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An Integral Associated With the Modified Saigo Operators Involving Two I-Functions and Generalized Polynomials

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Abstract: *In this paper, we obtained two fractional integrals involving the product of two I-functions [10], general class of polynomials and Gauss hypergeometric function. By making use of these integrals, we have obtained two theorems based on modified Saigo operators of fractional integration.*
Keywords: *fractional integral, Saigo operators, Riemann-Liouville operator, Weyl operator, Erdélyi- Kober operator, I-function, gauss hypergeometric function and general class of multivariable polynomial.*

I. Introducation

Modified Saigo Operators:

Let f, g, h be complex numbers and δ>0. The modified Saigo operators denoted by $\Delta_{0,z,\delta}^{f,g,h}$ and $\nabla_{z,\infty,\delta}^{f,g,h}$ are defined as following:

$$\Delta_{0,z,\delta}^{f,g,h}(\phi) = \delta z^{-\delta(f+g)} \int_0^z \left(z^\delta - u^\delta\right)^{f-1} {}_2F_1\left(f+g,-h;f;\frac{1-u^\delta}{z^\delta}\right).u^{\delta-1}\phi(u)du,$$

$$\mathrm{Re}(f) > 0 \qquad \ldots (1)$$

$$= \frac{d^n}{d\left(z^\delta\right)^n} \Delta_{0,z,\delta}^{f+n,g-n,h-n}(\phi), \qquad 0 < \mathrm{Re}(f) + n \le 1 \qquad \ldots (2)$$

Where ${}_2F_1$ (f +g+ h) is the Gauss hypergeometric function and

$$\nabla_{z,\infty,\delta}^{f,g,h}(\phi) = \frac{\delta}{\Gamma(f)} \int_z^\infty \left(u^\delta - z^\delta\right)^{f-1} u^{-\delta(f+g)} {}_2F_1\left(f+g,-h;f;\frac{1-z^\delta}{u^\delta}\right).u^{\delta-1}\phi(u)du,$$

$$\mathrm{Re}(f) > 0 \qquad \ldots (3)$$

$$= (-1)^n \frac{d^n}{d\left(z^\delta\right)^n} \nabla_{z,\infty,\delta}^{f+n,g-n,h-n}(\phi), \qquad 0 < \mathrm{Re}(f) + n \le 1 \qquad \ldots (4)$$

Sufficient conditions for the existence of (1) and (3) are δ>0,

$$\mathrm{Re}(f) > 1 - \frac{1}{2\delta}; \qquad \phi(z) \in L_2\left(R_+\right) \qquad \ldots (5)$$

and $\max.[0, \mathrm{Re}(g-h)] > 1 - \frac{1}{2\delta}$;

$$\min.[\mathrm{Re}(g), \mathrm{Re}(h)] > -\frac{1}{2\delta};$$

If these conditions are satisfied, then $\Delta_{0,z,\delta}^{f,g,h}\phi(z)$ and $\nabla_{z,\infty,\delta}^{f,g,h}\phi(z)$ both exist and also both $\in L_2\left(R_+\right)$

The operators $\Delta_{0,z,\delta}^{f,g,h}$ and $\nabla_{z,\infty,\delta}^{f,g,h}$ include as their special case g = -f, the fractional calculus of Riemann-Liouville and Weyl types:

$$\Delta_{0,z,\delta}^{f,-f,h}(\phi) = R_{0,z,\delta}^{f}(\phi); \qquad \nabla_{z,\infty,\delta}^{f,-f,h}(\phi) = W_{z,\infty,\delta}^{f}(\phi) \qquad \ldots (6)$$

When δ=1, we obtain the following identities and inverses:

$$\Delta_{0,z,\delta}^{0,0,h}(\phi) = \phi(z); \qquad \nabla_{z,\infty,\delta}^{0,0,h}(\phi) = \phi(z) \qquad \ldots (7)$$

$$\left[\Delta_{0,z,\delta}^{f,g,h}\right]^{-1} = \Delta_{0,z,\delta}^{-f,-g,f+h}; \qquad \left[\nabla_{z,\infty,\delta}^{f,g,h}\right]^{-1} = \nabla_{z,\infty,\delta}^{-f,-g,f+h} \qquad \ldots (8)$$

For the operators $\Delta_{0,z,\delta}^{f,g,h}$ and $\nabla_{z,\infty,\delta}^{f,g,h}$ there hold interesting results similar to the once derived in a series of earlier papers [11] to [22].

