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Intuitionistic Fuzzification of T-Ideals in Bci-Algebras

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Abstract: *The notions of intuitionistic fuzzy T-ideals in BCI-algebras are introduced.Conditions for an intuitionistic fuzzy ideal to be an intuitionistic fuzzy T-idealare provided. Using a collection of T-ideals, intuitionistic fuzzy T-ideals areestablished.*

Keywords: *T-ideal; intuitionistic fuzzy sub-algebra; (closed) intuitionistic fuzzy ideal; intuitionistic fuzzy T-ideal.*

I. Introduction

To develop the theory of BCI-algebras, the ideal theory plays an important role. Liu and Meng [6] introduced the notion of T-ideals and T-ideals in BCI- algebras. Liu and Zhang [7] discussed the fuzzification of T-ideals, gave relations between fuzzy ideals, fuzzy T-ideals and fuzzy p-ideals. They also considered characterizations of fuzzy T-ideals. Using the notion of fuzzy T-ideals, they provided characterization of associative BCI-algebras. After the introduction of fuzzy sets by Zadeh [9], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Aranassov [1, 2] is one among them. In this paper, we apply the concept of an intuitionistic fuzzy set to T-ideals in BCI-algebras. We introduce the notion of an intuitionistic fuzzy T-ideal of a BCI-algebra, and investigate some related properties. We provide relations between an intuitionistic fuzzy ideal and an intuitionistic fuzzy T-ideal. We give characterizations of an intuitionistic fuzzy T-ideal. Using a collection of T-ideals, we establish intuitionistic fuzzy T-ideals.

II. Preliminaries

Algebra (X; *, 0) of type (2, 0) is called a BCI-algebra if it satisfies the following conditions:

(I) $x, y, z \in X$, $((x * y) * (x * z)) * (z * y) = 0$,

(II) $x, y \in X$, $(x * (x * y)) * y = 0$,

(III) $x \in X$, $x * x = 0$,

(IV) $x, y \in X$, $X * y = 0, y * x = 0 \;\; x = y$,

We can define a partial order '≤' on X by $x \leq y$ if and only if $x * y = 0$. Any BCI-algebra X has the following properties:

(T1) $x \in X, x*0 = x$

(T2) $x, y, z \in X$, $(x * y) * z = (x * z) * y$,

(T3) $x, y, z \in X$, $x \leq y$, $\;\; x * z \leq y * z, z * y \leq z * x$,

A mapping $\mu: X \to [0, 1]$, where X is an arbitrary nonempty set, is called a fuzzy set in X. For any fuzzy set μ in X and any $t \in [0, 1]$ we define two sets $U(\mu; t) = \{x \in X: \mu(x) \geq t\}$ and $L(\mu; t) = \{x \in X: \mu(x) \leq t\}$, which are called an upper and lower t-level cut of μ and can be used to the characterization of μ. As an important generalization of the notion of fuzzy sets in X, Atanassov [1, 2] introduced the concept of an intuitionistic fuzzy set (IFS for short) defined on a nonempty set X as objects having the form $A = \{x, \mu_A(x), \lambda_A(x) : x \in X\}$ where the functions $\mu_A : X \to [0, 1]$ and $\lambda A : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\lambda_A(x)$) of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in X$. Such defined objects are studied by many authors (see for Example two journals: 1. Fuzzy Sets and Systems and 2. Notes on Intuitionistic Fuzzy Sets) and have many interesting applications not only in mathematics (see Chapter 5 in the book [3]). For the sake of simplicity, we shall use the symbol $A = <X, \mu_A, \lambda_A>$ for the intuitionistic fuzzy set $A = \{x, \mu_A(x), \lambda_A(x): x \in X\}$.

Definition 2.1: A nonempty subset A of a BCI-algebra X is called an idealof X if it satisfies:

(I1) $0 \in A$, (I2) $x, y \in X$, $y \in A$, $x * y \in A \Rightarrow x \in A$,

Definition 2.2A non-empty subset A of a BCI-algebra X is called a-ideal of X if it satisfies

(I1) and (I3) $x, y \in X$, $(z \in A)$ $((x * z) * (0 * y) \in A \Rightarrow y * x \in A)$

Definition 2.3: A non-empty subset I of BCI-algebra X is called an R-ideal of X, if
1.$0 \in I$, 2. $(x * z) * (z * y) \in I$ and $y \in I \Rightarrow x \in I$

Definition 2.4: A fuzzy subset μ in a BCK-algebra X is called a fuzzy p-ideal of X, if1.$\mu(0) \geq \mu(x)$,
2. $\mu(x) \geq \min\{\mu((x * z) * (z * y)), \mu(y)\}$, $\forall x, y, z \in X$.

Definition 2.5: Ideal I of a BCI-algebra (X, *, 0) is called closed if $0 * x \in I$, for all $x \in I$.
Definition 2.6: Let A and B be two fuzzy ideal of BCI algebra X. The fuzzy set $A \cap B$ with membership function
$\mu_{A\cap B}$ is defined by $\mu_{A\cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$, $\forall x \in X$

Definition 2.7: Let A and B be two fuzzy ideal of BCI algebra X. Thefuzzy set $A \cup B$ with membership function $\mu_{A\cup B}$ is defined by $\mu_{A\cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$, $\forall x \in X$.

Definition 2.8: Let A and B be two fuzzy ideal of BCI algebra X with membership functions μ_A and μ_B respectively. A is contained in B if $\mu_A(x) \leq \mu_B(x)$, $\forall x \in X$.

Definition 2.9: Let A be a fuzzy ideal of BCI algebra X. The fuzzy set Am with membership function $\mu_A{}^m$ isdefined by $\mu_A{}^m(x) = (\mu_A(x))^m$, $x \in X$

Definition 2.10: Let μ is a fuzzy set in X. The complement of μ is denoted by μ and is defined as
$\mu(x) = 1 - \mu(x)$, $\forall x \in X$.

Definition 2.11: Let $A = (X, \mu_A, \lambda_A)$ be an intuitionistic fuzzy set in X. Then (i) $\neg A = (X, \mu_A, \bar{\mu}_A)$ and
(ii). $\Diamond A = (X\, \bar{\lambda}_A, \lambda_A)$.

