alpha_{l}\ ,d_{ij}, w^{l}{}_{i}, ss^{l}{}_{ij}, h^{l}{}_{ij}, p^{l}{}_{ij}, t^{l}{}_{i}, t^{l}{}_{j}, t^{*l}{}_{j}, p, s^{*l}{}_{j}, a^{l}{}_{i}, s^{l}{}_{i} \ge 0, x_{l}\ , y^{l}{}_{j}\ are\ binary \quad \forall i,j,l$	(1.24)

For MIP model

In order to solve the formulated MIP model, let the new decision could be redefined as follows:

$$z_{l} = x_{l},\ for\ \ l = 1,.....,L$$

$$z^{l}{}_{j} = y^{l}{}_{j},\ for\ \ j = 1,....,n,\ \ l = 1,........,L$$

$$z^{l}{}_{ij} = Q^{l}{}_{ij},\ for\ i = 1,....,m,\ j = 1,....,n,\ l = 1,........,L$$

Since y^{l}_{j} and x_{l} are binary; as a result, z_{l} and z^{l}_{j} are become either zero or 1. Further, since, Q^{l}_{ij} is non negative, consequently, z^{l}_{ij} are also remaining non-negative. Therefore, MIP can be reformulated as follows:

Maximize: $\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L} z^{l}{}_{ij} S_{ij} - \sum_{l=1}^{L} x_{l}\alpha_{l} - (\sum_{l=1}^{L}\sum_{i=1}^{m} s^{l}{}_{i} a^{l}{}_{i} + \sum_{l=1}^{L}\sum_{i=1}^{m} t^{l}{}_{i}\ a^{l}{}_{i}) - \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z^{l}{}_{ij}\ p^{l}{}_{ij}$ $-\sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z^{l}{}_{ij}\ ss^{l}{}_{ij} - \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z^{l}{}_{ij}\ h^{l}{}_{ij}/2 - \sum_{l=}^{L}\sum_{j=1}^{n} p\, d_{ij} y^{l}{}_{j} (t^{l}{}_{j} - t^{*l}{}_{j}) g^{l}{}_{j} - \sum_{l=1}^{L}\sum_{j=1}^{n} t^{l}{}_{j}\, s^{*l}{}_{j}$	(1.25)

Subject to

$\sum_{l=1}^{L}\sum_{i=1}^{m} z^{l}{}_{ij} = \sum_{i=1}^{m} d_{ij} \ ,\forall j$	(1.26)
$\sum_{l=1}^{L}\sum_{j}^{n} z^{l}{}_{ij} = \sum_{j=1}^{n} d_{ij}, \forall i$	(1.27)

$\sum_{l=1}^{L} z^l_{ij} = d_{ij} \;, \forall i,j$	(1.28)
$\sum_{j=1}^{n} z^l_{ij} \le w^l_i \; \forall i,l$	(1.29)
$\sum_{j=1}^{n}\sum_{i=1}^{m} z^l_{ij} \le 10000\, z_l \;, \forall l$	(1.30)
$\sum_{l=1}^{L} z^l_j = 1, \forall j$	(1.31)
$\sum_{l=1}^{L} z_l\alpha_l + \sum_{l=1}^{L}\sum_{i=1}^{m} t^l_i\, d_i + \sum_{l=1}^{L}\sum_{i=1}^{m} s^l_i a^l_i + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z^l_{ij}\, p^l_{ij} + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z^l_{ij}\, ss^l_{ij} + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z^l_{ij}\, h^l_{ij}/2 + \sum_{i=1}^{m}\sum_{l=}^{L}\sum_{j=1}^{n} p\, d_{ij} z^l_j (t^l_j - t^{*l}_j) g^l_j + \sum_{l=1}^{L}\sum_{j=1}^{n} t^l_j\, s^{*l}_j = 1$	(1.32)

In order to analyze the effectiveness of the proposed models, a numerical example has been considered. It is assume that a vendor has 7 locations set, with 2 productions forecast for 2 buyers. The deterministic demand of unit products for buyers are (1700, 1400) and (1800, 1400), penalty cost of per unit in ($.) products for buyers are (0.10, 0.60) and (0.25, 0.40) respectively. Further, Table.1 describes additional information regarding the parameters of the MILFP and MIP models.