Here we shall study another generalization of (1) and (3) which is given as following:

$$\Delta_{0,z,\delta;t,s,r;k,\eta,\sigma}^{\mu;f',g',h';M,N,f,g,h}\{\phi(z)\}=\frac{\delta z^{-\delta}(f+g)}{\Gamma(f)}\int_0^z\left(z^{\delta}-u^{\delta}\right)^{f-1}{}_2F_1\left(f+g,-h;f;\frac{1-u^{\delta}}{z^{\delta}}\right)$$

$$.u^{\delta-1}zS_n^{f',g',h'}\left[xu^{\mu};t,s,r,M,N,k,\eta,\sigma\right]\phi(u)du \quad \ldots (9)$$

And

$$\nabla_{z,\infty,\delta;t,s,r;k,\eta,\sigma}^{\mu;f',g',h';M,N,f,g,h}\{\phi(z)\}=\frac{\delta}{\Gamma(f)}\int_z^{\infty}\left(u^{\delta}-z^{\delta}\right)^{f-1}u^{-\delta(f+g)}{}_2F_1\left(f+g,-h;f;\frac{1-z^{\delta}}{u^{\delta}}\right)$$

$$.u^{\delta-1}S_n^{f',g',h'}\left[xu^{\mu};t,s,r,M,N,k,\eta,\sigma\right]\phi(u)du \quad \ldots (10)$$

Where Re (f) > 0 and $S_n^{f',g',h'}$[x] stands for the generalized polynomial set defined by the following Rodrigues type formula [9, p.64, eq. (2.18)],

$$S_n^{f',g',h'}\left[z;t,s,r,M,N,k,\eta,\sigma\right]=(Mz+N)^{-f'}\left(1-M'z^{t}\right)^{\frac{-g'}{h'}}$$

$$.T_{\eta,\sigma}^{k+n}\left[(Mz+N)^{f'+rn}\left(1-M'z^{t}\right)^{\frac{g'}{u}+sn},\right] \quad \ldots (11)$$

With the differential operator $T_{\eta,\sigma}$ being defined as

$$T_{\eta,\sigma}\equiv z^{\sigma}\left(\eta+z\frac{d}{dz}\right) \quad \ldots (12)$$

An explicit form of this generalized polynomial set [9, p.71, eq. (2.3.4)] is given by

$$S_n^{f',g',h'}\left[z;t,s,r,M,N,k,\eta,\sigma\right]=B^{rn}z^{\sigma(k+n)}\left(1-M'z^{t}\right)^{sn}\sigma^{k+n}$$

$$.\sum_{\lambda=0}^{k+n}\sum_{k=0}^{\lambda=0}\sum_{y=0}^{k+n}\sum_{i=0}^{j}\frac{(-1)^{j}(-j_i)(f')_j(-\lambda)_k(-f'-rn)_j}{i!\,j!\,\lambda!(1-f'-j)_i}$$

$$.\left(-\frac{g'}{h'}-sn\right)_{\lambda}\left(\frac{i+\eta+tk}{\sigma}\right)_{k+n}\left(\frac{-h'z^{t}}{1-h'z^{t}}\right)\left(\frac{Mz}{N}\right)^{j} \quad \ldots (13)$$

It may be noted that the polynomial set defined by (11) is of general character and unifies and extends a number of classical polynomials introduced and studied by various authors, such as Chatterjea [2], Dhillon [4], Gould and Hopper [5], Krall and Frink [7], Singh [24], Singh and Srivastava [25], etc.

