IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 1 Ver. IV (Jan - Feb. 2015), PP 01-08
www.iosrjournals.org

On Some Types of Fuzzy Separation Axioms in Fuzzy Topological Space on Fuzzy Sets

Assist.Prof. Dr.Munir Abdul Khalik AL-Khafaji Gazwanhaider Abdul Hussein
AL-Mustinsiryah University \ College of Education \ Department ofMathematics

Abstract: *The aim of this paper to introduce and study fuzzy δ-open set and the relations of some other class of fuzzy open sets like (R-open set ,θ-open set , γ-open set , Δ-open set) , introduce and study some types of fuzzy δ-separation axioms in fuzzy topological space on fuzzy sets and study the relations between of themand study some properties and theorems on this subject*

I. Introduction

The concept of fuzzy set was introduced by Zedeh in his classical paper [1] in 1965.The fuzzy topological space was introduced by Chang [2] in 1968. Zahran [3] has introduced the concepts of fuzzy δ-open sets, fuzzy δ-closed sets, fuzzy regular open sets, fuzzy regular closed sets. And LuayA.Al.Swidi,AmedS.A.Oon [15] introduced the notion of γ-open set, fuzzy γ-closed set and studied some of its properties. N.V.Velicko[9] introduced the concept of fuzzy θ-open set, fuzzy θ-closed set,the fuzzy separation axioms was defined bySinha[10],And Ismail Ibedou[7]introduced anewsetting of fuzzy separation axioms. The purpose of the present paper is to introduce and study the concepts of fuzzy δ-open sets and some types of fuzzy open set and relationships between of them and study some types of fuzzy δ-separation axioms in fuzzy topological space on fuzzy sets and study the relationships between of them and we examine the validity of the standard results.

1 .fuzzy topological space on fuzzy set

Definition 1.1 [4]Let X be a non empty set, a fuzzy set $\tilde{A}$ in X is characterized bya function $\mu_{\tilde{A}}: X \rightarrow I$, where $I = [0,1]$ which is written as $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X, 0 \leq \mu_{\tilde{A}}(x) \leq 1\}$, the collection of all fuzzy sets in X will be denoted by I^X,that is $I^X = \{ \tilde{A} : \tilde{A}$ is a fuzzy sets in X$\}$ where $\mu_{\tilde{A}}$ is called the membership function .

Proposition1.2 **[5]**

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy sets in X with membership functions$\mu_{\tilde{A}}$ and $\mu_{\tilde{B}}$respectively then for all $x \in X$: -

1. $\tilde{A} \subseteq \tilde{B} \leftrightarrow \mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x)$.
2. $\tilde{A} = \tilde{B} \leftrightarrow \mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x)$.
3. $\tilde{C} = \tilde{A} \cap \tilde{B} \leftrightarrow C(x) = \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}$.
4. $\tilde{D} = \tilde{A} \cup \tilde{B} \leftrightarrow D(x) = \max\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}$.
5. $\tilde{B}^c$the complement of $\tilde{B}$ with membership function$\mu_{\tilde{B}^c}(x) = \mu_{\tilde{A}}(x) - \mu_{\tilde{B}}(x)$.

Definition1.3 [4]

A fuzzy point x_r is a fuzzy set such that :

$\mu_{x_r}(y) = r > 0$ if $x = y$, $\forall y \in X$ and $\mu_{x_r}(y) = 0$ if $x \neq y$, $\forall y \in X$

The family of all fuzzy points of $\tilde{A}$ will be denoted by $FP(\tilde{A})$.

Remark 1.4 [6] : Let $\tilde{A} \in I^X$ then $P(\tilde{A}) = \{ \tilde{B} : \tilde{B} \in I^X, \mu_{\tilde{B}}(x) \leq \mu_{\tilde{A}}(x) \} \forall x \in X$.

Definition 1.5 [4]

A collection $\tilde{T}$ of a fuzzy subsets of $\tilde{A}$, such that $\tilde{T} \subseteq P(\tilde{A})$ is said to be fuzzy topology on $\tilde{A}$ if it satisfied the following conditions

1. $\tilde{A}, \tilde{\phi} \in \tilde{T}$
2. If $\tilde{B}, \tilde{C} \in \tilde{T}$ then $\tilde{B} \cap \tilde{C} \in \tilde{T}$
3. If $\tilde{B}_j \in \tilde{T}$ then $\cup_j \tilde{B}_j \in \tilde{T}$, $j \in J$

$(\tilde{A}, \tilde{T})$ is said to be Fuzzy topological space and every member of $\tilde{T}$ is called fuzzy open set in $\tilde{A}$ and its complement is a fuzzy closed set .

2. On some types of fuzzy open set

Definition 2.1 [8,11,12,13,14]

A fuzzy set B̃ in a fuzzy topological space (Ã ,T̃) is said to be

1) Fuzzy δ-open [resp.Fuzzyδ-closed set] set if $\mu_{Int(Cl(\tilde{B}))}(x) \leq \mu_{\tilde{B}}(x)$ $[\mu_{\tilde{B}}(x) \leq \mu_{Cl(Int(\tilde{B}))}(x)]$The family of all fuzzy δ-open sets [resp. fuzzy δ-closed sets] in a fuzzy topological space (Ã ,T̃) will be denoted by FδO(Ã) [resp. FδC(Ã)]

2) Fuzzy regular open [Fuzzy regular closed] set if : $\mu_{\tilde{B}}(x)= \mu_{Int(Cl(\tilde{B}))}(x)[\mu_{\tilde{B}}(x)= \mu_{Cl(Int(\tilde{B}))}(x)]$, The family of all fuzzy regular open [fuzzy regular closed]set in Ã will be denoted by FRO(Ã)[FRC(Ã)].

