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A solution of the Burger's equation arising in the Longitudinal Dispersion Phenomena in Fluid Flow through Porous Media by Sumudu transform Homotopy perturbation Method

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Abstract: *The goal of the paper is to examine the concentration of the longitudinal dispersion phenomenon arising in fluid flow through porous media. The solution of the Burger's equation for the dispersion problem is presented by approach to Sumudu transformation. The solution is obtained by using suitable conditions and is more simplified under the standard assumptions.*
Keywords: *Longitudinal Dispersion phenomenon, porous media, Sumudu transform.*

I. Introduction

The present paper discusses the solution of Burger's equation [7] which represents longitudinal dispersion of miscible fluid flow through porous media.

Miscible displacement is one type of poly-phase flow in porous media, where both of the phases are completely soluble in each other. Hence the capillary forces between these two fluids do not come into effect. The miscible displacement could be described in a very simple form as follows:

The mixture of the fluids, under the conditions of complete miscibility, could be thought to behave as a single-phase fluid. Therefore it will obey the Darcy's law.

The change of connection would be caused by diffusion along the flow channels and thus be governed by diffusion of one fluid into the other.

The problem is to find the concentration as a function of time t and position x, as two miscible fluids flow through porous media on either sides of the mixed region, the single fluid equation describes the motion of the fluid. The problem becomes more complicated in one dimension with fluids of equal properties. Hence the mixing takes place longitudinally as well as transversely at time $t = 0$, a dot of fluid having C_0concentration is injected over the phase. It is shown in figure-1. The dot moves in the direction of flow as well as perpendicular to the flow. Finally, it takes the shape of ellipse with a different concentration C_n.

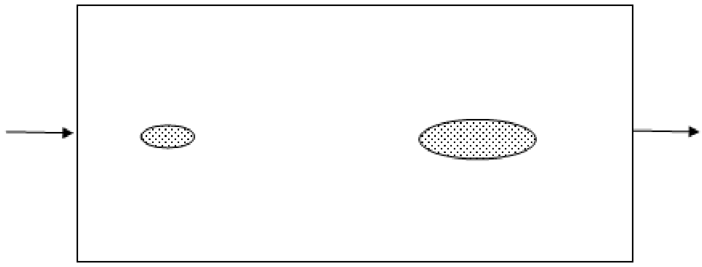


Figure-1 Longitudinal Dispersion Phenomenon

Many researchers have contributed in various physical phenomena. Patel and Mehta [14] have worked on Burger's equation for longitudinal dispersion of miscible fluid flow through porous media. Meher and Mehta [13] have discussed on a new approach to Backlund transformation to solve Burger's equation in longitudinal dispersion phenomenon. Borana, Pradhan and Mehta [12] have discussed numerical solution of Burger's equation.

To solve the differential equations, the integral transform is extensively applied and thus there are several works on the theory and application of integral transforms. In the sequence of these transforms, Watugala [4] introduced a new integral transform, named the Sumudu transform, and further applied it to the solution of ordinary differential equation in control engineering problems; see [4]. For further details and properties of Sumudu transform see [1,3,5,8-10]. The Sumudu transform is defined over the set of the functions

$$A = \left\{ f(t) / \exists M, \tau_1, \tau_2 > 0, |f(t)| < Me^{\frac{t}{\tau_j}}, \text{if } t\epsilon(-1)^j \times [0,\infty) \right\} \qquad (1)$$

by the following formula:

$$\bar{f}(u) = S[f(t); u] = \int_0^\infty f(ut)e^{-t}dt,\ u \in (-\tau_1, \tau_2) \tag{2}$$

II. Mathematical Formulation Of The Problem

According to Darcy's law, the equation of continuity for the mixture, in the case of compressible fluids is given by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{v}) = 0 \tag{3}$$

whereρ is the density for the mixture and $\bar{v}$ is the pore seepage velocity.

The equation of diffusion for a fluid flow through a homogeneous porous medium, without increasing or decreasing the dispersing material is given by

$$\frac{\partial C}{\partial t} + \nabla \cdot (C\bar{v}) = \nabla \cdot \left[\rho \bar{D} \nabla \left(\frac{C}{\rho}\right)\right] \tag{4}$$

Where C is the concentration of the fluid, $\bar{D}$ is the tensor coefficient of dispersion with nine components D_{ij}.

In a laminar flow through homogeneous porous medium at a constant temperature,ρ is constant.

Then$\nabla \cdot \bar{v} = 0$ (5)

Thus equation (4) becomes

$$\frac{\partial C}{\partial t} + \bar{v} \cdot \nabla C = \nabla \cdot (\bar{D} \nabla C) \tag{6}$$

When the seepage velocity is the along x-axis, the non-zero components are $D_{11} = D_L \cong x^2$ (coefficient of longitudinal dispersion, is a function of x along the x-axis) and other D_{ij} are zero. In this case equation (6) becomes

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = x^2 \frac{\partial^2 C}{\partial x^2} \tag{7}$$

where u is the component of velocity along the x-axis, which is time dependent, as well as concentration along the x-axis in $x > 0$ direction, and it is cross-sectional flow velocity of porous medium.

$$\therefore u = \frac{C(x,t)}{C_0}, \text{for } x > 0 \tag{8}$$

The boundary and initial conditions in longitudinal direction are

$$C(0,t) = C_1 (t > 0) \tag{9}$$

$$C(x,0) = C_0 (x > 0) \tag{10}$$

where C_0 is the initial concentration of the tracer (one fluid A) and C_1 is the concentration of the tracer (of the same fluid) at $x = 0$.