The Series I-Function

The I-function is defined in [10, p.2.99, eq. (3.1)] as follows:

$$I_{p_1,q_1}^{m_1,n_1}[x']=I_{p_1,q_1}^{m_1,n_1}\left[x'\left|\begin{matrix}(h'_1,\gamma'_1,M'_1),\ldots,(h'_p,\gamma'_p,M'_p)\\(l'_1,\theta'_1,N'_1),\ldots,(l'_p,\theta'_p,N'_q)\end{matrix}\right.\right]$$

$$=\frac{1}{2\pi i}\int_L\psi(w)dw \quad \ldots (14)$$

Where

$$\psi(w)=\frac{\prod_{j=1}^{m_1}\left\{\Gamma(\ell_j-\theta_j w)\right\}^{N_j}\prod_{j=1}^{m_1}\left\{\Gamma(1-h_j+\gamma_j w)\right\}^{M_j}}{\prod_{j=m_1+1}^{q_1}\left\{\Gamma(1-\ell_j+\theta_j w)\right\}^{N_j}\prod_{j=n_1+1}^{p_1}\left\{\Gamma(h_j-\gamma_j w)\right\}^{M_j}} \quad \ldots (15)$$

Here m_1, n_1, p_1, q_1 are non negative integers satisfying $1 \le m_1 \le q, 0 \le n_1 \le p, \quad h_i, \ell_i$ $(i = 1,..., p; j = 1,..., q)$ are complex numbers and $\gamma_i > 0, \theta_i > 0,$
L is a suitable contour. The sufficient conditions for the absolute convergence of the contour integral (15) are given in [10].

$$\Omega = \sum_{j=1}^{m_1} \left| N_j \theta_j \right| - \sum_{j=m_1+1}^{q_1} \left| N_j \theta_j \right| + \sum_{j=1}^{n_1} \left| M_j \gamma_j \right| - \sum_{j=n_1+1}^{p_1} \left| M_j \gamma_j \right| > 0 \qquad \ldots (16)$$

This condition provides exponential decay of the integrand in (14) and region of absolute convergence of the function defined by (14) is

$$\left| \arg x' \right| < \frac{1}{2} \pi \Omega \qquad \ldots (17)$$

Now we define the following representation of the I-function in a computable series [10] as following:

$$I(x') = I_{p_1,q_1}^{m_1,n_1}[x'] = \sum_{c=1}^{m_1} \sum_{s=0}^{\infty} \frac{(-1)^{\rho} R(w') x'^{w'}}{(\rho)! N'_c} \qquad \ldots (18)$$

Where

$$w' = \frac{(\theta'_c + s)}{N'_c}$$

Equation (18) exists for

$$0 < |x'| < \infty \text{ If } \mu^* = 0 \text{ and } 0 < |x'| < 1/\tau^*,$$

$$\mu^* = \sum_{j=1}^{m_1} \left| (N'_j) \right| + \sum_{j=m_1+1}^{q_1} \left| (N'_j \ell'_j) \right| - \sum_{j=1}^{p_1} \left| (M'_j h'_j) \right|,$$

$$\tau^* = \left[\prod_{j=1}^{m_1} (N'_j)^{-N'_j} \right] \left\{ \prod_{j=1}^{p_1} (M'_j)^{M'_j h'_n} \right\} \left\{ \prod_{j=m_1+1}^{q_1} (N'_j)^{-N'_j \ell'_n} \right\},$$

$$N'_c (\theta'_c + s'_1) \ne N'_j (\theta'_c + s'_2) \quad \text{For} \quad j \ne c (j, c = 1,..., m_1; s_1, s_2 = 0,1,2,....),$$

$$R(w') = [\psi(w')]_{\ell'_1,...\ell'_m=1} = \frac{\left[\prod_{j=1}^{m_1} \{\Gamma(\theta'_j - N'_j w')\} \right] \left[\prod_{j=1}^{n_1} \{\Gamma(1 - \gamma'_j + M'_j w')\} \right]^{h'_j}}{\prod_{j=m_1+1}^{q_1} \{\Gamma(1 - \theta'_j + N'_j w')\}^{\ell'_j} \prod_{j=n_1+1}^{p_1} \{\Gamma(\gamma'_j - M'_j w')\}^{h'_j}}$$