3) Fuzzy Δ-open set if for every point $x_r \in$ B̃ there exist a fuzzy regular semi-open set $\tilde{U}$ in Ã such that $\mu_{x_r}(x) \leq \mu_{\tilde{U}}(x) \leq \mu_{\tilde{B}}(x)$, B̃ is called [Fuzzy Δ-closed] set if its complement is FuzzΔ-open set the family of all Fuzzy Δ-open [Fuzzy Δ-closed] sets in Ã will be denoted by FΔO(Ã)[FΔC(Ã)].

4) Fuzzy $\gamma - open$ [$\gamma - closed$] set if $\mu_{\tilde{B}}(x) \leq$max $\{\mu_{Int(Cl(\tilde{B}))}(x), \mu_{Cl(int(\tilde{B}))}(x)\}$, $[\mu_{\tilde{B}}(x) \geq$ min $\{\mu_{Int(Cl(\tilde{B}))}(x), \mu_{Cl(int(\tilde{B}))}(x)\}]$ The family of all fuzzy γ – open [fuzzy γ – closed] sets in Ã will be denoted by FγO(Ã) [FγC(Ã)].

5) Fuzzy θ-open [θ-closed] set if $\mu_{\tilde{B}}(x) = \mu_{\theta Int(\tilde{B})}(x)$, [$\mu_{\tilde{B}}(x) = \mu_{\theta Cl(\tilde{B})}(x)$]

The family of all fuzzy θ-open (fuzzy θ-closed) sets in Ã will be denoted by FθO(Ã) [FθC(Ã)].

Proposition 2.2

Let (Ã , T̃) be a fuzzy topological space then :

1) Every fuzzy δ-open set (resp. fuzzy δ-closed set) is fuzzy Δ-open set (resp.fuzzy Δ-closed set) **[**fuzzy γ-open set (resp.fuzzy γ-closed set) **]**.

2) Every fuzzyθ-open set (resp. fuzzy θ-closed set) is fuzzy γ-open set (resp. fuzzy γ-closed set) **[**fuzzy δ-open set (resp. fuzzy δ-closed set , fuzzy Δ-open set (resp.fuzzy Δ-closed set) **]**

3) Every fuzzy regular open set (fuzzy regular closed set) is fuzzy δ-open set (resp. fuzzy δ-closed set) **[**fuzzy γ-open set(resp. fuzzy γ-closed set) , fuzzyΔ-open set(resp.fuzzy Δ-closed set)**]**

Proof : Obvious .

Remark 2.3

The converse of proposition (2.2) is not true in general as following examples shows

Examples 2.4

1) Let X = { a , b } and $\tilde{B}$, $\tilde{C}$, $\tilde{D}$ are fuzzy subset in $\tilde{A}$ where $\tilde{A}$ = {(a , 0.9) , (b , 0.9) } , $\tilde{B}$= { (a , 0.0) , (b , 0.7) } ,$\tilde{C}$ = { (a , 0.8) , (b , 0.0) },$\tilde{D}$ = { (a , 0.8) , (b , 0.7) , The fuzzy topology defined on $\tilde{A}$ is T̃ = { $\emptyset$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{D}$ }

- The fuzzy set $\tilde{D}$ is a fuzzy Δ-open set but not fuzzyδ – open set (fuzzy regular open set , fuzzy θ – open set).
- let X = { a , b , c } and $\tilde{B}$, $\tilde{C}$, $\tilde{D}$, $\tilde{E}$are fuzzy subset in $\tilde{A}$ where $\tilde{A}$ = {(a , 0.9) , (b , 0.9) , (c , 0.9) } , $\tilde{B}$= { (a , 0.3) , (b , 0.3) , (c , 0.4) }$\tilde{C}$ = { (a , 0.4) , (b , 0.3) , (c , 0.4) } , $\tilde{D}$ = { (a , 0.5) , (b , 0.5) , (c , 0.4) }$\tilde{E}$ = { (a , 0.6) , (b , 0.6) , (c , 0.7), The fuzzy topology defined on $\tilde{A}$ is T̃ = { $\emptyset$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{D}$, $\tilde{E}$ }
- The fuzzy set $\tilde{B}$ is a fuzzy γ – open set but not fuzzy δ – open set (fuzzy regular open set , fuzzy θ – open set).

2) Let X ={ a , b , c } and B̃ , C̃ , D̃ , Ẽ , $\tilde{F}$ be fuzzy subsets of Ã where: Ã = {(a ,0.8) ,(b ,0.8) , (c ,0.8)} , B̃ ={(a ,0.1) , (b ,0.1) , (c ,0.2)} , C̃ ={(a ,0.2) , (b , 0.1) , (c ,0.2)}$\tilde{D}$= {(a ,0.3) , (b ,0.3) , (c ,0.2)} ,Ẽ ={(a ,0.4) , (b ,0.4) , (c ,0.5)} ,$\tilde{F}$= {(a ,0.3) , (b ,0.3) , (c ,0.3)} The fuzzy topologies defined on Ã are T̃ = { $\tilde{\phi}$, Ã , B̃ , C̃ , D̃ }The fuzzy set $\tilde{E}$ is a fuzzy δ-open set but not fuzzy regular open set (fuzzy θ-open set).

Remark 2.5

Figure - 1 – illustrates the relation between fuzzy δ-open set and some types of fuzzy open sets.