Definition 2.12: An intuitionistic fuzzy set A in a non-empty set X is an object having the form$A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$, where the function $\mu_A: X \to [0, 1]$ and $\lambda_A: X \to [0, 1]$ denoted the degree of membership (namely$\mu_A(x)$) and the degree of non-membership(namely $\lambda_A(x)$) of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in X$.

Definition 2.13: An IFS $A =< X, \mu_A, \lambda_A>$in a BCI-algebra X is called an intuitionistic fuzzy sub-algebra of X if it satisfies: $x, y \in X$1.$\mu_A(x * y) \geq \min\{\mu_A(x), \mu_A(y)\})$, $\forall x, y \in X$
2.$\lambda_A(x * y) \leq \max\{\lambda_A(x), \lambda_A(y)\}$

Definition 2.14: An intuitionistic fuzzy set $A = (X, \mu_A, \lambda_A)$ in X is called anintuitionistic fuzzy ideal of X, if it satisfies the following axioms: (IF1) $\mu_A(0) \geq \mu_A(x)$ and $\lambda_A(0) \leq \lambda_A(x)$,
(IF2) $\mu_A(x) \geq \min\{\mu_A(x * y), \mu_A(y)\}$,
(IF3) $\lambda_A(x) \leq \max\{\lambda_A(x * y), \lambda_A(y)\}$, $\forall x, y \in X$

Definition 2.15: An intuitionistic fuzzy set $A = (X, \mu_A, \lambda_A)$ in X is called an intuitionistic fuzzy closed idealof X, if it satisfies (IF2), (IF3) and the following:(IF4) $\mu_A(0 * x) \geq \mu_A(x)$ and $\lambda_A(0 * x) \leq \lambda_A(x)$, $\forall x \in X$

Definition 2.16: An IFS $A = <X, \mu_A, \lambda_A>$ in X is called an intuitionistic fuzzy a-ideal of X.
If it satisfies (2.12) and $(x, y, z \in X)$ 1.$\mu_A(y * x) \geq \min\{\mu_A((x * z) * (0 * y)), \mu_A(z)\}$
2.$\lambda_A(y * x) \leq \max\{\lambda_A((x * z) * (0 * y)), \lambda_A(z)\}$

III. Intuitionistic Fuzzy T-Ideals

In what follows, let X denotes a BCI-algebra unless otherwise specified. We first consider the intuitionistic fuzzification of the notion of T-ideals in a BCI-algebra as follows.

Definition 3.1: An intuitionistic fuzzy set $A = (X, \mu_A, \lambda_A)$ in a BCI-algebra X is called an intuitionistic fuzzy T-ideal of X, if (IFT1) $\mu_A(0) \geq \mu_A(x)$ and $\lambda_A(0) \leq \lambda_A(x)$,
(IFT2) $\mu_A(x * z) \geq \min\{\mu_A(x * (y * z)), \mu_A(y)\}$,
(IFT3) $\lambda_A(x * z) \leq \max\{\lambda_A(x * (y * z)), \lambda_A(y)\}$, for all $x, y, z \in X$.

Definition 3.2: An intuitionistic fuzzy set A = (X, μ_A, λ_A) in a BCI-algebra X is called an intuitionistic fuzzy closed T-ideal of X, if it satisfies (IFT2), (IFT3) and the following:
(IFT4) $\mu_A(0 * x) \geq \mu A(x)$ and $\lambda_A(0 * x) \leq \lambda_A(x)$, $\forall x \in X$

Definition 3.3: Let A = (X, μ_A, λ_A) be an intuitionistic fuzzy set in a BCI-algebra X. The set U (μ_A; s) = {x ∈ X:$\mu_A(x) \geq s$} is called upper s-level of μ_A and the setL (λ_A; t) = {x ∈ X: $\lambda_A(x) \leq t$} is called lower t-level of λ_A.

Theorem3.4: Every intuitionisticic fuzzy T-ideal is an intuitionistic fuzzy ideal in BCI-Algebras.

Proof: ∀ x, y, z ∈ X.
1. we have $\mu A(0) \geq \mu A(x)$, $\lambda_A(0) \leq \lambda_A(x)$ and
2. $\mu_A(x*z) \geq \min\{\mu_A((x*y)*z), \mu_A((y)\}$, Putting z = y.
$\mu_A(x*y) \geq \min\{\mu_A((x*y)*y)), \mu_A(y)\}$
$\geq \min\{\mu_A(x*0), \mu_A(y)\}$,
$\mu_A(x*y) \geq \min\{\mu_A(x), \mu_A(y)$ for all x, y ∈ X
3. $\lambda_A(x*z) \leq \max\{\lambda_A(x*(y*y)), \lambda_A((y)\}$, Putting z = y.
$\lambda_A(x*y) \leq \max\{\lambda_A(x*0), \lambda_A(y)\}$
$\leq \max\{\lambda_A(x*0), \lambda_A(y)\}$,
$\lambda_A(x*y) \leq \max\{\lambda_A(x), \lambda_A(y)$, for all x, y ∈ X.

Theorem3.5: Every intuitionisticic fuzzy T-ideals is an intuitionistic fuzzy p-ideals in BCI-Algebras.

Proof: ∀ x, y, z ∈ X.
1. We have $\mu A(0) \geq \mu A(x)$, $\lambda_A(0) \leq \lambda_A(x)$
2. $\mu_A(x* z) \geq \min\{\mu_A((x*y)*z), \mu_A(y)\}$
$\mu_A(x* z) \geq \min\{\mu_A((x*z)*y), \mu_A(y)\}$
$\mu_A(x* z) \geq \min\{\mu_A((x*z)*(y*0)), \mu_A(y)\}$, put z = 0
$\mu_A(x* 0) \geq \min\{\mu_A((x*z)*(y*z)), \mu_A(y)\}$
$\mu_A(x) \geq \min\{\mu_A((x*z)*(y*z)), \mu_A(y)\}$
3. $\lambda_A(x* z) \leq \max\{\lambda_A((x*y)*z), \lambda_A(y)\}$
$\lambda_A(x* z) \leq \max\{\lambda_A((x*z)*y), \lambda_A(y)\}$
$\lambda_A(x* z) \leq \max\{\lambda_A((x*z)*(y*0)), \lambda_A(y)\}$, put z = 0
$\lambda_A(x* 0) \leq \max\{\lambda_A((x*z)*(y*z)), \lambda_A(y)\}$
$\lambda_A(x) \leq \max\{\lambda_A((x*z)*(y*z)), \lambda_A(y)\}$

Theorem3.6 Every intuitionisticic fuzzy T-ideals is an intuitionistic fuzzy H-ideals in BCI-Algebras.