MAIN THEOREMS We establish the main theorem based on I-function pertaining to the modified Saigo operators denoted by $\Delta_{0,z,\delta}^{f,g,h}$ and $\nabla_{z,\infty,\delta}^{f,g,h}$.

Theorem- 01
It will be shown here that

$$\Delta_{0,z,\delta;t,\gamma;r;k,\eta,\sigma}^{\mu;f',g',h';M,N,f,g,h} \left\{ u^{\mu'} I_{p_1,q_1}^{m_1,n_1} \left[f' u^{\delta'} \left| \begin{matrix} (h'_1, \gamma'_1, M'_1),...,(h'_p, \gamma'_p, M'_p) \\ (l'_1, \theta'_1, N'_1),...,(l'_p, \theta'_p, N'_q) \end{matrix} \right. \right] \right\} . S_V^U \left[u^{\upsilon'} \right]$$

$$. I_{p_2,q_2}^{m_2,n_2} \left[u^{\theta'} \left| \begin{matrix} (h'_1, \gamma'_1, M'_1),...,(h'_p, \gamma'_p, M'_p) \\ (l'_1, \theta'_1, N'_1),...,(l'_p, \theta'_p, N'_q) \end{matrix} \right. \right]$$

$$= \sum_{\lambda=0}^{m_1+n_1} \sum_{t=0}^{\lambda} \sum_{j=0}^{m_1+n_1} \sum_{i=0}^{j} \sum_{c=1}^{m_2} \sum_{\rho=0}^{\infty} \sum_{K=0}^{[V/U]} \frac{(-V)_{UK}}{K!} A_{V,K} \xi(i,j,k,r') x^{\sigma(m_1+n_1)+t\lambda+j} \frac{(-1)^{\rho} R(w')}{(\rho)! N'_c}$$

$$. z^{\mu' + \mu[\sigma(m_1+n_1)+t\lambda+j] + \theta' w' + \nu' K - \delta g} I_{p_1+2,q_1+2}^{m_1,n_1+2}$$

$$\left[f' z^{\delta'} \middle| \begin{matrix} \left(1-\frac{\beta}{\delta},\frac{\delta'}{\delta},1\right),\left(1-\frac{\beta}{\delta}+h-g,\frac{\delta^*}{\delta},1\right),(h'_1,\gamma'_1,M'_1),\dots,(h'_p,\gamma'_p,M'_p) \\ (\ell'_1,\theta'_1,N'_1),\dots,(\ell'_q,\theta'_q,N'_q),\left(1-\frac{\beta}{\delta}-g,\frac{\delta^*}{\delta},1\right),\left(1-\frac{\beta}{\delta}+h+f,\frac{\delta'}{\delta},1\right) \end{matrix} \right] \quad \dots (19)$$

Where

$$\xi(i,j,k,r') = (N')^{rn'} \sigma^{m_1+n_1} \frac{(-1)^j (-j_i)(f')_j (-\lambda)_k (-f'-rn)_i}{i!\, j!\, \lambda!\, k!\, (1-f'-j)_i}$$

$$\cdot\left(-\frac{\theta'}{h'}-wn'\right)_\lambda \left(\frac{i+r+tk}{\sigma}\right)_{m_1+n_1} \left(\frac{M'}{N'}\right)^j (-M')^\lambda \quad \dots (20)$$

And

$$\beta = \mu' + \delta + \mu\sigma(m_1+n_1) + \mu t\lambda + \theta' w' + \upsilon' K + \mu j \quad \dots (21)$$

