IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 1 Ver. 1 (Jan - Feb. 2015), PP 75-82
www.iosrjournals.org

Bulk Demand (S, s) Inventory System with Varying Environment

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Abstract: *This paper studies two stochastic bulk demand (S, s) inventory models A and B with randomly varying environment. In the models the maximum storing capacity of the inventory is S units and the order for filling up the inventory is placed when the inventory level falls to s or below. Demands for random number of units occur at a time and wait for supply forming a queue when the inventory has no stock. The inventory is exposed to changes in the environment. Inter occurrence time between consecutive bulk demands has exponential distribution and its parameter changes when the environment changes. Lead time for an order realization has exponential distribution and its parameter also changes when the environment changes. When an order is realized in the environment i, for $1 \leq i \leq k$, the inventory is filled up if no demand is waiting or if the number n of demands waiting is such that $0 \leq n \leq N_i$-S where $N_i$is the maximum number of units supplied in the environment i for an order. If n is such that with N_i-S < n < N_i all the n demands are cleared and N_i-n units become stocks for the inventory if N_i-n $\leq$ S and if N_i-n >S the inventory is filled up and the units in excess after filling up the inventory are returned. If $n \geq N_i$ demands are waiting, N_i demands are cleared reducing the demand level to n-N_i. In model A, $max_i\ M_i > max_i\ N_i$ where M_i is the maximum demand size in the environment i for$1 \leq i \leq k$. In model B the maximum demand size in all environments is less than maximum supply size in all environments. Matrix partitioning method is used to study the models. The stationary probabilities of demand length, its expected values, its variances and probabilities of empty levels are derived for the two models using the iterated rate matrix. Numerical examples are presented for illustration.*
Keywords: *Block Circulant Matrix, Block partitioning methods, Bulk Demands, Lead time, Matrix Geometric Methods.*

I. Introduction

In this paper two bulk demand (S, s) inventory systems with varying environment are treated using matrix geometric methods. Thangaraj and Ramanarayanan [1] have studied two ordering level and unit demand inventory systems using integral equations. Jacob and Ramanarayanan [2] have treated (S, s) inventory systems with server vacations. Ayyappan, Subramanian and Gopal Sekar [3] have analyzed retrial system using matrix geometric methods. Bini, Latouche and Meini [4] have studied numerical methods for Markov chains. Chakravarthy and Neuts [5] have discussed in depth a multi-server waiting model. Gaver, Jacobs and Latouche [6] have treated birth and death models with random environment. Latouche and Ramaswami [7] have studied Analytic methods. For matrix geometric methods and models one may refer Neuts [8]. Rama Ganesan, Ramshankar and Ramanarayanan [9] have analyzed M/M/1 bulk queues under varying environment. Fatigue failure models using Matrix geometric methods have been analyzed by Sundar [10]. The models considered in this paper are general compared to existing inventory models. Here at each environment in a demand epoch, random numbers of units are demanded and the maximum number of units demanded may be different in the various environments. When there is no stock in the inventory, after the lead time, realized orders can clear in each environment various number of waiting demands. Usually bulk arrival models have M/G/1 upper-Heisenberg block matrix structure with zeros below the first sub diagonal. The decomposition of a Toeplitz sub matrix of the infinitesimal generator is required to find the stationary probability vector. Matrix geometric structures have not been noted as mentioned by William J. Stewart [11] and even in such models the recurrence relation method to find the stationary probabilities is stopped at certain level in most general cases indicating limitations of such approach. Rama Ganesan and Ramanarayanan [12] have presented a special case where a generating function has been noticed in such a situation. But in this paper the partitioning of the matrix with blocks of size, which is the maximum of the maximum number of demands and the maximum of the order supply sizes in all the environments, exhibits the matrix geometric structure for the varying environment (S, s) inventory system. This shows (S, s) inventory systems of M/M/1 types with bulk arrivals of demands no matter how big the demand size is in number at each arrival epoch provided it is bounded above by a maximum and the supply for orders after the lead time no matter how big the supply size is in number provided it is bounded above by a maximum, have matrix geometric solutions.

Two models (A) and (B) of bulk demand (S, s) inventory systems with k varying environments and infinite storage spaces for demands are studied here using the block partitioning method and matrix geometric results are obtained. In the models considered here, the maximum demand sizes and the maximum order supply sizes are different for different environments. Model (A) presents the case when M, the maximum of all the maximum demand sizes in all the environments is bigger than N, the maximum of order supply sizes in all the environments. In Model (B), its dual case, N is bigger than M, is treated. In general in waiting line models, the state space of the system has the first co-ordinate indicating the number of customers in the system but here the demands in the system are grouped and considered as members of M sized blocks of demands (when M >N) or N sized blocks of demands (when N > M) for finding the rate matrix. Using the maximum of the bulk demand sizes or the maximum of the order supply sizes with grouping of the demands as members of the blocks for the partitioning of the infinitesimal generator is a new approach in this area. The matrices appearing as the basic system generators in these two models due to block partitions are seen as block circulants. The stationary probability of the number of demands waiting for service, the expectation, the variance and the probability of various levels of the inventory are derived for these models. Numerical cases are presented to illustrate their applications. The paper is organized in the following manner. In section II the (S, s) inventory system with bulk demand and order clearance after the lead time is studied for randomly varying environment in which maximum M is greater than maximum N. Section III treats the situation in which the maximum M is less than the maximum N. In section IV numerical cases are presented.