Proof: ∀ x, y, z ∈ X.
1. We have $\mu A(0) \geq \mu A(x)$, $\lambda_A(0) \leq \lambda_A(x)$
2. $\mu_A(x* z) \geq \min\{\mu_A((x*y)*z), \mu_A(y)\}$
$\mu_A(x* z) \geq \min\{\mu_A((x*z)*y), \mu_A(y)\}$
$\mu_A(x* z) \geq \min\{\mu_A((x*(z*0))*(y*0)), \mu_A(y)\}$, put z = 0
$\mu_A(x* 0) \geq \min\{\mu_A((x*(z*z))*(y*z)), \mu_A(y)\}$
$\mu_A(x) \geq \min\{\mu_A((x*0)*(y*z)), \mu_A(y)\}$
$\mu_A(x) \geq \min\{\mu_A((x*(y*z)), \mu_A(y)\}$
3. $\lambda_A(x* z) \leq \max\{\lambda_A((x*y)*z), \lambda_A(y)\}$
$\lambda_A(x* z) \leq \max\{\lambda_A((x*z)*y), \lambda_A(y)\}$
$\lambda_A(x* z) \leq \max\{\lambda_A((x*(z*0))*(y*0)), \lambda_A(y)\}$, put z = 0
$\lambda_A(x* 0) \leq \max\{\lambda_A((x*(z*z))*(y*z)), \lambda_A(y)\}$
$\lambda_A(x) \leq \max\{\lambda_A((x*0)*(y*z)), \lambda_A(y)\}$
$\lambda_A(x) \leq \max\{\mu_A((x*(y*z)), \lambda_A(y)\}$

Theorem 3.7: Let A = (X, μ_A, λ_A) be an intuitionistic fuzzy T-ideal of a BCI-algebra X. Then so is ¬A = (X, μ_A, $\bar{\mu}_A$).

Proof: 1. we have $\mu_A(0) \geq \mu A(x)$,
$\Rightarrow 1 - \mu_A(0) \leq 1 - \mu_A(x)$,
$\Rightarrow \bar{\mu}_A(0) \leq \bar{\mu}_A(x)$, ∀ x ∈ X

Let us Consider, $\forall$ x, y, z $\in$ X,
2. $\mu_A(x * z) \geq \min \{\mu_A (x*(y *z)), \mu_A(y)\}$
$\Rightarrow 1 - \mu_A (x * z) \leq$ 1-min $\{1 - \mu_A ((x*y) *z), 1 - \mu_A(y)\}$
$\Rightarrow \bar{\mu}_A(x * z) \leq 1 - \min \{1 - \mu_A ((x * y) *z), 1 - \mu_A(y)\}$
$\Rightarrow \bar{\mu}_A (x * z) \leq \max \{\bar{\mu}_A ((x *y) *z), \bar{\mu}_A(y)\}$,
Hence $\neg A = (X, \mu_A, \bar{\mu}_A)$ is an IFT-ideal of X.

Theorem 3.8: Let $A = (X, \mu_A, \lambda_A)$ be an intuitionistic fuzzy T-ideal of a BCI-algebra X.
Then so is $\Diamond A = (X, \bar{\lambda}_A, \lambda_A)$.

Proof: we have $\lambda_A (0) \leq \lambda_A(x)$
$\Rightarrow 1 - \lambda_A (0) \geq 1 - \lambda_A(x)$
$\Rightarrow \bar{\lambda}_A (0) \geq \bar{\lambda}_A(x), \forall x \in X$
Let us Consider, $\forall$ x, y, z $\in$ X,
$\lambda_A (x * z) \leq \max \{\lambda_A ((x*y) *z), \lambda_A(y)\}$
$\Rightarrow 1 - \lambda_A (x * z) \geq$ 1- max $\{1 - \lambda_A ((x*y) *z), 1 - \lambda_A(y)\}$
$\Rightarrow \bar{\lambda}_A(x * z) \geq 1 - \max \{1 - \lambda_A((x*y) *z), 1 - \lambda_A(y)\}$
$\Rightarrow \bar{\lambda}_A (x * z) \geq \min \{\bar{\lambda}_A ((x * y) *z), \bar{\lambda}_A(y)\}$
Hence $\Diamond A = (X, \bar{}_A, \lambda_A)$ is an IFT-ideal

Theorem 3.9: $A = (X, \mu_A, \lambda_A)$ be an intuitionistic fuzzy T-ideal of a BCI-algebra X if and only if $\neg A = (X, \mu_A, \bar{\mu}_A)$, $\Diamond A = (X, \bar{\lambda}_A, \lambda_A)$ and $B = (X, \bar{\mu}_A, \bar{\lambda}_A)$ are intuitionistic fuzzy T-ideals of a BCI-algebra X.

Proof: 1. we have $\mu_A (0) \geq \mu_A(x)$,
$\Rightarrow 1 - \mu_A (0) \leq 1 - \mu_A(x)$,
$\Rightarrow \bar{\mu}_A (0) \leq \bar{\mu}_A(x), \forall x \in X$

2. Let us Consider, $\forall$ x, y, z $\in$ X,
$\mu_A(x * z) \geq \min \{\mu_A ((x*y) *z), \mu_A(y)\}$
$\Rightarrow 1 - \mu_A (x * z) \leq$ 1-min $\{1 - \mu_A ((x*y) *z), 1 - \mu_A(y)\}$
$\Rightarrow \bar{\mu}_A(x * z) \leq 1 - \min \{1 - \mu_A ((x * y) *z), 1 - \mu_A(y)\}$
$\Rightarrow \bar{\mu}_A (x * z) \leq \max \{\bar{\mu}_A ((x *y) *z), \bar{\mu}_A(y)\}$,
Hence $\neg A = (X, \mu_A, \bar{\mu}_A)$ is an IFT-ideal of X.