Equation (19) holds true under the following conditions:

(i) $\left| M' x^t \right| < 1;$

(ii) Re (f) > 0;

(iii) $\Omega > 0, \left|\arg f'\right| < \left(\frac{1}{2}\right)\Omega\pi,$

Where

$$\Omega = \sum_{j=1}^{m_1} \left|N'_j \theta_j\right| - \sum_{j=m_1+1}^{q_1} \left|N'_j \theta_j\right| + \sum_{j=1}^{n_1} \left|M'_j \gamma_j\right| - \sum_{j=n_1+1}^{p_1} \left|M'_j \gamma_j\right|; \quad \dots (22)$$

(iv) U is an arbitrary positive integer and the coefficients $A_{V,K} (V, K \geq 0)$ are arbitrary constants, real or complex.

(v) The series occurring on the right hand side of (19) is absolutely convergent.

Proof:

In view of definition (9), the left hand side of (19)

$$= \frac{\delta z^{-\delta(f+g)}}{\Gamma(f)} \int_0^z \left(z^\delta - u^\delta\right)^{f-1} u^{\mu'+\delta-1} {}_2F_1\left(f+g,-h;f;\frac{1-u^\delta}{z^\delta}\right) I_{p_2,q_2}^{m_2,n_2}\left[u^{\theta'}\right] S_V^U [u^{\upsilon'}]$$

$$.I_{p_1,q_1}^{m_1,n_1}\left[f' u^{\delta'}\right] S_n^{f',g',h'}\left[x; u^\mu; t,s,r,M,N,k,\eta,\sigma\right] du \quad \dots (20)$$

Using (13), (14) and (18), we get the left hand side of (19)

$$= \sum_{\lambda=0}^{m_1+n_1} \sum_{k=0}^{\lambda} \sum_{j=0}^{m_1+n_1} \sum_{i=0}^{j} \sum_{c=1}^{m_2} \sum_{\rho=0}^{\infty} \sum_{K=0}^{[V/U]} \frac{(-V)_{UK}}{K!} A_{V,K} \xi(i,j,k,r') x^{\sigma(m_1+n_1)+t\lambda+j} \frac{(-1)^\rho R(w')}{(\rho)!\, N'_c} \frac{1}{2\pi i} \int_L \psi(y) f^y$$

$$\cdot\left[\frac{\delta z^{-\delta(f+g)}}{\Gamma(f)} \left\{ \int_0^z \left(z^\delta - u^\delta\right)^{f-1} u^{\mu'+\delta+\mu\sigma(m_1+n_1)+\mu\lambda t+\mu j+y\delta'+\theta' w'+\upsilon' K-1} \cdot {}_2F_1\left(f+g,-h;f;\frac{1-u^\delta}{z^\delta}\right) du \right\}\right] dy \quad \dots (21)$$

Where $\xi(i,j,k,r')$ and $\psi(y)$ are defined by (20) and (14) respectively and

$$\text{Re}\,(f') > 0, \text{Re}\,(\mu) > 0, \text{Re}\left(\frac{\mu}{M}+\varepsilon'-g'\right) > 0, M > 0, K = 0,1,2,.....$$

Now using the following result given by Saigo and Saxena [16, p.57, Eq. (4.16)]

$$M\int_L k^{\mu-1}\left(z^h-k^h\right)^{f'-1}{}_2F_1\left(f'+g',-\varepsilon';f';\frac{1-k^h}{z^h}\right)dk$$

$$=\frac{\Gamma(f')\Gamma\left(\frac{\mu}{M}\right)\Gamma\left(\frac{\mu}{M}+\varepsilon'-g'\right)}{\Gamma\left(\frac{\mu}{M}-g'\right)\Gamma\left(\frac{\mu}{M}+\varepsilon'+f'\right)}.z^{f'M-\mu-Mg'} \quad \ldots (22)$$

Now interpreting the result then obtained with the help of (18), we reach at the desired result (19).