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II. Model (A). Maximum Demand Size M is Greater Than The Maximum Supply Size N

2.1Assumptions for M > N.

i) There are k environments. The environment changes as per changes in a continuous time Markov chain with infinitesimal generator Q_1 of order k.

ii) Demands arrive for multiple numbers of units at demand epochs. The time between consecutive demand epochs has exponential distribution with parameterλ_i, in the environment i for $1 \le i \le k$. At each demand epoch in the environment i, χ_i units are demanded with probability given by P (χ_i= j) = p_j^i for $1 \le j \le M_i$ where M_i is the maximum number of units of demand and $\sum_{j=1}^{M_i} p_j^i$=1 for $1 \le i \le k$.

iii) The maximum capacity of the inventory to store units is S. Whenever the inventory level falls to s or below, orders are made for the supply of units for the inventory. Arriving demands are served till the inventory level falls to 0 after which the demands wait for order realization. During the lead time of an order, another order cannot be made. After the realization of an order if the inventory level becomes s or below s or the inventory is still empty with or without waiting demands, then the next order is made. When an order is realized in the environment i, for $1 \le i \le k$ the inventory is supplied with $N_i \ge S$ units. When the inventory level is in between 0 and s, the inventory is filled up and units which are in excess are returned immediately. When n demands are waiting for $0 \le n \le N_i$-S at the order realization epoch, n waiting demands are cleared, the inventory is filled up and units in excess are immediately returned. When the waiting number of demands n at the order realization epoch is such that N_i-S < n < N_i all the n demands are cleared and N_i-n units become stocks for the inventory if N_i-n ≤ S and if N_i-n > S the inventory is filled up and the units in excess are returned. If n ≥ N_i demands are waiting when an order is realized, N_i demands are cleared reducing the waiting demand level to n-N_i. The lead time distribution of an order has exponential distribution with parameter μ_i in the environment i for$1 \le i \le k$.

iv) iv)When the environment changes from i to j, the parameter λ_i of inter occurrence time of bulk demands and the parameter of the lead time of an order μ_i change to λ_j and μ_j respectively and the maximum demand size M_i and the order realization size N_i change to M_j and N_j respectively for$1 \le i, j \le k$.

v) The maximum of the maximum of demand sizes M = $\max_{1 \le i \le k} M_i$ is greater than the maximum of the order realization sizes N =$\max_{1 \le i \le k} N_i$.

2.2 Analysis

The state of the system of the continuous time Markov chain X (t) under consideration is presented as follows.

X (t) = {(n, j, i): for $0 \le j \le$ M-1; $1 \le i \le k$ and $n \ge 0$} (1)

The chain is in the state (0, j, i) when the S – j units are in the inventory for $0 \le j \le S$ and the environment is in state is i for $1 \le i \le k$. The chain is in the state (0, j, i) when j-S demands are waiting for S+1≤ j ≤ M-1 and the environment state is i, for$1 \le i \le k$. The chain is in the state (n, j, i) when the number of demands waiting for units is n M + j - S, for $0 \le j \le$ M-1, $1 \le n < \infty$ and the environment state is i for $1 \le i \le k$. When the number of demands in the system is r ≥ 1, then r is identified with (n, j) where r + S on division by M gives n as the quotient and j as the remainder. When the inventory level is r, for $0 \le r \le S$ without any waiting demand at level 0 then r is identified with (0, j) where j= S – r, for $0 \le j \le S$.

The chain X (t) describing model has the infinitesimal generator Q_A of infinite order which can be presented in block partitioned form given below.

$$Q_A = \begin{bmatrix} B_1 & A_0 & 0 & 0 & . & . & . & \cdots \\ A_2 & A_1 & A_0 & 0 & . & . & . & \cdots \\ 0 & A_2 & A_1 & A_0 & 0 & . & . & \cdots \\ 0 & 0 & A_2 & A_1 & A_0 & 0 & . & \cdots \\ 0 & 0 & 0 & A_2 & A_1 & A_0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots \end{bmatrix} \quad (2)$$

In (2) the states of the matrix are listed lexicographically as $\underline{0}, \underline{1}, \underline{2}, \underline{3}, \ldots . \underline{n}, \ldots$. Here the vector $\underline{n}$ is of type 1 x k M and $\underline{n}$ = ((n, 0, 1), (n, 0, 2) …(n, 0, k), (n, 1, 1), (n, 1, 2),…(n, 1, k),…(n, M-1, 1), (n, M-1, 2)…(n, M-1, k) for $n \ge 0$.

The matrices B_1 and A_1 have negative diagonal elements, they are of order Mk and their off diagonal elements are non- negative. The matrices A_0, and A_2 have nonnegative elements and are of order Mk and they are given below. Let the following be diagonal matrices of order k

Λ_j=diag $(\lambda_1 p_j^1, \lambda_2 p_j^2, \ldots, \lambda_k p_j^k)$ for $1 \le j \le M$; (3)

U_j is 0 matrix of order k for $1 \le j \le N$ except for $j = N_i$ for $1 \le i \le k$ and $S \le N_i \le N$ where U_{N_i} is a matrix of order k with only one non zero element $(U_{N_i})_{i,i} = \mu_i$ for $1 \le i \le k$ and its other elements are 0. (4)

$V_j = \sum_{i=j+1}^{N} U_i$ for $1 \le j \le N-1$ Λ = diag $(\lambda_1, \lambda_2, \ldots, \lambda_k)$; (5)

and $U = \text{diag}\,(\mu_1, \mu_2, \ldots, \mu_k) = \sum_{i=1}^{N} U_i$. (6)

Let $\mathcal{Q}_1' = \mathcal{Q}_1 - \Lambda - U$. (7)

Here $\mathcal{Q}_1$ is the infinitesimal generator of the Markov chain of the environment.

$$A_0 = \begin{bmatrix} \Lambda_M & 0 & \cdots & 0 & 0 & 0 \\ \Lambda_{M-1} & \Lambda_M & \cdots & 0 & 0 & 0 \\ \Lambda_{M-2} & \Lambda_{M-1} & \cdots & 0 & 0 & 0 \\ \Lambda_{M-3} & \Lambda_{M-2} & \ddots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ \Lambda_3 & \Lambda_4 & \cdots & \Lambda_M & 0 & 0 \\ \Lambda_2 & \Lambda_3 & \cdots & \Lambda_{M-1} & \Lambda_M & 0 \\ \Lambda_1 & \Lambda_2 & \cdots & \Lambda_{M-2} & \Lambda_{M-1} & \Lambda_M \end{bmatrix} (8)$$

$$A_2 = \begin{bmatrix} 0 & \cdots & 0 & U_N & U_{N-1} & \cdots & U_2 & U_1 \\ 0 & \cdots & 0 & 0 & U_N & \cdots & U_3 & U_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & U_N & U_{N-1} \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & U_N \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} (9)$$