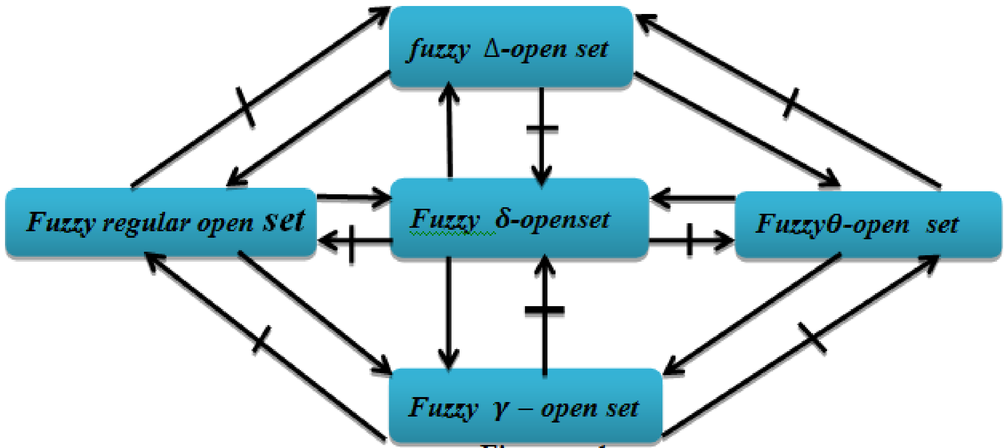

Figure - 1 –

III. Some Types Of Fuzzy Separation Axioms

Definition 3.1

A fuzzy topological space (Ã , T̃) is said to be **Fuzzy $\delta\,\tilde{T}_0$ − space(F$\delta\tilde{T}_0$)** if for every pair of distinct fuzzy points x_r , y_t in Ã there exist B̃ ∈ FδO(Ã) such that either $\mu_{x_r}(x) < \mu_{\tilde{B}}(x)$, $y_t \tilde{\bar{q}}$B̃ or $\mu_{y_t}(y) < \mu_{\tilde{B}}(y)$, $x_r \tilde{\bar{q}}$B̃.

Theorem 3.2

If (Ã , T̃) is a fuzzy$\delta\tilde{T}_0$- space then for every pair of distinct fuzzy points x_r ,$y_t$where $\mu_{x_r}(x) < \mu_{\tilde{A}}(x)$, $\mu_{y_t}(x) < \mu_{\tilde{A}}(x)$ then either δ- cl(x_r) $\tilde{\bar{q}}\, y_t$ or δ- cl(y_t) $\tilde{\bar{q}}\, x_r$.

Proof: -

Let x_r ,y_t be two distinct fuzzy points such that

$\mu_{x_r}(x) < \mu_{\tilde{A}}(x)$, $\mu_{y_t}(x) < \mu_{\tilde{A}}(x)$ then there exist a fuzzy δ- open set B̃ such that either $\mu_{x_r}(x) < \mu_{\tilde{B}}(x)$, B̃ $\tilde{\bar{q}}\, y_t$ or $\mu_{y_t}(x) < \mu_{\tilde{B}}(x)$, B̃ $\tilde{\bar{q}}\, x_r$

If $\mu_{x_r}(x) < \mu_{\tilde{B}}(x)$, $\tilde{B}\,\tilde{\bar{q}}\, y_t$ then $\tilde{B}^c\,\tilde{\bar{q}}\, x_r$, $\mu_{y_t}(x) \le \mu_{\tilde{B}^c}(x)$

Since $\tilde{B}^c$ is a fuzzy δ- closed set therefore $\mu_{\delta cl\,(y_t)}(x) \le \mu_{\tilde{B}^c}(x)$

Hence δ- cl(y_t) $\tilde{\bar{q}}\, x_r$

Similarly if $\mu_{y_t}(x) < \mu_{\tilde{B}}(x)$, B̃ $\tilde{\bar{q}}\, x_r$ ■

Definition 3.3 :

A fuzzy topological space (Ã , T̃) is said to be **Fuzzy $\delta\,\tilde{T}_1$ − space (F$\delta\tilde{T}_1$)** if for every pair of distinct fuzzy points x_r , y_t in Ã there exist two B̃ , $\tilde{C}$ ∈ FδO(Ã) such that $\mu_{x_r}(x) < \mu_{\tilde{B}}(x)$, $y_t \tilde{\bar{q}}$B̃ and $\mu_{y_t}(y) < \mu_{\tilde{C}}(y)$, $x_r\,\tilde{\bar{q}}\,\tilde{C}$.

Proposition 3.4 :

Every fuzzy $\delta\tilde{T}_1$ – space is a fuzzy $\delta\tilde{T}_0$ – space .

Proof : Obvious .

Remark 3.5 :

The converse of proposition (3.4) is not true in general as shown in the following example .

Example 3.6 :

Let X={ a , b } and $\tilde{B}$, $\tilde{C}$, $\tilde{D}$ are fuzzy subset of $\tilde{A}$ where: $\tilde{A}$ = {(a , 0.4) , (b , 0.4)} , $\tilde{B}$ = {(a , 0.4) , (b , 0.1)} ,$\tilde{C}$ = {(a , 0.1) , (b , 0.1)} , $\tilde{D}$ = {(a , 0.4) , (b , 0.2)} ,$\tilde{E}$ = {(a , 0.3) , (b , 0.1)}, $\tilde{T}$ = { $\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{D}$ } be a fuzzy topology on $\tilde{A}$ and the FδO($\tilde{A}$) = {$\tilde{\emptyset}$, $\tilde{A}$, $\tilde{C}$, $\tilde{E}$ } Then the space ($\tilde{A}$, $\tilde{T}$) is a fuzzy $\delta\tilde{T}_0$ - space but notfuzzy $\delta\tilde{T}_1$ – space

Theorem 3.7:

If (Ã , T̃) is a fuzzy Topological space then the following statements are equivalents :

1) (Ã , T̃) is a fuzzy $\delta\tilde{T}_1$ - space.

2) For every maximal fuzzy points x_r , y_t in Ã , there exists a fuzzy open nbhds sets Ũ and Ṽ o f x_r and y_t respectively in Ã such that $\mu_{x_r}(x)$= min {{ $\mu_{\tilde{U}}(x)$,$\mu_{\tilde{U}}(y)$}, { $\mu_{x_r}(x)$, $\mu_{y_t}(y)$}} and $\mu_{y_t}(y)$= min {{ $\mu_{(\tilde{V})}(x)$, $\mu_{(\tilde{V})}(y)$ } , { $\mu_{x_r}(x)$, $\mu_{y_t}(y)$ }}.
3) For every maximal fuzzy points x_r , y_t in Ã , there exists a fuzzy δ-open nbhds sets Ũ and Ṽ of x_r and y_t respectively in Ã such that $\mu_{x_r}(x)$= min {{ $\mu_{\tilde{U}}(x)$,$\mu_{\tilde{U}}(y)$}, { $\mu_{x_r}(x)$, $\mu_{y_t}(y)$}} and $\mu_{y_t}(y)$= min {{ $\mu_{(\tilde{V})}(x)$, $\mu_{(\tilde{V})}(y)$ } , { $\mu_{x_r}(x)$, $\mu_{y_t}(y)$ }}.