3. We have $\lambda_A (0) \leq \lambda_A(x)$
$\Rightarrow 1 - \lambda_A (0) \geq 1 - \lambda_A(x)$
$\Rightarrow \bar{\lambda}_A (0) \geq \bar{\lambda}_A(x), \forall x \in X$

4. Let us Consider, $\forall$ x, y, z $\in$ X,
$\lambda_A (x * z) \leq \max \{\lambda_A ((x*y) *z), \lambda_A(y)\}$
$\Rightarrow 1 - \lambda_A (x * z) \geq$ 1- max $\{1 - \lambda_A ((x* y) *z), 1 - \lambda_A(y)\}$
$\Rightarrow \bar{\lambda}_A(x * z) \geq 1 - \max \{1 - \lambda_A((x*y) *z), 1 - \lambda_A(y)\}$
$\Rightarrow \bar{\lambda}_A (x * z) \geq \min \{\bar{\lambda}_A ((x * y) *z), \bar{\lambda}_A(y)\}$,
Hence $\Diamond A = (X, \bar{\lambda}_A, \lambda_A)$ is an IFT-ideal

Theorem 3.10If $A = (X, \mu_A, \lambda_A)$ be an intuitionistic fuzzy closed T-ideal of a BCI-algebra X, then so is $\neg A = (X, \mu_A, \bar{\mu}_A)$.

Proof: $\forall$ x $\in$ X, we have $\mu_A (0 * x) \geq \mu_A(x)$,
$\Rightarrow 1 - \mu_A (0 * x) \leq 1 - \mu_A(x)$, $\Rightarrow \mu_A (0 * x) \leq \mu_A(x)$,
Hence $\neg A = (X, \mu_A, \bar{\mu}_A)$ is an intuitionistic fuzzy closed T-ideal of X.

Theorem 3.11: If $A = (X, \mu_A, \lambda_A)$ be an intuitionistic fuzzy closed T-ideal of a BCI-algebra X, then so is $\Diamond A = (X, \bar{\lambda}_A, \lambda_A)$

Proof: $\forall$ x $\in$ X, We have $\lambda A (0 * x) \leq \lambda A(x)$,
$\Rightarrow 1 - \bar{\lambda}_A (0 * x) \geq 1 - \bar{\lambda}_A(x)$, $\Rightarrow \bar{\lambda}_A (0 * x) \geq \bar{\lambda}_A(x)$,
Hence, $\Diamond A = (X, \bar{\lambda}_A, \lambda_A)$ is an intuitionistic fuzzy closed T-ideal of X.

Theorem 3.12: A = (X, μ_A, λ_A) be an intuitionistic fuzzy closed T-ideal of a BCI-algebra X. If and only if ¬A = (X, μ_A, $\bar{\mu}_A$), ◊A = (X, $\bar{\lambda}_A$, λ_A) and B = (X, $\bar{\lambda}_A$, λ_A) are Intuitionistic fuzzy closed T-ideals of BCI-algebra X.
Proof: 1. $\forall x \in X$, we have $\mu_A(0 * x) \geq \mu_A(x)$,
$\Rightarrow 1 - \mu_A(0 * x) \leq 1 - \mu_A(x)$, $\Rightarrow \mu A(0 * x) \leq \mu A(x)$,
Hence ¬A = (X, μ_A, $\bar{\mu}_A$) is an intuitionistic fuzzy closed T-ideal of X.
2. $\forall x \in X$, We have $\lambda_A(0 * x) \leq \lambda_A(x)$,
$\Rightarrow 1 - \bar{\lambda}_A(0 * x) \geq 1 - \bar{\lambda}_A(x)$
$\Rightarrow \bar{\lambda}_A(0 * x) \geq \bar{\lambda}_A(x)$,
Hence, ◊A = (X, $\bar{\lambda}_A$, λ_A) is an intuitionistic fuzzy closed T-ideal of X. And B = (X, $\bar{\lambda}_A$, λ_A) are Intuitionistic fuzzy closed T-ideals of BCI-algebra X.

Theorem 3.13: A = (X, μ_A, λ_A) be an intuitionistic fuzzy T-ideal of a BCI-algebra X if and only if the non-empty upper s-level cut U(μ_A; s) and the non-empty lower t-level cut L(λ_A; t) are T-ideals of X, for any s, t ∈ [0, 1].