Finally, in order to estimate the effect of the sensitivity of production cost, fixed opening cost and raw material price parameter we employ sensitivity on these costs of different location. Fig. 5 and 6, it is clear that MILFP provides optimal locations of the vendor for buyer-1 are 1, 2, 4, 5 and 6, whereas, MIP provides the optimal locations of the vendor for buyer-1 are 1, 2, 3, 4 and 5. The optimal locations are achieved both MILFP and MIP models of the vendor for buyer-2 is 2, 3, 4, 5 and 6. Therefore, from the distribution of different products by MILFP and MIP models, it is apparently recommended that vendor-7 is not remained optimal for the first case.

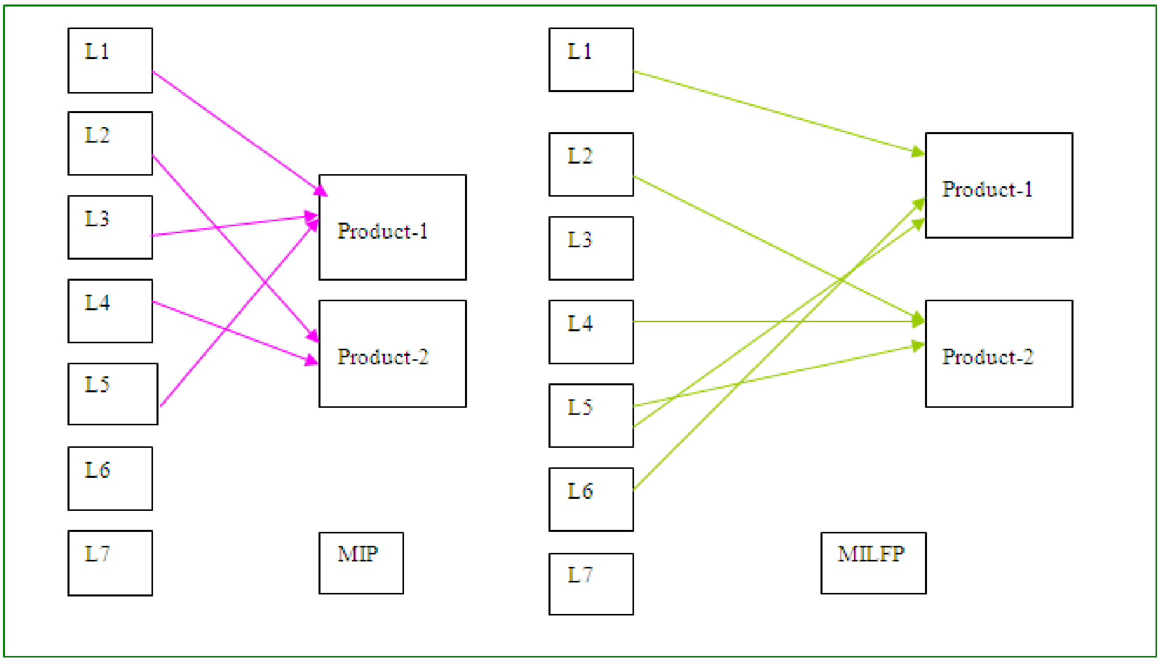


Figure: 5 Allocations for buyer-1 by (MIP) and (MILFP) models

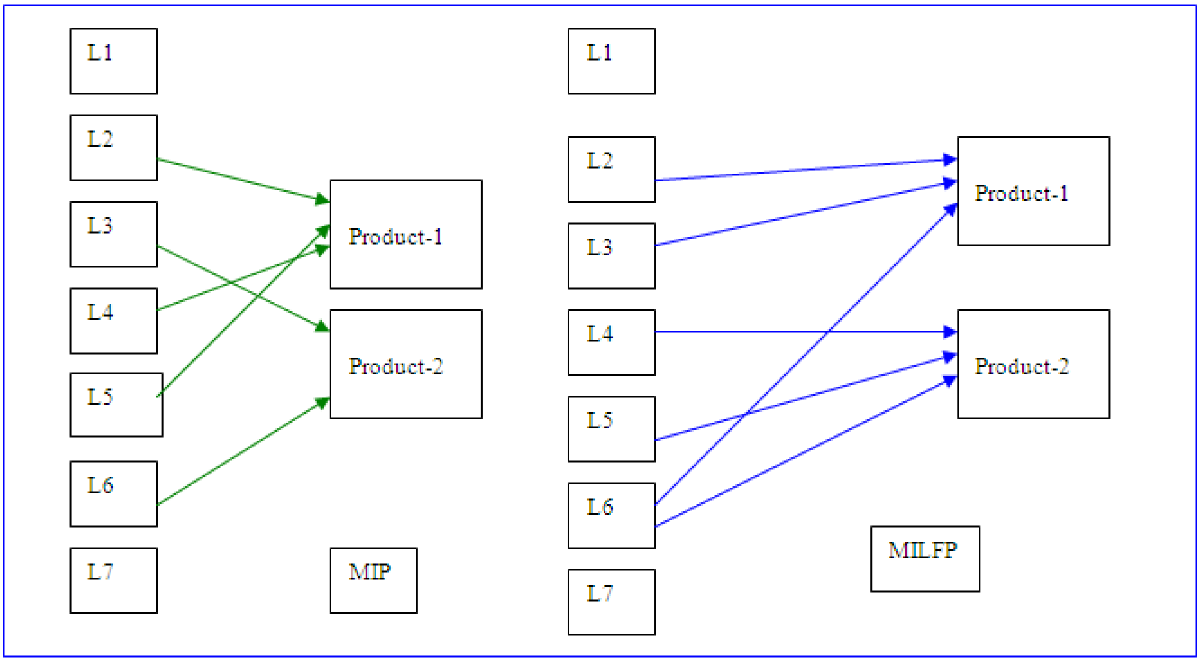


Figure: 6 Allocations for buyer-2 by (MIP) and (MILFP) models

Fig.7 and 8 describe the average demand of different products achieved by MILFP and MIP models for buyer-1. By both MILFP and MIP models, the highest demand of the product for buyer-1 is product 1 which is followed by product 2 as shown in Fig.7 and 8. The MILFP and MIP models satisfy the optimal demand of buyer-1 by the manufactures located at the location points 1, 2, 4, 5 and 6. MILFP model illustrates that vendor located at locations 5 is profitable for all two products. MIP model illustrates that locations 1 and 5 are profitable for product 1 and locations 2 and 4 are profitable for product 2. Further, both MILFP and MIP models describe that vendor-7 is not anyhow optimum for buyer-1 for all two products.