Theorem-02

It Will Be Shown Here That

$$\nabla^{\mu;f',g',h';M,N,f,g,h}_{z,\infty,\delta;t,\gamma',r;k,\eta,\sigma}\left\{u^{\mu'}I^{m_1,n_1}_{p_1,q_1}\left[f'u^{\delta'}\left|\begin{matrix}(h'_1,\gamma'_1,M'_1),\ldots,(h'_p,\gamma'_p,M'_p)\\(l'_1,\theta'_1,N'_1),\ldots,(l'_p,\theta'_p,N'_q)\end{matrix}\right.\right]\right\}.S^U_V\left[u^{\upsilon'}\right]$$

$$.I^{m_2,n_2}_{p_2,q_2}\left[u^{\theta'}\left|\begin{matrix}(h''_1,\gamma''_1,M''_1),\ldots,(h''_p,\gamma''_p,M''_p)\\(l''_1,\theta''_1,N''_1),\ldots,(l''_p,\theta''_p,N''_q)\end{matrix}\right.\right]\}$$

$$=\sum_{\lambda=0}^{m_1+n_1}\sum_{t=0}^{\lambda}\sum_{j=0}^{m_1+n_1}\sum_{i=0}^{j}\sum_{c=1}^{j}\sum_{\rho=0}^{m_2}\sum_{K=0}^{[V/U]}\frac{(-V)_{UK}}{K!}A_{V,K}\xi(i,j,k,r')x^{\sigma(m_1+n_1)+t\lambda+j}\frac{(-1)^{\rho}R(w')}{(\rho)!N'_c}$$

$$.z^{\mu'+\mu[\sigma(m_1+n_1)+t\lambda+j]+\theta'w'+\nu'K-\delta g}I^{m_1,n_1+2}_{p_1+2,q_1+2}$$

$$\left[f'z^{\delta'}\left|\begin{matrix}(h'_1,\gamma'_1,M'_1),\ldots,(h'_p,\gamma'_p,M'_p),\left(1-f-g-\frac{\beta'}{\delta},\frac{\delta'}{\delta},1\right),\left(1+h-\frac{\beta'}{\delta},\frac{\delta^*}{\delta},1\right)\\\left(1-f-\frac{\beta}{\delta},\frac{\delta'}{\delta},1\right),\left(1-f-g+h-\frac{\beta'}{\delta},\frac{\delta'}{\delta},1\right),(\ell'_1,\theta'_1,N'_1),\ldots,(\ell'_q,\theta'_q,N'_q)\end{matrix}\right.\right] \quad \ldots (23)$$

Where

$$\beta'=\mu'+\delta+\mu\sigma(m_1+n_1)+\mu t\lambda+\theta'w'+\upsilon'K+\mu j-\delta(f+g) \quad \ldots (24)$$

Proof:

In view of definition (10), the left hand side of (23)

$$=\frac{\delta}{\Gamma(f)}\int_z^{\infty}\left(u^{\delta}-z^{\delta}\right)^{f-1}u^{\mu'-\delta(f+g)+\delta-1}{}_2F_1\left(f+g,-h;f;\frac{1-z^{\delta}}{u^{\delta}}\right)I^{m_2,n_2}_{p_2,q_2}\left[u^{\theta'}\right]S^U_V[u^{\upsilon'}]$$

$$.I^{m_1,n_1}_{p_1,q_1}\left[f'u^{\delta'}\right]S^{f',g',h'}_n\left[x;u^{\mu};t,s,r,M,N,k,\eta,\sigma\right]du \quad \ldots (25)$$