Proof: Suppose A = (X, μ_A, λ_A) is an IFT-ideal of a BCI-algebra X. ∀ s, t ∈ [0, 1],
Define the sets U (μ_A; s) = {x ∈ X: $\mu_A(x) \geq s$} and L (λ_A; t) = {x ∈ X: $\lambda_A(x) \leq t$}.
Sine L (λ_A; t) ≠φ, for x ∈ L (λ_A; t) $\Rightarrow \lambda_A(x) \leq t \Rightarrow \lambda_A(0) \leq t \Rightarrow 0 \in$ L (λ_A; t)
Let ((x *y) *z) ∈ L (λ_A; t) and y ∈ L (λ_A; t) implies $\lambda_A((x*y)*z) \leq t$ and $\lambda_A(y) \leq t$.
Sine, ∀ x, y, z ∈ X, $\lambda_A(x) \leq \max\{\lambda_A((x*y)*z), \lambda_A(y)\} \leq \max\{t, t\} = t$, $\Rightarrow \lambda_A(x) \leq t$.
Therefore x * z ∈ L (λ_A; t), ∀ x, y, z ∈ X. Hence L (λ_A; t) is a T-ideal of X.
Similarly, we can prove U (μ_A; s) is a T-ideal of X.
Conversely, suppose that U (μ_A; s) and L (λ_A; t) are T-ideal of X, for any s, t ∈ [0, 1].If possible, assume $x_o \in X$ such that $\mu_A(0) < \mu_A(x_o)$ and $\lambda_A(0) > \lambda_A(y_o)$. Puts$s_o = 1/2\,[\mu_A(0) + \mu_A(x_o)]$
$\Rightarrow s_o < \mu_A(x_o)$, $0 \leq \mu_A(0) < s_o < 1 \Rightarrow$ xo ∈ U (μ_A; s_o). SinceU (μ_A; s_o) is a T-ideal of X, we have 0 ∈ U (μ_A; s_o)
$\Rightarrow \mu_A(0) \geq s_o$. This is contradiction. Therefore $\mu_A(0) \geq \mu_A(x)$, ∀ x ∈ X, Similarly by taking $t_o = 1/2\,[\lambda_A(0) + \lambda_A(y_o)]$, we can show $\lambda_A(0) \leq \lambda_A(y)$, ∀ y ∈ X. If possible assume x_o, y_o, $z_o \in X$ such that$\mu_A(x_o * z_o) < \min\{\mu_A((x_o * y_o) * z_0), \mu_A(y_o)\}$. Put $s_o = 1/2\,[\mu_A(x_o * z_o) + \min\{\mu_A((x_o * y_o) * z_0), \mu_A(y_o)\}] \Rightarrow s_o > \mu_A(x0)$, $S_o < \mu_A((x_o*y_o)*z_0)$, and $s_o < \mu_A(y_o) \Rightarrow x_o \in$ U (μ_A; s), (x_o *(y_o *z_0)) ∈ U (μ_A; s_o) and$y_o \in$ U (μ_A; s), which is contradiction to T-ideal U (μ_A; s). Therefore $\mu_A(x * z) \geq \min\{\mu_A((x* y) * z), \mu_A(y)\}$, ∀ x, y, z ∈ X, Similarly we can prove $\lambda_A(x * z) \leq \max\{\lambda_A((x *y) * z), \lambda_A(y)\}$, ∀ x, y, z ∈ X.
Hence A = (X, μ_A, λ_A) is an intuitionistic fuzzy T-ideal of a BCI-algebra X.

Theorem 3.14: A = (X, μ_A, λ_A) is an intuitionistic fuzzy closed T-ideal of a BCI-algebra X if and only if the non-empty upper s-level cut U(μ_A; s) and the non-empty lower t-level cut L(λ_A; t) are closed T-ideal of X, for any s, t ∈ [0, 1].

Proof: Suppose A = (X, μ_A, λ_A) is an intuitionistic fuzzy closed T-ideal of a BCI-algebra X. We have $\mu_A(0 * x) \geq \mu_A(x)$ and $\lambda_A(0 * x) \leq \lambda_A(x)$, for any x ∈ X. ∀ x ∈ U (μ_A; s), ⇒ x ∈ X and $\mu_A(x) \geq s \Rightarrow \mu_A(0*x) \geq s$, ⇒ 0*x ∈ U($\mu_A$; s). And x ∈ L ($\lambda_A$; t) ⇒ x ∈ X and $\lambda_A(x) \leq t \Rightarrow \lambda_A(0 * x) \leq t$ ⇒ 0 * x ∈ L (λ_A; t), Therefore U (μ_A; s) and L (λ_A; t) are closed T-ideals of X. Converse, it is enough to show that $\mu_A(0 * x) \geq \mu_A(x)$ and $\lambda_A(0 * x) \leq \lambda_A(x)$, ∀ x ∈ X. If possible, assume xo ∈ X such that $\mu_A(0 * x_o) < \mu_A(x_o)$. Take $s_o = 1/2\,[\mu_A(0 *x) + \mu_A(x_o)] \Rightarrow \mu_A(0 *x_o) < s_o < \mu_A(x_o)$.⇒ $x_o \in$ U (μ_A; s), but 0 * $x_o \in$ U (μ_A; s), which is contradiction to closed T-ideal. Hence $\mu_A(0 * x) \geq \mu_A(x)$, ∀ x ∀ X. Similarly we can prove that $\lambda_A(0 * x) \leq \lambda_A(x)$, ∀ x ∈ X
Corollary 3.15If A = (X, μ_A, λ_A) be an intuitionistic fuzzy closed T-ideal of X, then the sets J = {x ∈ X: $\mu_A(x) = \mu_A(0)$} and K = {x ∈ X: $\lambda_A(x) = \lambda_A(0)$} are T-ideal of X.

Proof: Since 0 ∈ X, $\mu_A(0) = \mu_A(0)$ and $\lambda_A(0) = \lambda_A(0)$ implies 0 ∈ J and 0 ∈ K, So J =Φ and K = Φ. Let ((x * y) * z) ∈ J and y ∈ J $\Rightarrow \mu_A((x * y) * z) = \mu_A(0)$ and $\mu_A(y) = \mu_A(0)$. Since $\mu_A(x * z) \geq \min\{\mu_A((x *y) * z), \mu_A(y)\} = \mu_A(0)$, $\Rightarrow \mu_A(x) \geq \mu_A(0)$, but $\mu_A(0) \geq \mu_A(x)$. It follows that x ∈ J, for all x, y, z ∈ X. Hence J is T-ideal of X. Similarly we can prove K is T-ideal of X.

Definition 3.16: Let f be a mapping on a set X and A = (X, μ_A, λ_A) an Intuitionistic fuzzy set in X. Then the fuzzy sets u and v on f(X) defined by U(y) = sup $\mu_A(x)$ and V(y) = inf. $\lambda_A(x)$,
$x \in f^{-1}(y)$ $x \in f^{-1}(y)$
∀ y ∈ f(X) is called image of A under f. If u, v are fuzzy sets in f(X) then the fuzzy sets μ_A = uo f and λ_A = vo f is called the pre-image of u and v under f.

Theorem 3.17: Let f: $X \rightarrow X^1$ is onto homomorphism of BCI-algebras. If $A^1 = (X^1, u, v)$ is an intuitionistic fuzzy T-ideal of X^1, then the pre-image of A^1 under f is an intuitionistic fuzzy T-ideal of X.