Hence equation (7) becomes

$$\frac{\partial C}{\partial t} + \frac{C}{C_0}\frac{\partial C}{\partial x} = x^2 \frac{\partial^2 C}{\partial x^2} \tag{11}$$

Consider the dimensionless variables

$$X = \frac{C_0}{L}x, T = \frac{t}{L};\ 0 \le x \le \frac{L}{C_0} \text{ and } t \ge 0 \tag{12}$$

Thus equation (11) becomes

$$\frac{\partial C}{\partial T} + C\frac{\partial C}{\partial X} = LX^2 \frac{\partial^2 C}{\partial X^2}$$

$$\therefore \frac{\partial C}{\partial T} + C\frac{\partial C}{\partial X} = \varepsilon \frac{\partial^2 C}{\partial X^2} (\text{where } \varepsilon = LX^2) \tag{13}$$

where $C(0,T) = C_1 (T > 0)$

$C(X,0) = C_0{}' (X > 0)$

Equation (13) is the non-linear Burger's equation for longitudinal dispersion arising in fluid flow through porous media.

Choose the transformation [2, 6,14]

$$C = \Psi_X, \Psi = -2\varepsilon \log \xi \tag{14}$$

which reduces equation (13) to the diffusion type Heat equation as

$$\xi_T = \varepsilon \xi_{XX} \tag{15}$$

where $\varepsilon = LX^2$

$$\therefore \frac{\partial \xi}{\partial T} = LX^2 \frac{\partial^2 \xi}{\partial X^2} \tag{16}$$

where $\xi(0, T) = 0$

$\xi(X, 0) = X^2$

Applying Sumudu transform to equation (16), we get

$$S\left[\frac{\partial \xi}{\partial T}\right] = LX^2 S\left[\frac{\partial^2 \xi}{\partial X^2}\right]$$

$$\therefore \frac{1}{u} S[\xi(X, T)] - \frac{1}{u} \xi(X, 0) = LX^2 S[\xi_{XX}]$$

$$\therefore S[\xi(X, T)] = X^2 + LuX^2 S[\xi_{XX}] \tag{17}$$

Applying inverse Sumudu transform on equation (17), we get

$$\xi(X, T) = X^2 + LX^2 S^{-1}\left[uS[\xi_{XX}]\right] \tag{18}$$

Now applying Homotopy Perturbation method, we get

$$\sum_{n=0}^{\infty} p^n \xi_n(X, T) = X^2 + p\left[LX^2 S^{-1}\left[uS[\xi_{XX}]\right]\right] \tag{19}$$

Comparing the coefficient of like powers of p, we get

$$p^0: \xi_0(X, T) = X^2 \tag{20}$$

$$p^1: \xi_1(X, T) = LX^2 S^{-1}\left[uS[(\xi_0)_{XX}]\right] = 2LX^2 T \tag{21}$$

$$p^2: \xi_2(X, T) = LX^2 S^{-1}\left[uS[(\xi_1)_{XX}]\right] = 2^2 L^2 X^2 \frac{T^2}{2!} \tag{22}$$

Proceeding in a similar manner, we get

$$p^3: \xi_3(X, T) = 2^3 L^3 X^2 \frac{T^3}{3!},$$

$$p^4: \xi_4(X, T) = 2^4 L^4 X^2 \frac{T^4}{4!}, \tag{23}$$

Thus the solution $\xi(X, T)$ is given by

$$\xi(X, T) = X^2\left(1 + 2LT + 2^2 L^2 \frac{T^2}{2!} + 2^3 L^3 \frac{T^3}{3!} + 2^4 L^4 \frac{T^4}{4!} + \cdots\right)$$

$$\xi(X, T) = X^2 e^{2LT} \tag{24}$$

Now to find C, using equation (14), we get

$$C = \frac{\partial}{\partial X}[-2\varepsilon \log \xi]$$

$$\therefore C = \frac{\partial}{\partial X}[-2LX^2 \log(X^2 e^{2LT})]$$

$$\therefore C = -4LX[1 + \log(X^2 e^{2LT})] \tag{25}$$

$$\text{OR } C = -4C_0 x\left[1 + 2\log\left(\frac{C_0}{L} x\right) + 2t\right]$$

The solution (25) represents the concentration of the longitudinal dispersion phenomenon for any value of x and for any time t.