Proof:
(1⟹ 2) :- Let x_r , $y_t \in$ MFP(Ã) , ∃ Ũ , Ṽ ∈ FδO(Ã)
such that $\mu_{x_r}(x) < \mu_{\tilde{U}}(x)$, $y_t \tilde{\bar{q}}$Ũ and $\mu_{y_t}(y) < \mu_{(\tilde{V})}(y)$, $x_r \tilde{\bar{q}}$ Ṽ.then$\mu_{x_r}(x)$=$\mu_{\tilde{U}}(x)$=$\mu_{\tilde{A}}(x)$, $\mu_{y_t}(y)$+$\mu_{\tilde{U}}(y) \leq \mu_{\tilde{A}}(y)$ and
$\mu_{y_t}(y)$=$\mu_{(\tilde{V})}(y)$=$\mu_{\tilde{A}}(y)$, $\mu_{x_r}(x)$+ $\mu_{(\tilde{V})}(x) \leq \mu_{\tilde{A}}(x)$then $\mu_{\tilde{U}}(y)$= 0 , $\mu_{(\tilde{V})}(x)$= 0 , and since Ũ , Ṽ ∈ FδO(Ã) then Ũ , Ṽ ∈ T̃
Therefore $\mu_{x_r}(x)$= min {{ $\mu_{\tilde{U}}(x)$,$\mu_{\tilde{U}}(y)$}, { $\mu_{x_r}(x)$, $\mu_{y_t}(y)$}}
and$\mu_{y_t}(y)$= min {{ $\mu_{(\tilde{V})}(x)$, $\mu_{(\tilde{V})}(y)$ } , { $\mu_{x_r}(x)$, $\mu_{y_t}(y)$ }}.
(2⟹ 3) :- Obvious.
(3 ⟹ 1) :- Let x_n , $y_m \in$ FP(Ã) **,** then every x_r , $y_t \in$ MFP(Ã) , there exist Ũ , Ṽ ∈ FδO(Ã) such that
$\mu_{x_r}(x)$= min {{ $\mu_{\tilde{U}}(x)$,$\mu_{\tilde{U}}(y)$}, { $\mu_{x_r}(x)$, $\mu_{y_t}(y)$}} and
$\mu_{y_t}(y)$= min {{ $\mu_{(\tilde{V})}(x)$, $\mu_{(\tilde{V})}(y)$ } , { $\mu_{x_r}(x)$, $\mu_{y_t}(y)$ }}.
then$\mu_{x_r}(x)$=$\mu_{\tilde{U}}(x)$=$\mu_{\tilde{A}}(x)$, $\mu_{\tilde{U}}(y)$= 0 and
$\mu_{y_t}(y)$=$\mu_{(\tilde{V})}(y)$= $\mu_{\tilde{A}}(y)$, $\mu_{(\tilde{V})}(x)$= 0
then$y_t \tilde{\bar{q}}$Ũ and $x_r \tilde{\bar{q}}$ Ṽ**,** Since $\mu_{x_n}(x) < \mu_{x_r}(x)$ and $\mu_{y_m}(y) < \mu_{y_t}(y)$, ∀ n , m ∈ I
then$\mu_{x_n}(x) < \mu_{\tilde{U}}(x)$, $y_m \tilde{\bar{q}}$Ũ and $\mu_{y_m}(y) < \mu_{(\tilde{V})}(y)$, $x_n \tilde{\bar{q}}$Ṽ Hence the space (Ã , T̃) is a fuzzy $\delta\tilde{T}_1$- space. ■

Definition 3.8:
A fuzzy topological space (Ã , T̃) is said to be **Fuzzy $\boldsymbol{\delta \tilde{T}_2 -}$ space (F$\boldsymbol{\delta\tilde{T}_2}$)** if for every pair of distinct fuzzy points x_r , y_t in Ã there exist two B̃ , $\tilde{C}$ ∈ FδO(Ã) such that $\mu_{x_r}(x) < \mu_{\tilde{B}}(x)$, $\mu_{y_t}(y) < \mu_{\tilde{C}}(y)$ and $\tilde{B}\tilde{\bar{q}}\,\tilde{C}$.

Theorem 3.9 :
A fuzzy topological space $(\tilde{A}, \tilde{T})$ is a fuzzy δ-$\tilde{T}_2$ –space if and only if min{$\mu_{\delta cl(\tilde{U})}$(x) : $\tilde{U}$ is a fuzzy δ-open set ,$\mu_{x_r}(x) < \mu_{\tilde{U}}(x)$ } $< \mu_{y_t}(x)$ any fuzzy point such that $\mu_{y_t}(x) < \mu_{\tilde{A}}(x)$.