Proof: Let A = (X, μ_A, λ_A), where μ_A = u o f and λ_A = v o f is the pre-image of $A^1 = (X^1, u, v)$ under f. Since $A^1 = (X^1, u, v)$ is an intuitionistic fuzzy T-ideal of X^1
We have $u(0^1) \geq u(f(x)) = \mu_A(x)$ and $v(0^1) \leq v(f(x)) = \lambda_A(x)$.
On other hand $u(0^1) = u(f(0)) = \mu A(0)$ and
$v(0^1) = v(f(0)) = \lambda_A(0)$.
Therefore $\mu_A(0) \geq \mu_A(x)$ and $\lambda_A(0) \leq \lambda_A(x)$, $\forall x \in X$
Now we show that 1) $\mu_A(x * z) \geq \min\{\mu_A((x * y) * z), \mu_A(y)\}$,
(2) $\lambda_A(x * z) \leq \max\{\lambda_A((x * y) * z), \lambda_A(y)\}$, $\forall$ x, y, z $\forall$ X,
We have $\mu_A(x * z) = u(f(x) * f(z)) \geq \min)\{u((f(x) * f(y)) * f(z)), u(y)\}$, $\forall y \in X$, since f is onto homomorphism, there is $y \in X$ such that $f(y) = y^1$
Thus $\mu_A(x * z) \geq \min\{u((f(x) * f(y)) * f(z)), u(y)\} = \min\{u((f(x) * f(y)) * f(z)), u(f(y))\}$
$= \min\{u(f((x * y) * z), u(f(y))\} = \min\{\mu_A((x * y) * z), \mu_A(y)\}$, $\forall$ x, y, z $\in$ X. Therefore the result $\mu_A(x * z) \geq \min\{\mu_A((x * y) * z), \mu_A(y)\}$, is true $\forall$ x, y, z $\in$ X, because y is an arbitrary element of X and f is onto mapping. Similarly we can prove $\lambda_A(x * z) \leq \max\{\lambda_A((x * y) * z), \lambda_A(y)\}$, $\forall$ x, y, z $\in$ X. Hence the pre-image A = (X, μ A, λ A), of A is an intuitionistic T-ideal of X

Theorem 3.18: If A is an intuitionistic fuzzy T-ideal of BCI-algebras X, then A^m is an intuitionistic fuzzy T-ideal of BCI-algebras X.

Proof: We have
1. $\mu_A(0) \geq \mu_A(x)$, $\{\mu_A(0)\}^m \geq \{\mu_A(x)\}^m$, $\mu_A(0)^m \geq \mu_A(x)^m$, $\mu_A^m(0) \geq \mu_A^m(x)$,
$\lambda_A(0) \leq \lambda_A(x)$, and $\{\lambda_A(0)\}^m \leq \{\lambda_A(x)\}^m$, $\lambda_A(0)^m \leq \lambda_A(x)^m$, $\lambda_A^m(0) \leq \lambda_A^m(x)$, $\forall x \in X$

2. $\mu_A(x * z) \geq \min\{\mu_A((x*y) * z), \mu_A(y)\}$,
$\{\mu_A(x * z)\}^m \geq \{\min\{\mu_A((x*)y * z), \mu_A(y)\}\}^m$
$\mu_A(x * z)$ m $\geq \min\{\mu_A((x*y*)z), \mu_A(y)\}^m$,
$\mu_A(x * z)$ m $\geq \min\{\mu_A((x*y) * z)^m, \mu_A(y)^m\}$
$\mu_A^m(x * z) \geq \min\{\mu_A((x*y) * z), \mu_A(y)^m\}$, $\forall$ x, y, z $\in$ X and

3. $\lambda_A(x * z) \leq \max\{\lambda_A((x*y)*z), \lambda_A(y)\}$,
$\{\lambda_A(x * z)\}^m \leq \{\max\{\lambda_A((x*y) * z), \lambda_A(y)\}\}^m$
$\lambda_A(x * z)^m \leq \max\{\lambda_A((x*y) * z), \lambda_A(y)\}^m$,
$\lambda_A(x * z)^m \leq \max\{\lambda_A((x*y) * z)^m, \lambda_A(y)^m\}$,
$\lambda_A^m(x * z) \leq \max\{\lambda_A((x*y) * z)^m, \lambda_A(y)^m\}$, $\forall$ x, y, z $\in$ X

Theorem 3.19: if A and B are two intuitionistic fuzzy T-ideal of BCI-algebras X, if one is contained another then prove that $A \cap B$ is an intuitionistic fuzzy T-ideal of BCI- algebra X.

Proof: We have1. $\mu_A(0) \geq \mu_A(x)$ and $\mu_B(0) \geq \mu_B(x)$, $\forall x \in X$
$\min\{\mu_A(0), \mu_B(0)\} \geq \min\{\mu_A(x), \mu_B(x)\}$,
$\mu_{A \cap B}(0) \geq \min\{\mu_A(x), \mu_B(x)\}$
$\mu_{A \cap B}(0) \geq \mu_{A\ B}(x)$, $\forall x \in X$ and
$\lambda_A(0) \leq \lambda_A(x)$ and $\lambda_B(0) \leq \lambda_B(x)$, $\forall x \in X$
$\min\{\lambda_A(0), \lambda_B(0)\} \leq \min\{\lambda_A(x), \lambda_B(x)\}$,
$\lambda_{A \cap B}(0) \leq \min\{\lambda_A(x), \lambda_B(x)\}$
$\lambda_{A \cap B}(0) \leq \lambda_{A\ B}(x)$, $\forall x \in X$. We have
2. $\mu_A(x * z) \geq \min\{\mu_A((x*y) * z), \mu_A(y)\}$ and $\mu_B(x * z) \geq \min\{\mu_B(x*(y * z)), \mu_B(y)\}$
$\min\{\mu_A(x * z), \mu_B(x * z)\} \geq \min\{\min\{\mu_A((x*y) * z), \mu_A(y)\}, \min\{\mu_B((x*y) * z), \mu_B(y)\}\}$
$\mu_{A \cap B}(x * z) \geq \min\{\min\{\mu_A((x*y) * z), \mu_B((x*y) * z)\}, \min\{\mu_A(y), \mu_A(y),\}\}$
$\mu_{A \cap B}(x * z) \geq \min\{\mu_{A \cap B}((x*y) * z), \mu_{A \cap B}(y)\}$, $\forall$ x, y, z $\in$ X
3. $\lambda_A(x * z) \leq \max\{\lambda_A((x*y) * z), \lambda_A(y)\}$ and $\lambda_B(x * z) \leq \max\{\lambda_B((x*y) * z), \lambda_B(y)\}$
$\min\{\lambda_A(x * z), \lambda_B(x * z)\} \leq \min\{\max\{\lambda_A((x*y) * z), \lambda_A(y)\}, \max\{\lambda_B((x*y) * z), \lambda_B(y)\}\}$,
if one is contained another
$\lambda_{A \cap B}(x * z) \leq \max\{\min\{\lambda_A((x*y) * z), \lambda_B((x*y) * z)\}, \min\{\lambda_A(y), \lambda_A(y),\}\}$
$\lambda_{A \cap B}(x * z) \leq \max\{\lambda_{A\ B}((x*y) * z), \lambda_{A\ B}(y)\}$, $\forall$ x, y, z $\in$ X