$$A_1 = \begin{bmatrix} \mathcal{Q}_1' & \Lambda_1 & \Lambda_2 & \cdots & \Lambda_{M-N-2} & \Lambda_{M-N-1} & \Lambda_{M-N} & \cdots & \Lambda_{M-2} & \Lambda_{M-1} \\ U_1 & \mathcal{Q}_1' & \Lambda_1 & \cdots & \Lambda_{M-N-3} & \Lambda_{M-N-2} & \Lambda_{M-N-1} & \cdots & \Lambda_{M-3} & \Lambda_{M-2} \\ U_2 & U_1 & \mathcal{Q}_1' & \cdots & \Lambda_{M-N-4} & \Lambda_{M-N-3} & \Lambda_{M-N-2} & \cdots & \Lambda_{M-4} & \Lambda_{M-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ U_N & U_{N-1} & U_{N-2} & \cdots & \mathcal{Q}_1' & \Lambda_1 & \Lambda_2 & \cdots & \Lambda_{M-N-2} & \Lambda_{M-N-1} \\ 0 & U_N & U_{N-1} & \cdots & U_1 & \mathcal{Q}_1' & \Lambda_1 & \cdots & \Lambda_{M-N-3} & \Lambda_{M-N-2} \\ 0 & 0 & U_N & \cdots & U_2 & U_1 & \mathcal{Q}_1' & \cdots & \Lambda_{M-N-4} & \Lambda_{M-N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & U_N & U_{N-1} & U_{N-2} & \cdots & \mathcal{Q}_1' & \Lambda_1 \\ 0 & 0 & 0 & \cdots & 0 & U_N & U_{N-1} & \cdots & U_1 & \mathcal{Q}_1' \end{bmatrix} (10)$$

$$B_1 = \begin{bmatrix} \mathcal{Q}_1 - \Lambda & \Lambda_1 & \Lambda_2 & \cdots & \Lambda_{S-s-1} & \Lambda_{S-s} & \Lambda_{S-s+1} & \cdots & \Lambda_{M-N-2} & \Lambda_{M-N-1} & \cdots & \Lambda_{M-2} & \Lambda_{M-1} \\ 0 & \mathcal{Q}_1 - \Lambda & \Lambda_1 & \cdots & \Lambda_{S-s-2} & \Lambda_{S-s-1} & \Lambda_{S-s} & \cdots & \Lambda_{M-N-3} & \Lambda_{M-N-2} & \cdots & \Lambda_{M-3} & \Lambda_{M-2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \mathcal{Q}_1 - \Lambda & \Lambda_1 & \Lambda_2 & \cdots & \Lambda_{M-N-S+s-1} & \Lambda_{M-N-S+s} & \cdots & \Lambda_{M-(S-s+1)} & \Lambda_{M-(S-s)} \\ V_{S-s-1} & U_{S-s-1} & U_{S-s-2} & \cdots & U_1 & \mathcal{Q}_1' & \Lambda_1 & \cdots & \Lambda_{M-N-S+s-2} & \Lambda_{M-N-S+s-1} & \cdots & \Lambda_{M-(S-s+2)} & \Lambda_{M-(S-s+1)} \\ V_{S-s} & U_{S-s} & U_{S-s-1} & \cdots & U_2 & U_1 & \mathcal{Q}_1' & \cdots & \Lambda_{M-N-S+s-3} & \Lambda_{M-N-S+s-2} & \cdots & \Lambda_{M-(S-s+3)} & \Lambda_{M-(S-s+2)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \cdots & \vdots & \vdots \\ V_{N-1} & U_{N-1} & U_{N-2} & \cdots & U_{N-S+s} & U_{N-S+s-1} & U_{N-S+s-2} & \cdots & U_1 & \mathcal{Q}_1' & \cdots & \Lambda_{M-N-2} & \Lambda_{M-N-1} \\ 0 & U_N & U_{N-1} & \cdots & U_{N-S+s+1} & U_{N-S+s} & U_{N-S+s-1} & \cdots & U_2 & U_1 & \cdots & \Lambda_{M-N-3} & \Lambda_{M-N-2} \\ 0 & 0 & U_N & \cdots & U_{N-S+s+2} & U_{N-S+s+1} & U_{N-S+s} & \cdots & U_3 & U_2 & \cdots & \Lambda_{M-N-4} & \Lambda_{M-N-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & U_N & U_{N-1} & \cdots & \mathcal{Q}_1' & \Lambda_1 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & U_N & \cdots & U_1 & \mathcal{Q}_1' \end{bmatrix} (11)$$