Fig.9 and 10 depict the average demand of different products obtained by MILFP and MIP models for buyer-2. By MILFP model the maximum demand for the product of buyer-2 is product 2 which is followed by product 1 as shown in Fig.9 and MIP models, the maximum demand for the product of buyer-2 is product 1 which is followed by product 2 as shown in Fig.10. The MILFP and MIP models perform the optimal demand of buyer-2 by the manufactures located at the location points 2, 3, 4, 5 and 6. MILFP model illustrates that all vendor located is not profitable and can not satisfy the optimal demand of all two products. MIP model illustrates that location 6 is profitable for all two products. Further, both MILFP and MIP models explain that vendor-1 and 7 is not anyway profitable for buyer-2 for all the products.

Fig.11, 12 and 13 describes the sensitive of the production, raw material and fixed opening cost on the total ratio of return on investment obtained different cases by the MILFP and MIP models. The proportion of the return and investment obtained by both MILFP and MIP models are not differing much. In addition, all cases the profit achieved by MILFP model is slightly higher than that of by MIP model as shown in Fig.11, 12 and 13. The sensitivity of the production, raw material and fixed opening cost demonstrates that all the cases the increment of the production cost decrease the profit by both MILFP and MIP models since this additional cost increases the investment as well as cost. The production cost changes the profit more than the raw material cost and fixed opening cost of the product and raw material cost changes the profit more than the fixed opening cost of the product.