Proof :
(⟹) Let $(\tilde{A}, \tilde{T})$ be a fuzzy δ-$\tilde{T}_2$ –space and x_r , y_t be a distinct fuzzy points in $\tilde{A}$ such that $\mu_{x_r}(x) < \mu_{\tilde{A}}(x)$, $\mu_{y_t}(x) < \mu_{\tilde{A}}(x)$ and
$\mu_{y_n}(x) = \mu_{\tilde{A}}(x)$ - $\mu_{y_t}(x)$then$\mu_{y_n}(x) < \mu_{\tilde{A}}(x)$ and there exists two fuzzy δ-open set $\tilde{U}$, $\tilde{G}$ in $\tilde{A}$ such that $\mu_{x_r}(x) < \mu_{\tilde{U}}(x)$, $\mu_{y_n}(x) < \mu_{\tilde{G}}(x)$, $\tilde{U}\tilde{\bar{q}}\,\tilde{G}$, $\mu_{\tilde{U}}(x) \leq \mu_{\tilde{G}^c}(x)$ and
$\mu_{\delta cl(\tilde{U})}(x) \leq \mu_{\delta cl(\tilde{G}^c)}(x) = \mu_{\tilde{G}^c}(x)$
Since $\mu_{y_n}(x) < \mu_{\tilde{G}}(x)$ and $\mu_{y_n}(x) = \mu_{\tilde{A}}(x)$ - $\mu_{y_t}(x)$,Then
$\mu_{\tilde{A}}(x)$ - $\mu_{y_t}(x) < \mu_{\tilde{G}}(x)$,$\mu_{\tilde{G}^c}(x) < \mu_{\tilde{A}}(x)$ - $\mu_{y_n}(x)$ and $\mu_{\tilde{G}^c}(x) < \mu_{y_t}(x)$
Since $\mu_{\delta cl(\tilde{U})}(x) \leq \mu_{\tilde{G}^c}(x) < \mu_{y_t}(x)$ then $\mu_{\delta cl(\tilde{U})}(x) < \mu_{y_t}(x)$
Hence min{$\mu_{\delta cl(\tilde{U}i)}$(x) : i = 1,……, n }< $\mu_{y_t}(x)$
(⟸) Suppose that given condition hold ,x_r , y_t are distinct fuzzy points in $\tilde{A}$ such that $\mu_{x_r}(x) < \mu_{\tilde{A}}(x)$, $\mu_{y_t}(x) < \mu_{\tilde{A}}(x)$ and
Let $\mu_{y_n}(x) = \mu_{\tilde{A}}(x)$ - $\mu_{y_t}(x)$ then $\mu_{y_n}(x) < \mu_{\tilde{A}}(x)$
And $\mu_{\delta cl(\tilde{U})}(x) < \mu_{y_n}(x)$ for every $\mu_{x_r}(x) < \mu_{\tilde{U}}(x) \leq \mu_{\delta cl(\tilde{U})}(x)$
, since $\mu_{y_n}(x) = \mu_{\tilde{A}}(x)$ - $\mu_{y_t}(x)$
Then $\mu_{\delta cl(\tilde{U})}(x) < \mu_{\tilde{A}}(x)$ - $\mu_{y_t}(x)$ and $\mu_{\tilde{A}}(x)$ - $\mu_{y_n}(x) < \mu_{\delta int(\tilde{U}^c)}(x)$hence $\mu_{y_t}(x) < \mu_{\delta int(\tilde{U}^c)}(x)$
let$\mu_{\tilde{G}}(x)$ = $\mu_{\delta int(\tilde{U}^c)}(x)$ and since $\mu_{\delta int(\tilde{U}^c)}(x) \leq \mu_{(\tilde{U}^c)}(x)$,Then
$\mu_{y_t}(x) < \mu_{\tilde{G}}(x)and\mu_{\tilde{G}}(x) \leq \mu_{(\tilde{U}^c)}(x)$ we get $\tilde{U}\tilde{\bar{q}}\,\tilde{G}$
Hence the space $(\tilde{A}, \tilde{T})$ is a fuzzy δ-$\tilde{T}_2$ –space ■

Proposition 3.10 :
Every fuzzy $\delta\tilde{T}_2$ – space is a fuzzy $\delta\tilde{T}_1$ – space .
Proof : Obvious .

Remark 3.11 :
The converse of proposition (3.10) is not true in general as shown in the following example .
Example 3.12 :
Let X={ a , b } and $\tilde{B}$, $\tilde{C}$, $\tilde{D}$,$\tilde{E}$ are fuzzy subset of $\tilde{A}$ where: $\tilde{A}$ = {(a , 0.7) , (b , 0.9)},$\tilde{B}$ = {(a , 0.5) , (b , 0.0)} ,$\tilde{C}$ = {(a , 0.0) , (b , 0.7)} ,$\tilde{D}$ = {(a , 0.5) , (b , 0.7)},$\tilde{E}$= {(a , 0.1) , (b , 0.8)} , $\tilde{F}$= {(a , 0.6) , (b , 0.1)} , $\tilde{T}$ = { $\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$,$\tilde{D}$} be a fuzzy topology on $\tilde{A}$ and the FδO($\tilde{A}$) = {$\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{D}$, $\tilde{E}$,$\tilde{F}$} then the space ($\tilde{A}$, $\tilde{T}$) is a fuzzy $\delta\tilde{T}_1$ - space but not fuzzy$\delta\tilde{T}_2$ – space

Definition 3.13:
A fuzzy topological space (Ã , T̃) is said to be **Fuzzy $\boldsymbol{\delta\,\tilde{T}_{2\frac{1}{2}}}$ − space (F$\boldsymbol{\delta\tilde{T}_{2\frac{1}{2}}}$)** if for every pair of distinct fuzzy points x_r , y_t in Ã there exist two B̃ , $\tilde{C}$ ∈ FδO(Ã) such that $\mu_{x_r}(x) < \mu_{\tilde{B}}(x)$, $\mu_{y_t}(x) < \mu_{\tilde{C}}(x)$ and $\delta cl(\tilde{B})\,\tilde{\bar{q}}\,\delta cl(\tilde{C})$.

Proposition 3.14 :
Every fuzzy $\delta\tilde{T}_{2\frac{1}{2}}$- space is a fuzzy $\delta\tilde{T}_2$- space .