$$
\mathcal{Q}_A'' = \begin{bmatrix}
\mathcal{Q}_1' + \Lambda_M & \Lambda_1 & \cdots & \Lambda_{M-N-2} & \Lambda_{M-N-1} & \Lambda_{M-N} + U_N & \cdots & \Lambda_{M-2} + U_2 & \Lambda_{M-1} + U_1 \\
\Lambda_{M-1} + U_1 & \mathcal{Q}_1' + \Lambda_M & \cdots & \Lambda_{M-N-3} & \Lambda_{M-N-2} & \Lambda_{M-N-1} & \cdots & \Lambda_{M-3} + U_3 & \Lambda_{M-2} + U_2 \\
\Lambda_{M-2} + U_2 & \Lambda_{M-1} + U_1 & \cdots & \Lambda_{M-N-4} & \Lambda_{M-N-3} & \Lambda_{M-N-2} & \cdots & \Lambda_{M-4} + U_3 & \Lambda_{M-3} + U_3 \\
\vdots & \vdots & \vdots\vdots\vdots & \vdots & \vdots & \vdots & \vdots\vdots\vdots & \vdots & \vdots \\
\Lambda_{M-N+2} + U_{N-2} & . & \cdots & . & . & . & \cdots & \Lambda_{M-N} + U_N & \Lambda_{M-N+1} + U_{N-1} \\
\Lambda_{M-N+1} + U_{N-1} & . & \cdots & . & . & . & . & \Lambda_{M-N-1} & \Lambda_{M-N} + U_N \\
\Lambda_{M-N} + U_N & . & \cdots & \mathcal{Q}_1' + \Lambda_M & \Lambda_1 & \Lambda_2 & \cdots & \Lambda_{M-N-2} & \Lambda_{M-N-1} \\
\Lambda_{M-N-1} & \Lambda_{M-N} + U_N & \cdots & \Lambda_{M-1} + U_1 & \mathcal{Q}_1' + \Lambda_M & \Lambda_1 & \cdots & \Lambda_{M-N-3} & \Lambda_{M-N-2} \\
\Lambda_{M-N-2} & \Lambda_{M-N-1} & \cdots & \Lambda_{M-2} + U_2 & \Lambda_{M-1} + U_1 & \mathcal{Q}_1' + \Lambda_M & \cdots & \Lambda_{M-N-4} & \Lambda_{M-N-3} \\
\vdots & \vdots & \vdots\vdots\vdots & \vdots & \vdots & \vdots & \vdots\vdots\vdots & \vdots & \vdots \\
\Lambda_2 & \Lambda_3 & \cdots & \Lambda_{M-N} + U_N & \Lambda_{M-N+1} + U_{N-1} & \Lambda_{M-N+2} + U_{N-2} & \cdots & \mathcal{Q}_1' + \Lambda_M & \Lambda_1 \\
\Lambda_1 & \Lambda_2 & \cdots & \Lambda_{M-N-1} & \Lambda_{M-N} + U_N & \Lambda_{M-N+1} + U_{N-1} & \cdots & \Lambda_{M-1} + U_1 & \mathcal{Q}_1' + \Lambda_M
\end{bmatrix} \quad (12)
$$

The basic generator $\mathcal{Q}_A''$ of the system, which is concerned with only the demand and supply, is a matrix of order Mk given above in (12) where $\mathcal{Q}_A'' = A_0 + A_1 + A_2$ (13)

Its probability vector w gives, $w\mathcal{Q}_A'' = 0$ and w e = 1 (14)

It is well known that a square matrix in which each row (after the first) has the elements of the previous row shifted cyclically one place right, is called a circulant matrix. It is very interesting to note that the matrix $\mathcal{Q}_A'' = A_0 + A_1 + A_2$ is a block circulant matrix where each block matrix is rotated one block to the right relative to the preceding block partition. Let the probability vector of the environment generator $\mathcal{Q}_1$ be π. Then $\pi\mathcal{Q}_1 = 0$ and π e =1. It can be seen in (13) that the first block-row of type k x Mk is,

$W = (\mathcal{Q}_1' + \Lambda_M, \Lambda_1, \Lambda_2, \ldots \Lambda_{M-N-2}, \Lambda_{M-N-1}, \Lambda_{M-N} + U_N, \ldots \Lambda_{M-2} + U_2, \Lambda_{M-1} + U_1)$

This gives as the sum of the blocks $(\mathcal{Q}_1' + \Lambda_M) + \Lambda_1 + \Lambda_2 + \ldots + \Lambda_{M-N-2} + \Lambda_{M-N-1} + \Lambda_{M-N} + U_N + \ldots \ldots + \Lambda_{M-2} + U_2 + \Lambda_{M-1} + U_1 = \mathcal{Q}_1$. So $\pi\ \mathcal{Q}_1 = \pi(\mathcal{Q}_1' + \Lambda_M) + \pi\sum_{i=1}^{M-N-1}\Lambda_i + \pi\ \sum_{i=1}^{N}(\Lambda_{M-i} + U_i) = 0$ which implies ($\pi, \pi \ldots \pi, \pi$). W = 0 = ($\pi, \pi \ldots \pi, \pi$) W' where W' is the transpose of vector W. Since all blocks, in any block-row are seen somewhere in each and every column block due to block circulant structure, the above equation shows the left eigen vector of the matrix $\mathcal{Q}_A''$ is ($\pi, \pi \ldots \pi$). Using (14)

$$w = \left(\frac{\pi}{M}, \frac{\pi}{M}, \frac{\pi}{M}, \ldots, \frac{\pi}{M}\right). \quad (15)$$

Neuts [8], gives the stability condition as, $w A_0 e < w A_2 e$ where w is given by (15). Taking the sum cross diagonally using the structure in (8) and (9) for the A_0 and A_2 matrices, it can be seen that

$w A_0 e = \frac{1}{M}\pi\left(\sum_{n=1}^{M} n\Lambda_n\right)e = \frac{1}{M}\pi\,.\,(\lambda_1 E(\chi_1), \lambda_2 E(\chi_2), \ldots, \lambda_k E(\chi_k)) < w A_2 e = \frac{1}{M}\pi\left(\sum_{n=1}^{N} nU_n\right)e = \frac{1}{M}\pi\,.\,(\mu_1 M_1, \mu_2 M_2, \ldots, \mu_k M_K)$.

Taking the probability vector of the environment generator $\mathcal{Q}_1$ as $\pi = (\pi_1, \pi_2, \ldots, \pi_{k-1}, \pi_k)$, the inequality reduces to $\sum_{i=1}^{k}\pi_i\ \lambda_i E(\chi_i) < \sum_{i=1}^{k}\pi_i\ \mu_i M_i$. (16)

This is the stability condition for (S, s) inventory system under random environment with bulk demand where maximum of the maximums of demand sizes in all environments is greater than the maximum of order realization sizes in all environments. When (16) is satisfied, the stationary distribution exists as proved in Neuts [8].

Let π (n, j, i), for $0 \le j \le$ M-1, $1 \le i \le k$ and $0 \le n < \infty$ be the stationary probability of the states in (1) and π_n be the vector of type 1xMk with, π_n= (π(n, 0, 1), π(n, 0, 2) … π(n, 0, k), π(n, 1, 1), π(n, 1, 2),… π(n, 1, k)... π(n, M-1, 1), π(n, M-1, 2)…π(n, M-1, k)), for $n \ge 0$.