$A \cap B$ is an intuitionistic fuzzy T-ideal of BCI- algebra X

Theorem 3.20: If Aand B are two intuitionistic fuzzy T-ideal of BCI-algebras X, if one is contained another then prove that $A \cup B$ is an intuitionistic fuzzy T-ideal of BCI-algebras X.

Proof: We have
1. $\mu_A(0) \geq \mu_A(x)$ and $\mu_B(0) \geq \mu_B(x)$,
$\max\{\mu_A(0), \mu_B(0)\} \geq \max\{\mu_A(x), \mu_B(x)\}$,
$\mu_{A\cup B}(0) \geq \max\{\mu_A(x), \mu_B(x)\}, \forall x \in X$,
$\mu_{A\cup B}(0) \geq \mu_{A\cup B}(x), \forall x \in X$, and
$\lambda_A(0) \leq \lambda_A(x)$ and $\lambda_B(0) \leq \lambda_B(x)$ for all $x \in X$,
$\max\{\lambda_A(0), \lambda_B(0)\} \leq \max\{\lambda_A(x), \lambda_B(x)\}$
$\lambda_{A\cup B}(0) \leq \max\{\lambda_A(x), \lambda_B(x)\}$,
$\lambda_{A\cup B}(0) \leq \lambda_{A\cup B}(x), \forall x \in X$
2. $\mu_A(x * z) \geq \min\{\mu_A((x*y)*z), \mu_A(y)\}$ and $\mu_B(x * z) \geq \min\{\mu_B((x*y)*z), \mu_B(y)\}$
$\max\{\mu_A(x * z), \mu_B(x * z)\} \geq \max\{\min\{\mu_A((x*y)*z), \mu_A(y)\}, \min\{\mu_B((x*y)*z), \mu_B(y)\}\}$

If one is contained another
$\mu_{A\cup B}(x * z) \geq \min\{\max\{\mu_A((x*y)*z), \mu_B((x*y)*z)\}, \max\{\mu_A(y), \mu_A(y),\}\}$
$\mu_{A\cup B}(x * z) \geq \min\{\mu_{A B}((x*y)*z), \mu_{A\cup B}(y)\}, \forall x, y, z \in X$ and
3. $\lambda_A(x * z) \leq \max\{\lambda_A((x*y)*z), \lambda_A(y)\}$ and $\lambda_B(x* z) \leq \max\{\lambda_B((x*y)*z), \lambda_B(y)\}$
$\max\{\lambda_A(x*z), \lambda_B(x*z)\} \leq \max\{\max\{\lambda_A((x*y)*z), \lambda_A(y)\}, \max\{\lambda_B((x*y)*z), \lambda_B(y)\}\}$

If one is contained another
$\lambda_{A\cup B}(x*z) \leq \max\{\max\{\lambda_A((x*y)*z), \lambda_B((x*y)*z)\}, \max\{\lambda_A(y), \lambda_A(y),\}\}$
$\lambda_{A\cup B}(x*z) \leq \max\{\lambda_{A\cup B}((x*y)*z), \lambda_{A\cup B}(y)\}, \forall x, y, z \in X$
$A\cup B$ is an intuitionistic fuzzy T-ideal of BCI-algebras X.

Theorem3.21: An IFS $A = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy T-ideals of X if and only if the fuzzy sets α_A and β_A are fuzzy T- ideals of X.

Proof: $A = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy T-ideals of X.
Clearly, α_A is a fuzzy T-ideals of X. $\forall x, y, z \in X$,
We have $\beta_A(0) = 1- \beta_A(0) \geq 1- \beta_A(x) = \beta_A(x)$,
$\beta_A(x*z) = 1- \beta_A(x*z)$
$\geq 1- \max\{\beta_A((x * y) * z), \beta_A(y)\}$
$= \min\{1- \beta_A((x * y) * z), 1- \beta_A(y)\}$
$\beta_A(x*z) = \min\{\beta_A((x *y) * z), \beta_A(y)\}$,
Hence β_A is a fuzzy T-ideal of X.
Conversely, assume that α_A, β_A are fuzzy T- ideals of X. $\forall x, y, z \in X$,
We get $\alpha_A(0) \geq \alpha_A(x)$, $1- \beta_A(0) = \beta_A(0) \geq \beta_A(x) = 1 - \beta_A(x)$
$\beta_A(0) \leq \beta_A(x)$; $\alpha_A(x) \geq \min\{\alpha_A((x * y) *z), \alpha_A(y)\}$ and
$1-\beta_A(x*z) = \beta_A(x*z) \geq \min\{\beta_A((x * y) *z), \beta_A(y)\}$
$= \min\{1- \beta_A((x *y) * z), 1- \beta_A(y)\}$
$= 1- \max\{\beta_A((x * y) *z), \beta_A(y)\}$
$\beta_A(x*z) = \max\{\beta_A((x * y) * z), \beta_A(y)\}$. Hence $A = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy T-ideals of X