Using (13), (14) and (18), we get the left hand side of (23)

$$=\sum_{\lambda=0}^{m_1+n_1}\sum_{k=0}^{\lambda}\sum_{j=0}^{m_1+n_1}\sum_{i=0}^{j}\sum_{c=1}^{m_2}\sum_{\rho=0}^{\infty}\sum_{K=0}^{[V/U]}\frac{(-V)_{UK}}{K!}A_{V,K}\xi(i,j,k,r')x^{\sigma(m_1+n_1)+t\lambda+j}\frac{(-1)^{\rho}R(w')}{(\rho)!N'_c}\frac{1}{2\pi i}\int_L\psi(y)f^y$$

$$.\left[\frac{\delta}{\Gamma(f)}\left\{\int_z^{\infty}\left(u^{\delta}-z^{\delta}\right)^{f-1}u^{\mu'+\delta+\mu\sigma(m_1+n_1)+\mu\lambda t+\mu j+y\delta'+\theta'w'+\upsilon'K+\delta(f+g)-1}.{}_2F_1\left(f+g,-h;f;\frac{1-z^{\delta}}{u^{\delta}}\right)du\right\}\right]dy$$

$$\ldots (26)$$

Where $\xi(i,j,k,r')$ and $\psi(y)$ are defined by (20) and (14) respectively and

$$\mathrm{Re}\,(f')>0,\mathrm{Re}\left(1-f'-\frac{\mu}{M}'\right)>0,\mathrm{Re}\left(1-f'-g'+\varepsilon'-\frac{\mu}{M}'\right)>0, M>0, K=0,1,2,.....$$

Now using the following result given by Saigo and Saxena [16, p.57, Eq. (4.17)]

$$M\int_z^{\infty} k^{\mu-1}\left(k^h-z^h\right)^{f'-1}{}_2F_1\left(f'+g',-\varepsilon';f';\frac{1-z^h}{k^h}\right)dk$$

$$=\frac{\Gamma(f')\Gamma\left(1-f'-\frac{\mu}{M}\right)\Gamma\left(1-f'-g'-\frac{\mu}{M}+\varepsilon'\right)}{\Gamma\left(1-f'-g'-\frac{\mu}{M}\right)\Gamma\left(1+\varepsilon'-\frac{\mu}{M}\right)}.z^{f'M-\mu-Mg'} \qquad \ldots (27)$$

Now interchanging the order of integration and summation, we reach at the desired result (26).

Special Cases

1. Taking $M_j = N_j = 1\ \forall$ j in (19), the I-function reduces to well known Fox's H-function [3] as following:

$$\Delta_{0,z,\delta;t,\gamma;r;k,\eta,\sigma}^{\mu;f',g',h';M,N,f,g,h}\left\{u^{\mu'}I_{p_2,q_2}^{m_2,n_2}\left[u^{\theta'}\right]H_{p_1,q_1}^{m_1,n_1}\left[f'u^{\delta'}\right]S_V^U\left[u^{\upsilon'}\right]\right\}$$

$$=\left[A^*\right]\times H_{p_1+2,q_1+2}^{m_1,n_1+2}\left[f'z^{\delta'}\left|\begin{matrix}\left(1-\frac{\beta}{\delta},\frac{\delta'}{\delta}\right),\left(1-\frac{\beta}{\delta}+h+g,\frac{\delta'}{\delta}\right),\left(h_j,\gamma_j\right)_{1,p_1}\\ \left(\ell_j,\theta_j\right)_{1,q_1},\left(1-\frac{\beta}{\delta}+g,\frac{\delta'}{\delta}\right),\left(1-\frac{\beta}{\delta}+h+f,\frac{\delta'}{\delta}\right)\end{matrix}\right.\right] \qquad \ldots (27)$$

Where

$$\left[A^*\right]=\sum_{\lambda=0}^{m_1+n_1}\sum_{t=0}^{\lambda}\sum_{j=0}^{m_1+n_1}\sum_{i=0}^{j}\sum_{c=1}^{m_2}\sum_{\rho=0}^{\infty}\sum_{K=0}^{[V/U]}\frac{(-V)_{UK}}{K!}A_{V,K}\xi(i,j,k,r')x^{\sigma(m_1+n_1)+t\lambda+j}\frac{(-1)^{\rho}R(w')}{(\rho)!N'_c}$$

$$.z^{\mu'+\mu[\sigma(m_1+n_1)+t\lambda+j]+\theta'w'+\nu'K-\delta g} \qquad \ldots (28)$$