The stationary probability vector $\pi = (\pi_0, \pi_1, \pi_3, \ldots\ldots)$ satisfies $\pi Q_A = 0$, and πe=1. (17)

From (17), it can be seen $\pi_0 B_1 + \pi_1 A_2 = 0$. (18)

$\pi_{n-1}A_0 + \pi_n A_1 + \pi_{n+1}A_2 = 0$, for $n \ge 1$. (19)

Introducing the rate matrix R as the minimal non-negative solution of the non-linear matrix equation $A_0 + RA_1 + R^2A_2 = 0$, (20)

it can be proved (Neuts [8]) that π_n satisfies $\pi_n = \pi_0 R^n$ for $n \ge 1$. (21)

Using (17) and (18), π_0 satisfies $\pi_0\ [B_1 + RA_2] = 0$ (22)

Now π_0 can be calculated up to multiplicative constant by (22). From (17) and (21) $\pi_0\ (I - R)^{-1}e = 1$. (23) Replacing the first column of the matrix multiplier of π_0 in equation (22) by the column vector multiplier of π_0 in (23), a matrix which is invertible may be obtained. The first row of the inverse of that same matrix is π_0 and this gives along with (21) all the stationary probabilities of the system. The matrix R given in (20) is computed by substitutions in the recurrence relation

$R(0) = 0;\ R(n+1) = -A_0A_1^{-1} - R^2(n)A_2A_1^{-1}$, $n \ge 0$. (24)

The iteration may be terminated to get a solution of R at an approximate level where $\|R(n+1) - R(n) < \varepsilon$ where ε is very small number.

2.3. Performance Measures

(1) The probability of the demand length L = r > 0, P (L = r), can be seen as follows. Let n ≥ 0 and j for 0 ≤ j ≤ M-1 be non-negative integers such that r = n M + j - S. Then using (21) (22) and (23) it is noted that P (L=r) $=\sum_{i=1}^{k} \pi\ (n, j, i\ )$, where r = n M + j - S.

(2) P (waiting demand length = 0) = P (L = 0) = $\sum_{j=0}^{S} \sum_{i=1}^{k} \pi$(0, j, i) and P (Inventory level is r) = P (INV= r) = $\sum_{i=1}^{k} \pi(0, S-r, i)$ for 0 ≤ r ≤ S.

(3) The expected demand length E (L), can be calculated as follows. Demand length L = 0 when there is stock in the inventory or when the inventory becomes empty without a waiting demand. They are described for various environments in (1) and with probabilities in the elements of π_0 by j for 0 ≤ j ≤ S for n=0. Now for L > 0, π (n, j, i) = P [L = M n + j - S, and environment state = i], for n ≥ 0, and 0 ≤ j ≤ M-1 and 1 ≤ i ≤ k, which shows E(L) = 0 P(L=0) + $\sum_{j=S+1}^{M-1} \sum_{i=1}^{k} \pi$(0,j,i) (j-S) + $\sum_{n=1}^{\infty} \sum_{j=0}^{M-1} \sum_{i=1}^{k} \pi(n, j, i)\ (Mn + j - S)$ $=\pi_0 \delta_1$ – S$\pi_0 \delta_2$+M $\sum_{n=1}^{\infty} n\, \pi_n\ e$ + $\sum_{n=1}^{\infty} \pi_n\ \delta_3$- S (1 - $\pi_0 e$) where $\delta_1, \delta_2, and\ \delta_3$ are type Mk x 1 column vectors defined as follows. δ_1= (0,0,…0,1,1,…1,2,2,…2,…,M-1-S,M-1-S,…,M-1-S)'. Here in the vector, the number 0 appears (S+1)k times, and the numbers 1,2,3,.., (M-1-S) appear k times one by one in order. The vector δ_2= (0,0,…,0,1,1…1)' where the number 0 appears (S+1)k times and the number 1 appears (M-1-S)k times. The vector δ_3= (0,0,…0,1,1,…1,2,2,…2,…,M-1, M-1, …, M-1)' where all the numbers 0 to M-1 appear k times. On simplification E (L) = $\pi_0 \delta_1$ – S$\pi_0 \delta_2$ +M π_0(I-R)$^{-2}$Re + π_0(I-R)$^{-1}$Rδ_3- S(1-π_0e). (25)

(4) Variance of the demand length can be seen using VAR (L) = E (L^2) – E(L)^2. Let δ_4 be column vector $\delta_4 = [0, .. 0, 1^2, \dots 1^2\ 2^2, .. 2^2, \dots\ (M-1-S)^2, \dots\ (M-1-S)^2]'$ of type Mkx1 where the number 0 appears (S+1)k times, and the square of numbers 1,2,3,.., (M-1-S) appear k times one by one in order and let $\delta_5 = [0, .. 0, 1^2, \dots 1^2\ 2^2, .. 2^2, \dots\ (M-1)^2, \dots\ (M-1)^2]'$ of type Mkx1 where the number 0 appears k times, and the square of numbers 1,2,3,.., (M-1) appear k times one by one in order. It can be seen that the second moment, E (L^2) $=\sum_{j=S+1}^{M-1} \sum_{i=1}^{k} \pi$(0, j, i) (j-S)^2 + $\sum_{n=1}^{\infty} \sum_{j=0}^{M-1} \sum_{i=1}^{k} \pi(n, j, i)\ [M\, n + j - S]^2$. Using Binomial expansion in the second series it may be noted E (L^2) $= \pi_0 \delta_4 + M^2 [\sum_{n=1}^{\infty} n(n-1) \pi_n\, e + \sum_{n=1}^{\infty} n \pi_n\ e] + \sum_{n=1}^{\infty} \pi_n\ \delta_5\ + 2M \sum_{n=1}^{\infty} n\, \pi_n \delta_3$-2S$\sum_{n=1}^{\infty} \sum_{j=0}^{M-1} \sum_{i=1}^{k} \pi(n, j, i)\ (Mn + j)$+ $S^2 \sum_{n=1}^{\infty} \sum_{j=0}^{M-1} \sum_{i=1}^{k} \pi(n, j, i)$. After simplification,

E(L^2)= $\pi_0 \delta_4$+$M^2 [\pi_0 (I-R)^{-3} 2R^2\ e + \pi_0 (I-R)^{-2} Re]$+$\pi_0 (I-R)^{-1} R \delta_5 + 2M\ \pi_0 (I-R)^{-2} R \delta_3$
-2S [M π_0(I-R)$^{-2}$Re + π_0(I-R)$^{-1}$Rδ_3] +S^2 (1-$\pi_0 e$)
Using (25) and (26) variance of L can be written. (26)

(5) The above partition method may also be used to study the case in which the supply for an order is a finite valued discrete random variable by suitably redefining the matrices U_j for 1 ≤ j ≤ N as presented in Rama Ganesan, Ramshankar and Ramanarayanan for M/M/1 bulk queues. [9].