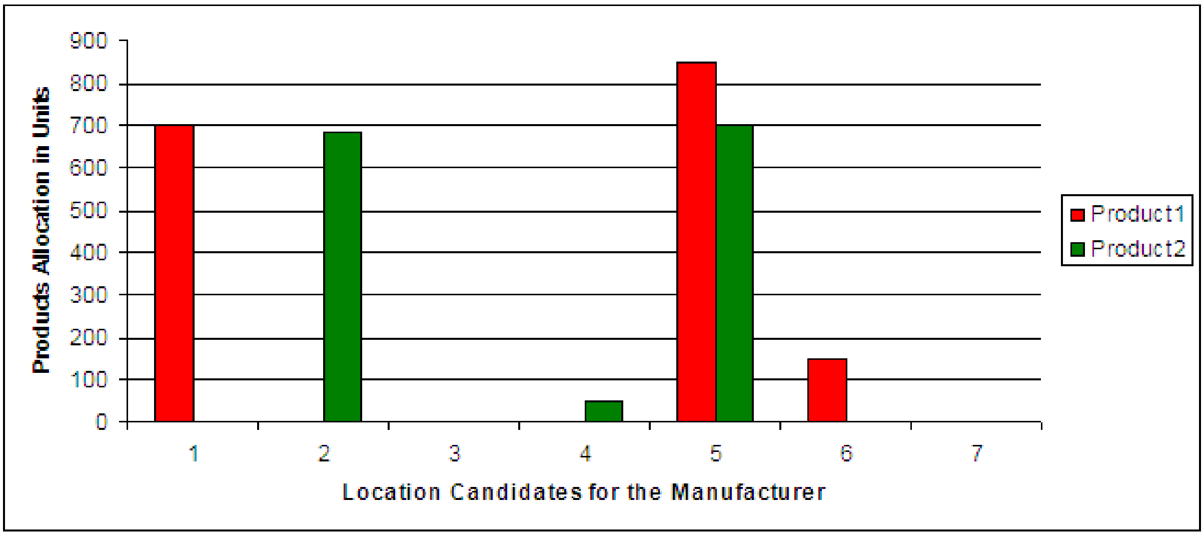


Figure: 7 Demand of different products at different locations for buyer-1 by MILFP