2. If we take M_j (j= n_1+1,…,p) = N_j (j= 1,…,m_1) in (19), the I-function reduces to $\overline{H}$-function defined by Innayat-Hussain [6], as following:

$$\Delta_{0,z,\delta;t,\gamma;r;k,\eta,\sigma}^{\mu;f',g',h';M,N,f,g,h}\left\{u^{\mu'}I_{p_2,q_2}^{m_2,n_2}\left[u^{\theta'}\right]\overline{H}_{p_1,q_1}^{m_1,n_1}\left[f'u^{\delta'}\right]S_V^U\left[u^{\upsilon'}\right]\right\}$$

$$=\left[A^*\right]\times \overline{H}_{p_1+2,q_1+2}^{m_1,n_1+2}\left[f'z^{\delta'}\left|\begin{matrix}\left(1-\frac{\beta}{\delta}+h-g,\frac{\delta'}{\delta},1\right),\left(h_j,\gamma_j,M_j\right)_{1,n_1},\left(1-\frac{\beta'}{\delta},\frac{\delta'}{\delta},1\right),\left(h_j,\gamma_j\right)_{n_1+1,p_1}\\ \left(\ell_j,\theta_j\right)_{1,m_1},\left(\ell_j,\theta_j,N_j\right)_{m_1+1,q_1},\left(1-\frac{\beta}{\delta}-g,\frac{\delta'}{\delta},1\right),\left(1-\frac{\beta}{\delta}+h+f,\frac{\delta'}{\delta},1\right)\end{matrix}\right.\right] \qquad \ldots (29)$$

Where β is defined in (21) and A^* is defined in (28).

3. If we use the identity $\Delta_{0,z,0}^{f,-f,h}f=R_{0,z,\delta}^{f}f$ in (19), then we obtain the result for the Riemann-Liouville operator.

$$R_{0,z,\delta;t,\gamma;r;k,\eta,\sigma}^{\mu;f',g',h';M,N,f}\left\{u^{\mu'}I_{p_1,q_1}^{m_1,n_1}\left[f'u^{\delta'}\right]\right\}.S_V^U\left[u^{\upsilon'}\right]I_{p_2,q_2}^{m_2,n_2}\left[u^{\theta'}\right]$$

$$=\sum_{\lambda=0}^{m_1+n_1}\sum_{k=0}^{\lambda}\sum_{j=0}^{m_1+n_1}\sum_{i=0}^{j}\sum_{c=1}^{m_2}\sum_{\rho=0}^{\infty}\sum_{K=0}^{[V/U]}\frac{(-V)_{UK}}{K!}A_{V,K}\xi(i,j,k,r')x^{\sigma(m_1+n_1)+t\lambda+j}\frac{(-1)^{\rho}R(w')}{(\rho)!N'_c}$$

$$.z^{\mu'+\mu[\sigma(m_1+n_1)+t\lambda+j]+\theta'w'+v'K+\delta g} I^{m_1,n_1+1}_{p_1+1,q_1+1}$$

$$\left[f'z^{\delta'} \left| \begin{matrix} \left(1-\frac{\beta}{\delta},\frac{\delta'}{\delta},1\right),(h'_1,\gamma'_1,M'_1),\ldots,(h'_p,\gamma'_p,M'_p) \\ (\ell'_1,\theta'_1,N'_1),\ldots,(\ell'_q,\theta'_q,N'_q),\left(1-\frac{\beta}{\delta}+f,\frac{\delta'}{\delta},1\right) \end{matrix} \right. \right] \quad \ldots (30)$$

II. Conclusion

The results derived in this paper are useful for preparing the table of Riemann-Liouville operator, Weyl operator, Erdélyi- Kober operator and Saigo operator of fractional calculus.

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