III. Model.(B). Maximum Demand Size M is Less Than the Maximum Supply Size N

In this Model (B) the dual case of Model (A), namely the case, M < N is treated. (When M =N both models are applicable and one can use any one of them.) The assumption (v) of Model (A) is modified and all its other assumptions are unchanged.

3.1Assumption.

v) The maximum of the maximum demands sizes in all the environments M = $max_{1 \leq i \leq k}\ M_i$ is less than the maximum of the order clearance sizes in all the environments N =$max_{1 \leq i \leq k}\ N_i$ where the maximum demands and order clearance sizes are M_i and N_i respectively in the environment i for 1 ≤ i ≤ k.

3.2. Analysis

Since this model is dual, the analysis is similar to that of Model (A). The differences are noted below. The state space of the chain is defined as follows in a similar way.
X (t) = {(n, j, i): for 0 ≤ j ≤ N-1; 1 ≤ i ≤ k and n ≥ 0} (27)

The chain is in the state (0, j, i) when the S – j units are in the inventory for 0 ≤ j ≤ S and the environment state is i for 1 ≤ i ≤ k. The chain is in the state (0, j, i) when the inventory is empty and j-S demands are waiting for order realization where S+1≤ j ≤ N-1 and 1 ≤ i ≤ k. The chain is in the state (n, j, i) when the number of demands waiting for units is n N + j - S, for 0 ≤ j ≤ N-1, 1 ≤ n < ∞ and the environment state is i for 1 ≤ i ≤ k. When the number of demands in the system is r ≥ 1, then r is identified with (n, j) where r + S on division by N gives n as the quotient and j as the remainder. The infinitesimal generator Q_B of the model has the same block partitioned structure given in (4) for Model (A) but the inner matrices are of different orders and elements.

$$Q_B = \begin{bmatrix} B'_1 & A'_0 & 0 & 0 & . & . & . & \cdots \\ A'_2 & A'_1 & A'_0 & 0 & . & . & . & \cdots \\ 0 & A'_2 & A'_1 & A'_0 & 0 & . & . & \cdots \\ 0 & 0 & A'_2 & A'_1 & A'_0 & 0 & . & \cdots \\ 0 & 0 & 0 & A'_2 & A'_1 & A'_0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots \end{bmatrix} \quad (28)$$

In (28) the states of the matrix are listed lexicographically as $\underline{0}, \underline{1}, \underline{2}, \underline{3}, \ldots.\underline{n}, \ldots$.Here the state vector is given as follows. $\underline{n}$ = ((n, 0, 1),…(n, 0, k),(n, 1, 1),…(n, 1, k),(n, 2, 1),…(n, 2, k),…(n, N-1, 1),…(n, N-1, k)), for $0 \leq n < \infty$. The matrices, $B'_1,\ A'_0\ ,A'_1\ and\ A'_2$ are all of order Nk. The matrices $B'_1\ and\ A'_1$ have negative diagonal elements and their off diagonal elements are non- negative. The matrices $A'_0\ and\ A'_2$ have nonnegative elements. They are all given below. As in model (A), letting $\Lambda_j, for\ 1 \leq j \leq M, and\ U_j, V_j\ for\ 1 \leq j \leq N$, Λ and U as matrices of order k given by (3), (4), (5) and (6) and letting $Q_1' = Q_1 - \Lambda - U$, the partitioning matrices are defined as follows

$$A'_0 = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \Lambda_M & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \Lambda_{M-1} & \Lambda_M & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \Lambda_2 & \Lambda_3 & \cdots & \Lambda_M & 0 & 0 & \cdots & 0 \\ \Lambda_1 & \Lambda_2 & \cdots & \Lambda_{M-1} & \Lambda_M & 0 & \cdots & 0 \end{bmatrix} (29) \qquad A'_2 = \begin{bmatrix} U_N & U_{N-1} & U_{N-2} & \cdots & U_3 & U_2 & U_1 \\ 0 & U_N & U_{N-1} & \cdots & U_4 & U_3 & U_2 \\ 0 & 0 & U_N & \cdots & U_5 & U_4 & U_3 \\ 0 & 0 & 0 & \ddots & U_6 & U_5 & U_4 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & U_N & U_{N-1} & U_{N-2} \\ 0 & 0 & 0 & \cdots & 0 & U_N & U_{N-1} \\ 0 & 0 & 0 & \cdots & 0 & 0 & U_N \end{bmatrix} (30)$$

$$A'_1 = \begin{bmatrix} Q_1' & \Lambda_1 & \Lambda_2 & \cdots & \Lambda_M & 0 & 0 & \cdots & 0 & 0 \\ U_1 & Q_1' & \Lambda_1 & \cdots & \Lambda_{M-1} & \Lambda_M & 0 & \cdots & 0 & 0 \\ U_2 & U_1 & Q_1' & \cdots & \Lambda_{M-2} & \Lambda_{M-1} & \Lambda_M & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ U_{N-M-1} & U_{N-M-2} & U_{N-M-3} & \cdots & Q_1' & \Lambda_1 & \Lambda_2 & \cdots & \Lambda_{M-1} & \Lambda_M \\ U_{N-M} & U_{N-M-1} & U_{N-M-2} & \cdots & U_1 & Q_1' & \Lambda_1 & \cdots & \Lambda_{M-2} & \Lambda_{M-1} \\ U_{N-M+1} & U_{N-M} & U_{N-M-1} & \cdots & U_2 & U_1 & Q_1' & \cdots & \Lambda_{M-3} & \Lambda_{M-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ U_{N-2} & U_{N-3} & U_{N-4} & \cdots & U_{N-M-2} & U_{N-M-3} & U_{N-M-2} & \cdots & Q_1' & \Lambda_1 \\ U_{N-1} & U_{N-2} & U_{N-3} & \cdots & U_{N-M-1} & U_{N-M-2} & U_{N-M-1} & \cdots & U_1 & Q_1' \end{bmatrix} \quad (31)$$