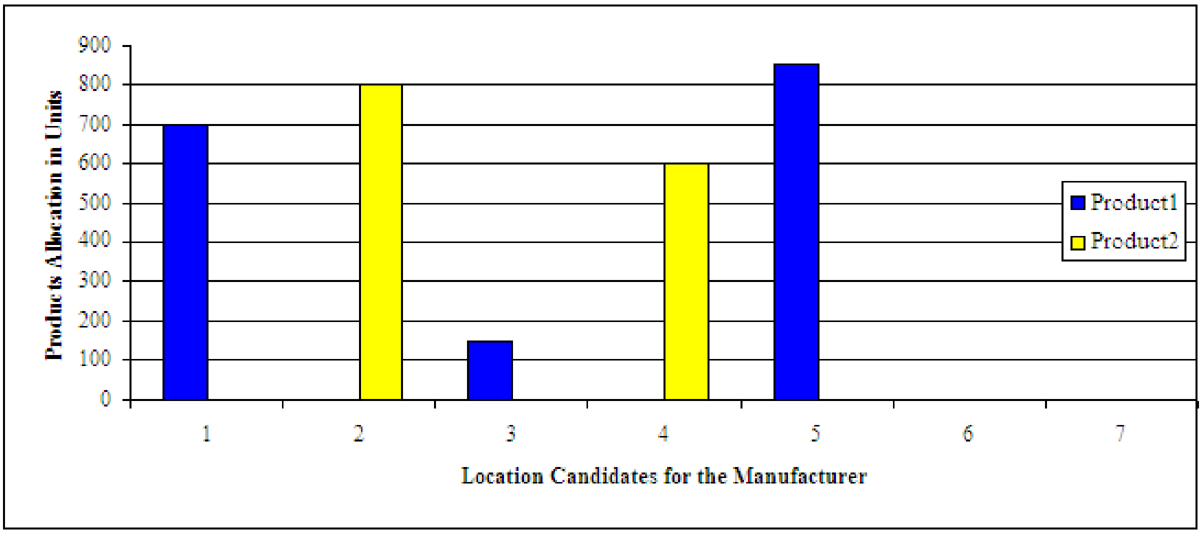


Figure: 8 Demand of different products at different locations for buyer-1 by MIP

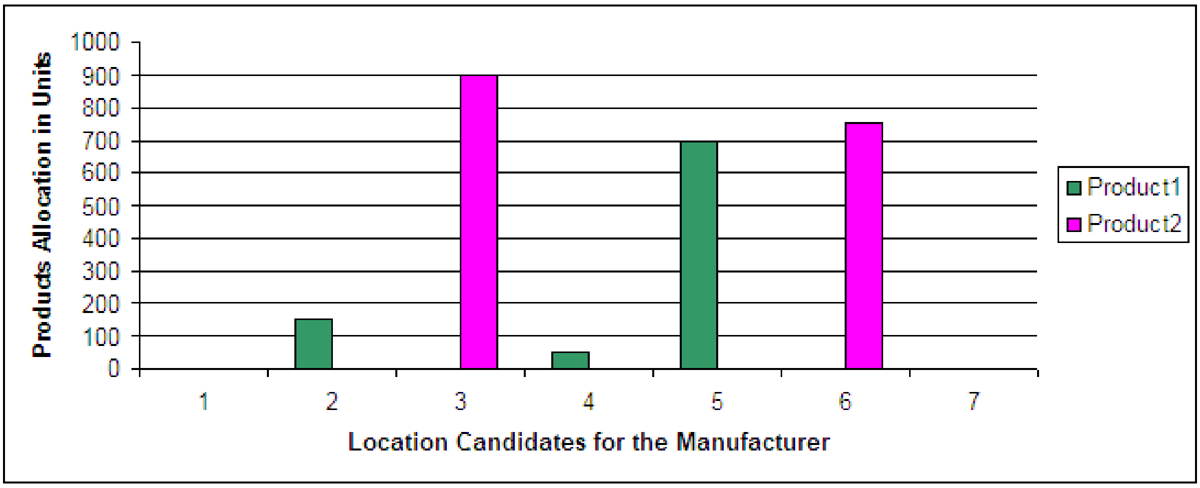


Figure: 9 Demand of different products at different locations for buyer-2 by MILFP

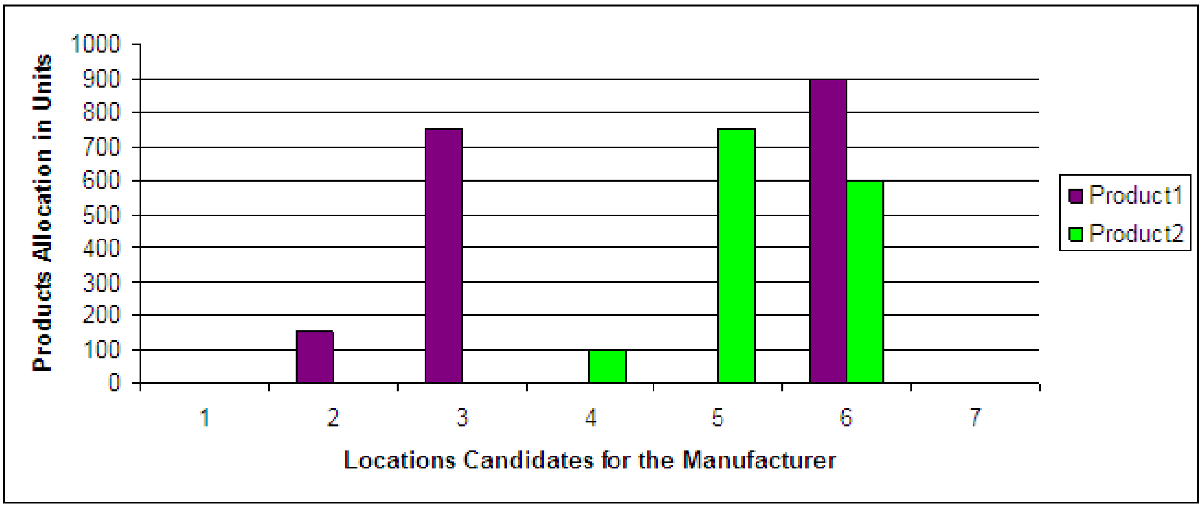


Figure: 10 Demand of different products at different locations for buyer-2 by MIP