Proof :
Let ($\tilde{A}$, $\tilde{T}$) be a fuzzy $\delta\tilde{T}_{2\frac{1}{2}}$- space , then every pair of distinct fuzzy points x_r , y_t in Ã there exist two B̃ , $\tilde{C}$ ∈ FδO(Ã) such that $\mu_{x_r}(x) < \mu_{\tilde{B}}(x)$,$\mu_{y_t}(x) < \mu_{\tilde{C}}(x)$ and $\delta cl(\tilde{B})\,\tilde{\bar{q}}\,\delta cl(\tilde{C})$
Since $\mu_{\tilde{B}}(x) \le \delta cl(\tilde{B})$, $\mu_{\tilde{C}}(x) \le \delta cl(\tilde{C})$
Then We get $\tilde{B}\,\tilde{\bar{q}}\,\tilde{C}$, hence ($\tilde{A}$, $\tilde{T}$) is a fuzzy $\delta\tilde{T}_2$- space

Remark 3.15 :
The converse of proposition (3.14) is not true in general as shown in the following example

.Example 3.16 :
Let X={ a , b } and , $\tilde{B}_1$, $\tilde{B}_2$, $\tilde{B}_3$, $\tilde{B}_4$,$\tilde{B}_5$, $\tilde{B}_6$, $\tilde{B}_7$, $\tilde{B}_8$, are fuzzy subset of $\tilde{A}$ where: $\tilde{A}$ = {(a , 0.9) , (b , 0.9)} , $\tilde{B}_1$ = {(a , 0.8) , (b , 0.1)} , $\tilde{B}_2$ = {(a , 0.0) , (b , 0.7)} , $\tilde{B}_3$ = {(a , 0.8) , (b , 0.7)}, $\tilde{B}_4$ = {(a , 0.0) , (b , 0.1)} , $\tilde{B}_5$ = {(a , 0.0) , (b , 0.9)} , $\tilde{B}_6$ = {(a , 0.8) , (b , 0.9)} , $\tilde{B}_7$ = {(a , 0.0) , (b , 0.8)} $\tilde{B}_8$ = {(a , 0.8) , (b , 0.0)} , $\tilde{T}$ = { $\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}_1$, $\tilde{B}_2$, $\tilde{B}_3$, $\tilde{B}_4$,$\tilde{B}_5$, $\tilde{B}_6$ } be a fuzzy topology on $\tilde{A}$ and the FδO($\tilde{A}$) = {$\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}_1$, $\tilde{B}_2$, $\tilde{B}_4$, $\tilde{B}_7$, $\tilde{B}_8$ }, then the space ($\tilde{A}$, $\tilde{T}$) is a fuzzy $\delta\tilde{T}_2$- space but not fuzzy $\delta\tilde{T}_{2\frac{1}{2}}$ –space

Definition 3.17: A fuzzy topological space (Ã , T̃) is said to be **Fuzzy $\boldsymbol{\delta}$- regular space (F$\boldsymbol{\delta}$R)** if for each fuzzy point x_r in Ã and each fuzzy closed set $\tilde{F}$ with $x_r\,\tilde{\bar{q}}\,\tilde{F}$ there exists $\tilde{B}$, $\tilde{C}$ ∈FδO(Ã) such that $\mu_{x_r}(x) \le \mu_{\tilde{B}}(x)$, $\mu_{\tilde{F}}(x) \le \mu_{\tilde{C}}(x)$∀ x ∈ X and $\tilde{B}\,\tilde{\bar{q}}\,\tilde{C}$

Definition 3.18 :
A fuzzy topological space (Ã , T̃) is said to be **fuzzy $\boldsymbol{\delta^*}$- regular space (F$\boldsymbol{\delta^*}$R)** if for each fuzzy point x_r in Ã and each fuzzy δ- closed set $\tilde{F}$ with $x_r\,\tilde{\bar{q}}\,\tilde{F}$ there exists$\tilde{B}$, $\tilde{C}$ ∈FδO(Ã) such that $\mu_{x_r}(x) \le \mu_{\tilde{B}}(x)$, $\mu_{\tilde{F}}(x) \le \mu_{\tilde{C}}(x)$∀ x ∈ X and $\tilde{B}\,\tilde{\bar{q}}\,\tilde{C}$

Proposition 3.19 :
Every fuzzy δ- regular space is a fuzzy δ^*- regular space.

Proof: Obvious .

Remark 3.20 :
The converse of proposition (3.19) is not true in general as shown in the following example
Example 3.21 :
Let X={ a , b } and $\tilde{B}$, $\tilde{C}$, $\tilde{D}$,$\tilde{E}$, $\tilde{F}$ is a fuzzy subset of $\tilde{A}$ where:
$\tilde{A}$ = {(a , 0.7) , (b , 0.8)} , $\tilde{B}$ = {(a , 0.0) , (b , 0.7)} ,$\tilde{C}$ = {(a , 0.6) , (b , 0.0)} , $\tilde{D}$ = {(a , 0.6) , (b , 0.7)}, $\tilde{E}$ = {(a , 0.7) , (b , 0.0)} , $\tilde{F}$ = {(a , 0.0) , (b , 0.8)}

$\tilde{T}$ = {$\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{D}$ }be a fuzzy topology on $\tilde{A}$ and the FδO($\tilde{A}$) = {$\tilde{\emptyset}$, $\tilde{A}$,$\tilde{B}$, $\tilde{C}$, $\tilde{D}$,$\tilde{E}$, $\tilde{F}$ } Then the space ($\tilde{A}$, $\tilde{T}$) is a fuzzyδ^*- regular space but not fuzzy δ- regular space.

Definition 3.22:A fuzzy topological space (Ã , T̃) is said to be **Fuzzy $\delta\,\tilde{T}_3 -$ space (F$\delta\tilde{T}_3$)** if it isδ- regular space (FδR)as well as fuzzy $\delta\,\tilde{T}_1 -$ space (F$\delta\tilde{T}_1$) .