$$B'_1 = \begin{bmatrix} Q_1 - \Lambda & \Lambda_1 & \cdots & \Lambda_{S-s-1} & \Lambda_{S-s} & \Lambda_{S-s+1} & \cdots & \Lambda_M & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & Q_1 - \Lambda & \cdots & \Lambda_{S-s-2} & \Lambda_{S-s-1} & \Lambda_{S-s} & \cdots & \Lambda_{M-1} & \Lambda_M & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & \Lambda_{S-s-3} & \Lambda_{S-s-2} & \Lambda_{S-s-1} & \cdots & \Lambda_{M-2} & \Lambda_{M-1} & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & Q_1 - \Lambda & \Lambda_1 & \Lambda_2 & \cdots & \Lambda_{M-(S-s)} & \Lambda_{M-(S-s-1)} & \cdots & \Lambda_M & 0 & \cdots & 0 & 0 \\ V_{S-s-1} & U_{S-s-1} & \cdots & U_1 & Q_1' & \Lambda_1 & \cdots & \Lambda_{M-(S-s+1)} & \Lambda_{M-(S-s)} & \cdots & \Lambda_{M-1} & \Lambda_M & \cdots & 0 & 0 \\ V_{S-s} & U_{S-s} & \cdots & U_2 & U_1 & Q_1' & \cdots & \Lambda_{M-(S-s+2)} & \Lambda_{M-(S-s+1)} & \cdots & \Lambda_{M-2} & \Lambda_{M-1} & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ V_{N-M-2} & U_{N-M-2} & \cdots & U_{N-M-S+s} & U_{N-M-S+s-1} & U_{N-M-S+s-2} & \cdots & Q_1' & \Lambda_1 & \cdots & \Lambda_{S-s} & \Lambda_{S-s+1} & \cdots & \Lambda_{M-1} & \Lambda_M \\ V_{N-M-1} & U_{N-M-1} & \cdots & U_{N-M-S+s+1} & U_{N-M-S+s} & U_{N-M-S+s-1} & \cdots & U_1 & Q_1' & \cdots & \Lambda_{S-s-1} & \Lambda_{S-s} & \cdots & \Lambda_{M-2} & \Lambda_{M-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots & \vdots & \vdots \\ V_{N-3} & U_{N-3} & \cdots & U_{N-S+s-1} & U_{N-S+s-2} & U_{N-S+s-3} & \cdots & U_{N-M-2} & U_{N-M-3} & \cdots & U_{N-M-S+s-2} & U_{N-M-S+s-3} & \cdots & Q_1' & \Lambda_1 \\ V_{N-2} & U_{N-2} & \cdots & U_{N-S+s} & U_{N-S+s-1} & U_{N-S+s-2} & \cdots & U_{N-M-1} & U_{N-M-2} & \cdots & U_{N-M-S+s-1} & U_{N-M-S+s-2} & \cdots & U_1 & Q_1' \end{bmatrix} (32)$$

$$Q_B'' = \begin{bmatrix} Q_1' + U_N & \Lambda_1 + U_{N-1} & \cdots & \Lambda_{M-1} + U_{N-M+1} & \Lambda_M + U_{N-M} & U_{N-M-1} & \cdots & U_2 & U_1 \\ U_1 & Q_1' + U_N & \cdots & \Lambda_{M-2} + U_{N-M+2} & \Lambda_{M-1} + U_{N-M+1} & \Lambda_M + U_{N-M} & \cdots & U_3 & U_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ U_{N-M-2} & U_{N-M-3} & \cdots & Q_1' + U_N & \Lambda_1 + U_{N-1} & \Lambda_2 + U_{N-2} & \cdots & \Lambda_M + U_{N-M} & U_{N-M-1} \\ U_{N-M-1} & U_{N-M-2} & \cdots & U_1 & Q_1' + U_N & \Lambda_1 + U_{N-1} & \cdots & \Lambda_{M-1} + U_{N-M+1} & \Lambda_M + U_{N-M} \\ \Lambda_M + U_{N-M} & U_{N-M-1} & \cdots & U_2 & U_1 & Q_1' + U_N & \cdots & \Lambda_{M-2} + U_{N-M+2} & \Lambda_{M-1} + U_{N-M+1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \Lambda_2 + U_{N-2} & \Lambda_3 + U_{N-3} & \cdots & U_{N-M-1} & U_{N-M-2} & U_{N-M-3} & \cdots & Q_1' + U_N & \Lambda_1 + U_{N-1} \\ \Lambda_1 + U_{N-1} & \Lambda_2 + U_{N-2} & \cdots & \Lambda_M + U_{N-M} & U_{N-M-1} & U_{N-M-2} & \cdots & U_1 & Q_1' + U_N \end{bmatrix} (33)$$

The basic generator (33) which is concerned with only the demands and supply is $Q_B'' = A'_0 + A'_1 + A'_2$. This is also block circulant. Using similar arguments given for Model (A) it can be seen that its probability vector is $\left(\frac{\pi}{N}, \frac{\pi}{N}, \frac{\pi}{N}, \ldots, \frac{\pi}{N}\right)$ and the stability condition remains the same. Following the arguments given for Model (A), one can find the stationary probability vector for Model (B) also in matrix geometric form. All performance measures given in section 2.3 including the expectation of waiting demands for supply and its variance for Model (B) have the same form as given in Model (A) except M is replaced by N.