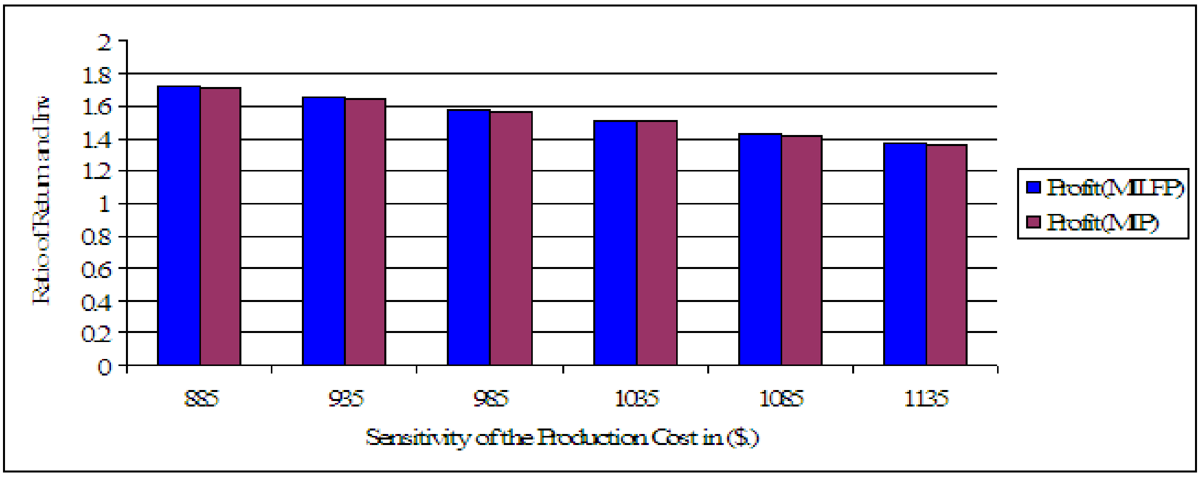


Figure: 11 Comparison between return and investment obtained by MILFP and MIP models, which effect of the sensitivity analysis of production cost on profit.

Figure: 12 Comparison between return and investment obtained by MILFP and MIP models, which effect of the sensitivity analysis of raw material cost on profit.

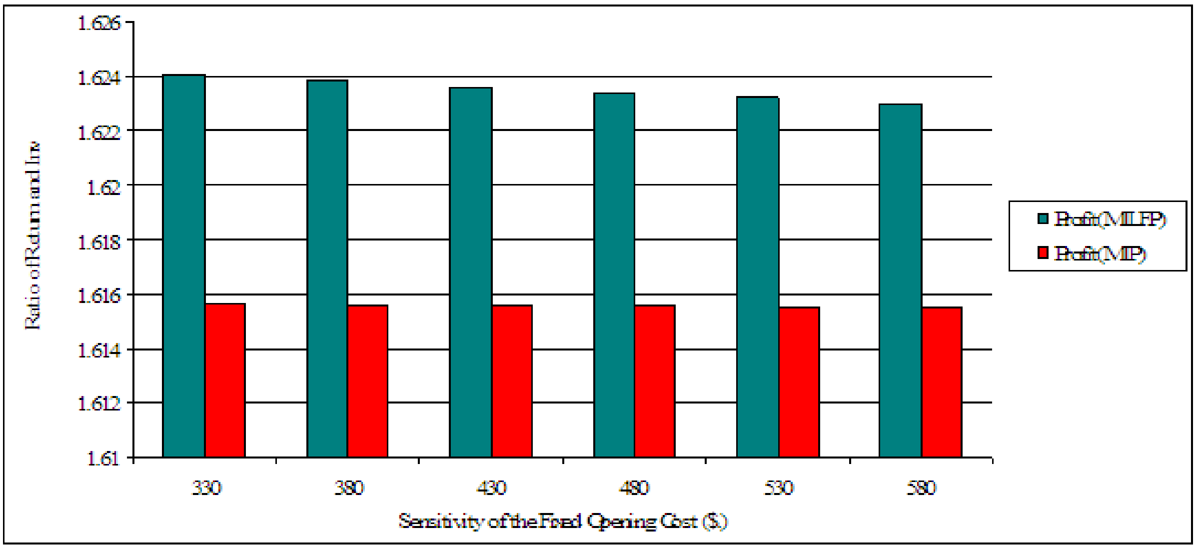


Figure: 13 Comparison between return and investment obtained by MILFP and MIP models, which effect of the sensitivity analysis of fixed opening cost on profit.

V. Conclusion

In this study, an MILFP based model is developed for the integrated supply chain network and using the suitable transformation the model is solved by AMPL. The formulated model simultaneously maximizes the ratio of return on investment. Further, in order to demonstrate the significance of MILFP model, an MIP based model is also formulated. Some of the significance findings can be summarized as follows:

Firstly, the illustrated numerical example apparently shows that both MILFP and MIP are provides very similar distribution pattern for the integrated multi-product, multi-facility, and multi-buyer location production supply chain network, which is worthy to the developed MILFP model. Secondly, the optimums locations of the warehouse are obtained by both of the models are very similar and reject the same location. The optimal demands for different products by the buyer are almost analogous by both MILFP and MIP models. The differences of the ratio of the return on investment achieved by the both models are less than 0.72%. Moreover, from the sensitivity analysis of the production cost and raw material cost, it is concluded that production cost and raw material cost is one of the momentous factors to increase and decrease the profit of a vendor. Further, the production cost and raw material cost has negative influence on the total profit. Therefore, MILFP model could be one of the relevant approaches in a logistic model which seeks to find the optimum manufacturer as well as optimum distribution with profit maximization and cost minimization.

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