Proposition 3.23 :
Every fuzzy $\delta\tilde{T}_3$- space is a fuzzy $\delta\tilde{T}_{2\frac{1}{2}}$- space .**Proof :**

Let ($\tilde{A}$, $\tilde{T}$) be a fuzzy $\delta\tilde{T}_3$ - space ,
Then ($\tilde{A}$, $\tilde{T}$) be a fuzzy δ - regular space, for every fuzzy point x_r ∈ FP(Ã) and $\tilde{F}$ ∈ FC(Ã)
Such that $x_r \,\tilde{\bar{q}}\, \tilde{F}$, $\tilde{F} = \delta cl(\tilde{F})$
And since ($\tilde{A}$, $\tilde{T}$) be a fuzzy $\delta\tilde{T}_1$ - space then We get $\{x_r\}$ is a fuzzy δ-closed set
Let $\{x_r\} = \tilde{B}$is a fuzzy δ-closed set
Then $\delta cl(\tilde{B})\,\tilde{\bar{q}}\,\delta cl(\tilde{F})$, hence ($\tilde{A}$, $\tilde{T}$) is a fuzzy $\delta\tilde{T}_{2\frac{1}{2}}$- space■

Remark 3.24 :The converse of proposition (3.23) is not true in general as shown in the following example .
Example 3.25 : Let X={ a , b } and , $\tilde{B}_1$, $\tilde{B}_2$, $\tilde{B}_3$, $\tilde{B}_4$,$\tilde{B}_5$, $\tilde{B}_6$, $\tilde{B}_7$, $\tilde{B}_8$, $\tilde{B}_9$, $\tilde{B}_{10}$, $\tilde{B}_{11}$, are fuzzy subset of $\tilde{A}$ where:$\tilde{A}$ = {(a , 0.8) , (b , 0.9)} , $\tilde{B}_1$ = {(a , 0.8) , (b , 0.0)} , $\tilde{B}_2$ = {(a , 0.0) , (b , 0.7)} , $\tilde{B}_3$ = {(a , 0.8) , (b , 0.7)}, $\tilde{B}_4$ = {(a , 0.1) , (b , 0.9)} , $\tilde{B}_5$ = {(a , 0.6) , (b , 0.0)} , $\tilde{B}_6$ = {(a , 0.1) , (b , 0.0)} , $\tilde{B}_7$ = {(a , 0.6) , (b , 0.9)} $\tilde{B}_8$ = {(a , 0.1) , (b , 0.7)} , $\tilde{B}_9$ = {(a , 0.6) , (b , 0.7)} , $\tilde{B}_{10}$= {(a , 0.0) , (b , 0.8)} , $\tilde{B}_{11}$ = {(a , 0.7) , (b , 0.0)} ,$\tilde{T}$ = { $\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}_1$, $\tilde{B}_2$, $\tilde{B}_3$, $\tilde{B}_4$,$\tilde{B}_5$, $\tilde{B}_6$, $\tilde{B}_7$, $\tilde{B}_8$, $\tilde{B}_9$ } be a fuzzy topology on $\tilde{A}$ and the FδO($\tilde{A}$) = {$\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}_1$, $\tilde{B}_2$, $\tilde{B}_4$, $\tilde{B}_5$, $\tilde{B}_6$, $\tilde{B}_7$, $\tilde{B}_{10}$, $\tilde{B}_{11}$ }, then the space ($\tilde{A}$, $\tilde{T}$) is a fuzzy $\delta\tilde{T}_{2\frac{1}{2}}$ - space but not fuzzyfuzzy $\delta\tilde{T}_3$ – space .

Definition 3.26 :A fuzzy topological space (Ã , T̃) is said to be **fuzzy $\delta^*\tilde{T}_3 -$ space(F$\delta^*\tilde{T}_3$)** if it isδ^*- regular space (Fδ^*R) as well as fuzzy $\delta\,\tilde{T}_1 -$ space (F$\delta\tilde{T}_1$)

Proposition 3.27 : Every fuzzy $\delta\tilde{T}_3$- space is a fuzzy $\delta^*\tilde{T}_3 -$ space.

Proof: Obvious
Proposition 3.28 : Every fuzzy $\delta^*\tilde{T}_3$- space is a fuzzy $\delta\tilde{T}_{2\frac{1}{2}}$- space .

Proof: Obvious

Remark 3.29: The converse of proposition (3.27)and (2.28) is not true in general

Definition 3.30:
A fuzzy topological space (Ã , T̃) is said to be Fuzzy **δ- normal space (FδN)** if for each two fuzzy closed sets $\tilde{F}_1$ and $\tilde{F}_2$in Ã such that $\tilde{F}_1\,\tilde{\bar{q}}\,\tilde{F}_2$, there exists
$\tilde{U}_1$, $\tilde{U}_2$ ∈FδO(Ã) such that $\mu_{\tilde{F}_1}(x) \le \mu_{\tilde{U}_1}(x)$, $\mu_{\tilde{F}_2}(x) \le \mu_{\tilde{U}_2}(x)$ and $\tilde{U}_1\,\tilde{\bar{q}}\,\tilde{U}_2$.

Definition 3.31:
A fuzzy topological space (Ã , T̃) is said to be **Fuzzy δ^*- normal space (Fδ^*N)** if for each two fuzzy δ-closed sets $\tilde{F}_1$ and $\tilde{F}_2$in Ã such that $\tilde{F}_1\,\tilde{\bar{q}}\,\tilde{F}_2$, there exists $\tilde{U}_1$, $\tilde{U}_2$ ∈FδO(Ã) such that $\mu_{\tilde{F}_1}(x) \le \mu_{\tilde{U}_1}(x)$, $\mu_{\tilde{F}_2}(x) \le \mu_{\tilde{U}_2}(x)$ and $\tilde{U}_1\,\tilde{\bar{q}}\,\tilde{U}_2$.

Proposition 3.32:
Every fuzzy δ- normal space is a fuzzy δ^*- normal space

Proof: Obvious .

Definition 3.33:
A fuzzy topological space (Ã , T̃) is said to be **Fuzzy $\delta\,\tilde{T}_4 -$ space (F$\delta\tilde{T}_4$)** if it is
δ-normal space (FδN)as well as fuzzy $\delta\,\tilde{T}_1 -$ space (F$\delta\tilde{T}_1$) .