Theorem 3.22: Let f: $X \rightarrow Y$ is a Homo of BCI-algebra. If μ_A and λ_A is a intuitionistic fuzzy T-ideal of Y, then $\mu^f_A$is an intuitionistic fuzzy T-ideal of X

Proof: For any $x \in X$, we have $\mu^f_A(x) = \mu_A[f(x)] \leq \mu_A(0^1) = \mu^f(0)] = \mu^f_A(x) = \mu^f_A(0)$,
Thus $\mu^f_A(x) \leq \mu^f_A(0), \forall x, \in X$,
Let x, y, z, $\in X$. Then $T[\mu^f_A[((x*y)*z), \mu^f_A(y) = T[\mu_A[f((x*y)*z), \mu_A(y)]] = T[\mu_A[(f(x)* f(y)) *f(z)), f(y)\}]] \leq \mu_A[f(x) = \mu_A f(x *z)]$
$T[\mu^f_A[((x*y)*z), \mu^f_A(y)] \leq \mu^f_A(x *z)], \forall x, y, z \in X$
$\lambda^f_A(x) = \lambda_A[f(x)] \geq \lambda_A(0^1) = \lambda^f_A(0)] = \lambda^f_A(x) = \lambda^f_A(0), \forall x \in X$
Thus $\lambda^f_A(x) \geq \lambda^f_A(0), \forall x, \in X$
Let x, y, z, $\in X$ .Then $T[\lambda^f_A[((x*y)*z), \lambda^f_A(y) = T[\lambda_A[f((x*y) * z), \lambda A(y)]]$

$= T [\lambda_A [((f(x) *f (y)) *f (z)), \lambda A [f(y)]\}$

$\geq\lambda_A [f(x *z)]$

$= \lambda^f_A(x *z)$

$T [\lambda^f_A [((x*y)*z), \lambda^f_A(y) \geq \lambda^f_A(x *z)], \forall x, y, z \in X$

Theorem3.23Let f: X → Y is an epimorphism of BCI-algebra. If μ^f_A is an intuitionistic fuzzy T-ideal of X, then μ_A is an intuitionistic fuzzy T-ideal of Y.

Proof: For any $x\in X$, We have $\mu^f_A(x) = \mu_A\{f(x)\} \leq \mu_A (0) = \mu^f_A (f (0)) =\mu^f_A (0^1)$ and $\lambda^f_A(x) = \lambda_A \{f(x)\} \leq \lambda_A (0) = \lambda^f_A (f (0)) =\lambda^f_A (0^1)$,
Thus $\mu^f_A(x) \leq \mu^f_A (0)$, and $\lambda^f_A (x) \geq\lambda^f_A(0),\forall x, \in X$.
Let x, y, z, ∈ X. Then there exists a, b, c ∈ X such that f (a) = x, f (b) = y, f(c) = z. it follows that$\mu_A (x *z) = \mu_A(f (a) * f(c)) = \mu^f_A (a *c) \geq \min \{\mu^f_A ((a*b) * c), \mu^f_A (b)\}= \min \{\mu_A\{((f (a) *f(b))*f(c))\}, \mu_A f(b)\}, \geq\min \{\mu_A ((x* y) *z), \mu_A(y)\}$,
$\lambda_A (x * z) = \lambda_A(f (a) * f(c)) = \lambda^f_A (a *c)\leq \max \{\lambda^f_A ((a*b) *c), \lambda^f_A (b)\}$
$= \max \{\lambda_A ((f (a)*f (b))*f(c)), \lambda_A f (b)\}\leq \max \{\lambda_A ((x*y)*z), \lambda_A(y)\}$
Therefore μ_A is an intuitionisticic fuzzy T-ideal of Y.

Theorem 3.24: Let f: X→ Y be onto BCI – homomorphism. If an intuitionistic fuzzy subset B of Y with membership Function μ_B is an intuitionistic fuzzy T-ideal, then the fuzzy subset $f^{-1}(B)$ is also an intuitionistic fuzzy T-ideal of X.

Proof: Let y ∈ Y Since f into, there exists x ∈X. Y= f (x) since B is an intuitionistic fuzzy T-ideal of Y. It follows that $\mu_B (0) \geq\mu_B (y)$, $\mu_B f (0) \geq\mu_B (x)$, then by definition $\mu^{-1}_{f (B)} (x)$ for all x ∈ X. Next B is an intuitionistic fuzzy T-ideal. Therefore for any y_1, y_2, y_3 in Y
$\mu_B (y_1 *y_3) \geq \min\{ \mu_B ((y_1*y_2)*y_3), \mu_B (y_2)\} = \min\{ \mu_B \{(f(x_1)*f(x_2)) *f(x_3))\}, \mu_B (f (x_2))\}$,
$= \min \{\mu_B ((f(x_1)*f(x_2))*f(x_2)), \mu_B (f (x_2))\}$,
$\mu_B f(x_1*x_3) \geq \min \{\mu^{-1}_{f (B)} ((x_1*x_2)*x_3), \mu^{-1}_{f (B)} (x_2)\}$,
It follows that $\lambda_B (0) \geq\lambda_B (y)$ $\lambda_B f (0) \geq\lambda_B (x)$,
Then by definition $\lambda^{-1}_{f (B)} (x)$ for all x ∈ X.
Next B is an intuitionistic fuzzy T-ideal.
Therefore for any y_1, y_2, y_3 in Y
$\lambda_B (y_1 *y_3) \leq \max \{ \lambda_B ((y_1*y_2)*y_3), \lambda_B (y_2)\}=\max \{ \lambda_B ((f(x_1)*f(x_2))* f(x_3)), \lambda_B (f (x_2))\}$,
$\lambda_B f(x_1*x_3) \geq \min \{\lambda^{-1}_{f (B)} ((x_1*x_2)*x_3), \lambda^{-1}_{f (B)} (x_2)\}$,
Which proves that f^{-1} (B) is an intuitionistic fuzzy T-ideal of X.

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