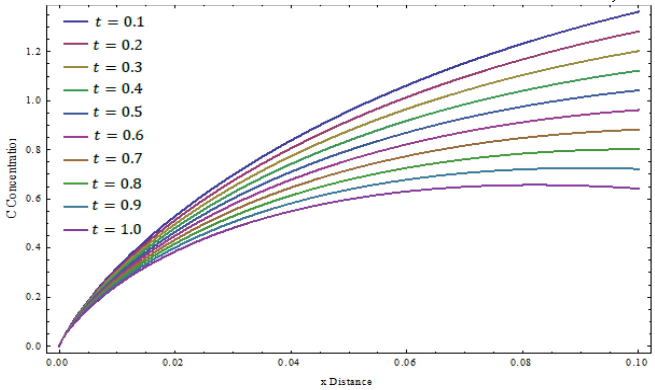
III. Concluding Remark

Expression (25) represents the solution of Burger's equation arising in longitudinal dispersion phenomenon in fluid flow through porous media which is the concentration for any time $= 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$ and 1.0. The graph shows the concentration versus distance x when time t is fixed, and it is observed over here is that after distance 0.01, concentration was increased scatterely.

x → / t ↓	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
0.1	0.3204	0.5299	0.6976	0.8380	0.9583	1.0624	1.1532	1.2325	1.3017	1.3621
0.2	0.3124	0.5139	0.6736	0.8060	0.9183	1.0144	1.0972	1.1685	1.2297	1.2821
0.3	0.3044	0.4979	0.6496	0.7740	0.8783	0.9664	1.0412	1.1045	1.1577	1.2021

0.4	0.2964	0.4819	0.6256	0.7420	0.8383	0.9184	0.9852	1.0405	1.0857	1.1221
0.5	0.2884	0.4659	0.6016	0.7100	0.7983	0.8704	0.9282	0.9765	1.0137	1.0421
0.6	0.2804	0.4499	0.5776	0.6780	0.7583	0.8224	0.8732	0.9125	0.9417	0.9621
0.7	0.2724	0.4339	0.5536	0.6460	0.7183	0.7744	0.8172	0.8485	0.8697	0.8821
0.8	0.2644	0.4179	0.5296	0.6140	0.6783	0.7264	0.7612	0.7845	0.7977	0.8021
0.9	0.2564	0.4019	0.5056	0.5820	0.6383	0.6784	0.7052	0.7205	0.7257	0.7221
1.0	0.2484	0.3859	0.4816	0.5520	0.5983	0.6304	0.6492	0.6565	0.6537	0.6421

Table-1 The value of concentration for different values of distance, x and time, t



Graph-1: Concentration, C versus distance, x.

References

[1]. Kilicman, V. G. Gupta and B. Shrma, On the solution of fractional Maxwell equations by Sumudu transform, J. of Math Research 2(4) (2010) 147-151.

[2]. E. Hopf, The Partial Differential Equation $u_t + uu_x = \mu u_{xx}$ comm, Pure Appl. Math. 3 (1950) 201-230.

[3]. F. B. M. Belgacem, A. A. Karabali and S. L. Kalla, Analytical investigations of the Sumudu transform and applications to integral production equations, Math. Prob. In Engg. 3 (2003) 103-118.

[4]. G. K. Watugala, Sumudu transform: a new integral transform to solve differential equations and control engineering problems, Int. J. of Math. Edu. In Sci. and Tech. 24(1) (1993) 35-43.

[5]. H. Eltayeb, A. Kilicman and B. Fisher, A new integral transform and associated distributions, Int. Tr. and Sp. Fn.21(5-6) (2010) 367-379.

[6]. J. D. Cole, On a Quasilinear Parabolic Equation occurring in Aerodynamic, Q. Appl. Math. 9 (1951) 225-236.

[7]. J. M. Burger, A Mathematical Model Illustrating the Theory of Turbulence, Adv. Appl. Mech. 45 (1948) 171-199.

[8]. M. A. Asiru, Classroom note: application of the Sumudu transform to discrete dynamic systems, Int. J. of Math. Edu. In Sci. and Tech. 34 (6) (2003) 944-949.

[9]. M. A. Asiru, Further properties of the Sumudu transform and its applications, Int. J. of Math. Edu. In Sci. and Tech. 33(3) (2002) 441-449.

[10]. M. A. Asiru, Sumudu transform and the solution of integral equations of convolution type, Int. J. of Math. Edu. In Sci. and Tech. 32(6) (2001) 906-910.

[11]. M. N. Mehta and A. P. Verma, A Singular Perturbation Solution of the Double Phase Flow due to Differential Wettability, Indian J. Pure Appl. Math. 8 (1977) 523-526.

[12]. R. Borana, V. Pradhan and M. Mehta, Numerical solution of Burger's equation arising in longitudinal dispersion phenomena in fluid flow through porous media by Crank-Nicolson scheme, Engg. Con. Int. ECI Digital Archieves (2014).

[13]. R. Meher and M. N. Mehta, A new approach to Backlund transformations of Burger equation arising in longitudinal dispersion of miscible fluid flow through porous media, Int. J. Appl. Math. And Com. 2(3) (2010) 17-24.

[14]. T. Patel and M. N. Mehta, A Solution of the Burger's Equation for Longitudinal Dispersion of Miscible Fluid Flow Through Porous Media, Ind. J. of Petroleum Geology 14 (2) (2005) 49-54.