Definition 3.34:
A fuzzy topological space ($\tilde{A}$, $\tilde{T}$) is said to be fuzzy $\boldsymbol{\delta^{*}\widetilde{T}_4 -}$ **space** ($\boldsymbol{F\delta^{*}\widetilde{T}_4}$) if it is$\delta^{*}$- normal space ($F\delta^{*}N$) as well as fuzzy $\delta\,\widetilde{T}_1 -$ space ($F\delta\widetilde{T}_1$)

Proposition 3.35:
Every fuzzy $\delta\widetilde{T}_4$ – space is a fuzzy $\delta^{*}\widetilde{T}_4 -$ space

Proof: Obvious

Proposition 3.36 :
Every fuzzy $\delta\widetilde{T}_4$ – space is a fuzzy $\delta\widetilde{T}_3 -$ space

Proof: Obvious

Remark 3.37:

The converse of proposition (3.35) and (3.36) is not true in general
Definition 3.38:
A fuzzy topological space ($\tilde{A}$, $\tilde{T}$) is said to be **Fuzzy δ-completely normal** if for any two fuzzy δ-separated sets $\widetilde{B}$, $\tilde{C}$ in $\tilde{A}$ there exist $\widetilde{D}$, $\widetilde{E}$ $\in F\delta O(\tilde{A})$ such that $\mu_{\widetilde{B}}(x) \leq \mu_{\widetilde{D}}(x)$, $\mu_{\widetilde{C}}(x) \leq \mu_{\widetilde{E}}(x)$ and $\widetilde{D}\,\tilde{\bar{q}}\,\widetilde{E}$

Definition 3.39: A fuzzy topological space ($\tilde{A}$, $\tilde{T}$) is said to be Fuzzy $\boldsymbol{\delta\,\widetilde{T}_5 -}$ **space** ($\boldsymbol{F\delta\widetilde{T}_5}$) if it isδ-completely normal space as well as fuzzy $\delta\,\widetilde{T}_1 -$ space ($F\delta\widetilde{T}_1$) .

Proposition 3.40:
Every fuzzy $\delta\widetilde{T}_5$ – space is a fuzzy $\delta\widetilde{T}_4 -$ space

Proof: Obvious

Remark 3.41 :
The converse of proposition (3.40) is not true in general
Remark 3.42 :
Figure (2) illustrate the relations among a certain types of fuzzy
$\delta\widetilde{T}_i$ – space , i = 0 , 1 , 2 , $2\frac{1}{2}$,3,4,5.

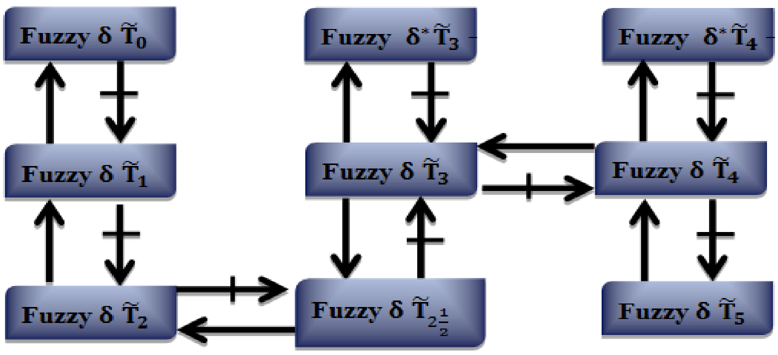

Figure - 2 –

References

[1]. Zadeh L. A."Fuzzy sets", Inform.Control 8, 338-353 (1965).
[2]. Chang, C. L. "Fuzzy Topological Spaces", J. Math. Anal.Appl., Vol.24, pp.182-190, (1968).
[3]. A.M.Zahran "fuzzy δ-continuous , fuzzy almost Regularity (normality) on fuzzy topology No fuzzy set"fuzzy mathematics,Vol.3,No.1,1995,pp.89-96
[4]. Kandil1 A. , S. Saleh2 and M.M Yakout3 "Fuzzy Topology On Fuzzy Sets: Regularity and Separation Axioms" American Academic & Scholarly Research Journal Vol. 4, No. 2, March (2012).
[5]. Mashhour A.S. and Ghanim M.H."Fuzzy closure spaces"J.Math.Anal.And Appl.106,pp.145-170(1985).
[6]. Chakraborty M. K. and T. M. G. Ahsanullah"Fuzzy topology on fuzzy sets and tolerance topology " Fuzzy Sets and Systems, 45103-108(1992).

[7]. Ismail Ibedou," A New Setting of fuzzy separation axioms"Department of Mathematics, Faculty of Science, Benha University, Benha, (13518), Egypt

[8]. ShymaaAbdAlhassan A " On fuzzy semi-separation Axioms in fuzzy topological space on fuzzy sets"M.Sc Thesis , College of Eduction , Al-Mustansiritah University (2013).

[9]. N.V.Velicko" H-closed topological space"Amer.math.soc.Transl.(2),78(1968), 103-118.

[10]. Sinha,S.P." separation axioms in fuzzy topological spaces"suzzy sets and system,45:261-270(1992).

[11]. S.S.Benchalli,R.S.Wali and BasavarajM.Ittanagi " On fuzzy rw-closed sets And fuzzy rw-open sets in fuzzy topological spaces"Int.J.mathematical sciences and Applications , Vol.1,No.2,May 2011.

[12]. Shahla H. K. “On fuzzy Δ - open sets in fuzzy topological spaces” M. Sc. Thesis , college of science , Salahddin Univ. (2004).

[13]. R.UshaParameswari and K.Bageerathi" On fuzzy γ-semi open sets and fuzzy γ-semi closed sets in fuzzy topological spaces" IOSR Journal of mathematics ,Vol 7, pp 63-70, (May-Jun. 2013).

[14]. M.E. El-shafei and A. Zakari," θ-generalized closed sets in fuzzy topological spaces " The Arabian Journal for science and Engineering 31(2A) (2006), 197-206.

[15]. LuayA.Al.Swidi,AmedS.A.Oon, Fuzzy γ-open sets and fuzzy γ-closed sets " Americal Journal of Scientific research,27(2011),62-67.