IV. Numerical Illustration

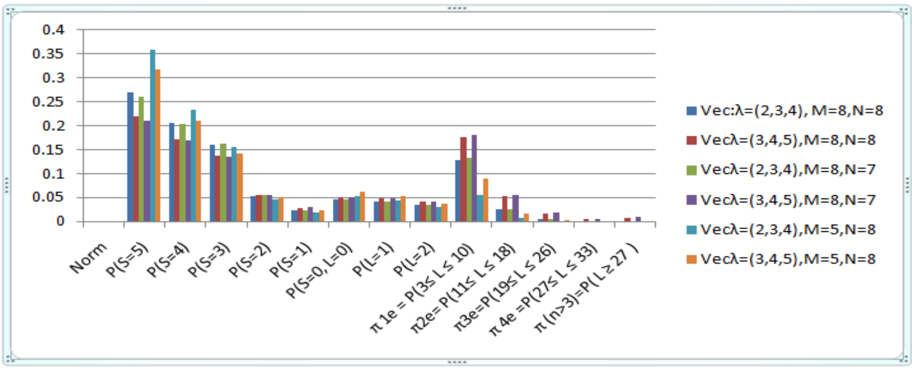
Six numerical examples are studied to exhibit the usefulness of this above study since so far bulk demand inventory models have not been treated at any depth. The results obtained in the examples are presented in table 1. The varying environment considered here has three states whose infinitesimal generator is $Q_1 = \begin{bmatrix} -3 & 1 & 2 \\ 2 & -3 & 1 \\ 1 & 2 & -3 \end{bmatrix}$. In the six cases the maximum capacity of the inventory and the ordering level are S=5 and s=2. The lead time parameter $\underline{\mu}$ = (3, 4, 5) for the three environments. The maximum demand size is 8 in the first four examples and the demand size probabilities in the three environments (i=1, 2, 3), P (χ_i= j) = p_j^i for 1≤ j ≤ 8 are given by p_1^1= .5, p_5^1= .4 and p_8^1= .1; p_1^2= .6, p_5^2= .3, p_8^2= .1; p_1^3=.7, p_5^3= .2, p_8^3= .1 and p_j^i = 0 for i = 1,2,3 and j ≠ 1,5,8. In the last two examples the maximum demand size is 5 and the demand size probabilities in the three environments (i=1,2,3), P(χ_i=j)=p_j^i for 1≤ j ≤ 5 are given by p_1^1= .5, p_5^1= .5 ; p_1^2= .6, p_5^2= .4 ; p_1^3=.7, p_5^3= .3 and p_j^i = 0 for i = 1,2,3 and j ≠ 1,5. The maximum supply for an order after the lead time is N= 8 in examples 1,2,5,6 and N=7 in examples 3 and 4. For three cases (M=8, N=8), (M=8, N=7) and (M=5, N=8) two sets arrival rates for bulk demands are considered as follows $\underline{\lambda}$= (2, 3, 4) and $\underline{\lambda}$ = (3, 4, 5) respectively.

Using the theory mentioned, fifteen iterations are performed to find the rate matrix R to present the probabilities and various performance measures. The various inventory level probabilities P(S=i) for 1 ≤ i ≤ 5, probability of empty stock and empty queue P(S=0,L=0), queue length probabilities for L=1, L=2 and for various ranges, the expected queue length of demands, the variances and standard deviations obtained are listed. Figure 1 compares the results obtained for the six examples.

Table1. Performance Measures

	$\underline{\lambda}$=(2,3,4), M=8,N=8	$\underline{\lambda}$=(3,4,5), M=8,N=8	$\underline{\lambda}$=(2,3,4), M=8,N=7	$\underline{\lambda}$ =(3,4,5), M=8,N=7	$\underline{\lambda}$=(2,3,4), M=5,N=8	$\underline{\lambda}$=(3,4,5), M=5,N=8
Norm	0	0.000001	0	0.000001	0	0
P(S=5)	0.269622	0.219981	0.26065	0.210089	0.359018	0.318294
P(S=4)	0.207264	0.17197	0.205028	0.168154	0.233524	0.209436
P(S=3)	0.161299	0.136563	0.162088	0.13585	0.156201	0.142097
P(S=2)	0.05346	0.054522	0.0553	0.055719	0.04548	0.049533
P(S=1)	0.023172	0.027688	0.024909	0.029284	0.017267	0.022354
P(S=0, L=0)	0.046097	0.049506	0.046132	0.0494	0.053424	0.061912
P(L=1)	0.042828	0.047583	0.043132	0.047672	0.04312	0.051824
P(L=2)	0.035658	0.041055	0.036269	0.041553	0.030437	0.037623
π 1e = P(3≤ L ≤ 10)	0.128616	0.175832	0.132342	0.180817	0.054615	0.088512
π2e= P(11≤ L ≤ 18)	0.025489	0.052471	0.027033	0.055906	0.0063	0.01569
π3e=P(19≤ L ≤ 26)	0.005175	0.015903	0.005649	0.017533	0.000554	0.00229
π 4e =P(27≤ L ≤ 33)	0.001052	0.004824	0.001182	0.005503	0.000056	0.000368
π (n>3)=P(L ≥ 27)	0.00132	0.006926	0.001495	0.008022	0.000062	0.000435
P(L ≥ 0)	0.285183	0.389276	0.292025	0.400903	0.18851	0.258285
P(1≤ S ≤5)	0.714817	0.610724	0.707975	0.599097	0.81149	0.741715
E(L)	0.914466	1.980259	0.971758	2.13345	0.122178	0.439724
VAR(L)	52.143939	99.496507	54.70922	106.202096	17.177379	33.288072
Std(L)	7.221076028	9.974793582	7.396568	10.30544	4.14456	5.769582

Figure1. Probabilities of Waiting Length of Demands.



V. Conclusion

The (S, s) inventory system with bulk demand under varying environment has been studied. The difficulties involved in finding the stationary demand length probabilities are resolved by considering the maximum of the demands and the supply sizes for partitions of the infinitesimal generator. Using the Matrix partition method of Neuts, the demand length probabilities have been presented for bulk demand (S, s) inventory system with randomly varying environment explicitly using the rate matrix. Various performance measures are derived. Two general models are presented considering the demand size is bigger than the supply size and the supply size is bigger than the demand size. Numerical examples are treated to illustrate the usefulness of the method. For future studies, models with catastrophic demands and supplies may present further useful results.

Acknowledgement

The first author thanks ANSYS Inc., USA, for providing facilities. The contents of the article published are the responsibilities of the authors.

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