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Critical Paths Identification on Fuzzy Network Project

Alaulden N. A.[1] and Saad M. S.[2]

Professor, Department of Mathematics and Computer Applications, College of Science, Al-Nahrain University, Baghdad-Iraq[1]
Ass.Prof., Department of Mathematics and Computer Applications, College of Science, Al-Nahrain University, Baghdad-Iraq[2]

Abstract: *In this paper, a new approach for identifying fuzzy critical path is presented, based on converting the fuzzy network project into deterministic network project, by transforming the parameters set of the fuzzy activities into the time probability density function PDF of each fuzzy time activity. A case study is considered as a numerical tested problem to demonstrate our approach.*
Keywords: *Project network, Fuzzy Network, Defuzzification Techniques.*

I. Introduction

In recent years, the range of project management applications, under uncertainty are extremely critical for many organizations has greatly expanded. Project management concerns the scheduling and controlling of activities (tasks) in such a way that the project can be completed in as little time as possible.

As a result, the uncertainty associated with such risky projects should be reduced. The problem of identifying critical activities in deterministic problems is well understood. Since a project could be delayed if these activities were not completed in the scheduled time, Standard Critical Path Method (CPM) analyses can be used to identify the longest path(s), known as the critical path(s), in an activity network. However, there are many cases where the activity times may not be presented in a precise manner. To deal quantitatively with imprecise data, the (PERT) based on the probability theory can be employed, and the probability distributions of each activities is needed, it is difficult to use in some situations when the priori data of the activity probability distributions are absence.

Identifying critical activities in a fuzzy project is difficult problems. Several methods had been proposed contain series draw backs which lead to identifying critical fuzzy activities incorrectly, leaving project mangers without means to identify the most probable sources of project delays. A new direction for identifying fuzzy critical path activities in stochastic project is based on different philosophy, than in deterministic project, where each critical activity must correspond to zero time slack activity, while such condition need not to be necessary. We immediately encounter difficulties developing concepts analogous to total slack and "critical" activities for stochastic project. Such concept is a criticality index, defined as the probability that an activity will lie on a critical path. However, an activity may lie on a critical path without introducing risk of project delay. Based on such concept, a critical degree is constructed to schedule fuzzy critical paths, [1].

Fuzzy set theory has been proposed to handle non crisp parameters (fuzzy) by generalizing the notion of membership in a set. Essentially, in a fuzzy set each element is associated with a point value selected from the unit interval [0,1], which is an arbitrary grade of truth referred to as the grade of membership in the set. The main objective in FLP is to find the best solution possible with imprecise, vague, uncertain or incomplete information.

Many previous studies on fuzzy project management network are reviewed before. In [2], Prade (1979) first applied fuzzy set theory into the project scheduling problem. Furthermore, in [3], [4], [5], [6] and [7], various types of project scheduling problems with fuzzy activity duration times are discussed.

An alternative way to deal with imprecise data is to employ the concept of fuzziness; the main advantages of methodologies based on fuzzy theory are that they do not require prior predictable regularities or posterior frequency distributions . The problems of computing the intervals of possible values of the latest starting times and floats of activities with imprecise durations represented by fuzzy or interval numbers, and many solution methods have been proposed, in which most of them are straightforward extension of deterministic CPM. In [8], a nice literature survey about this subject mentioned the works had been presented.

Most of the systems work with fuzzy values, which have to be mapped to non-fuzzy (crisp) values after conversion processing called defuzzification. Various defuzzification methods have been proposed in [9], [10] & [11]. Many researchers attempted to understand the logic of the defuzzification process from the perspective of invariant transformation between different uncertainty paradigms, including basic defuzzification distribution, semi-linear defuzzification and generalized level set defuzzification. from the perspective of invariant transformation between different uncertainty paradigms, including basic defuzzification distribution, semi-linear defuzzification and generalized level set defuzzification. s from the perspective of invariant transformation between different uncertainty paradigms, including basic defuzzification distribution, semi-linear defuzzification and generalized level set defuzzification, see [12], [13], [14] & [15]. In [16] they attempt to understand the defuzzification problem from the scope of optimal selection of an element from a fuzzy set. They used the concepts of interaction, variability, and voting techniques to compute an optimal solution. In [17], they proposed a fuzzy clustering based defuzzification method. In [18], they proposed a defuzzification method with most typical values. In [19], he proposed a method for defuzzification with weighted distance. Smith [20], proposed a dynamic switching defuzzification method for fuzzy control. In [21], he proposed a defuzzification method with the "nearest" symmetric triangular fuzzy number of a fuzzy set. In [22], he proposed a procedure to defuzzify fuzzy subsets and interval values by employing the concept of sensitivity analysis with a kind min-max principle. There are also researchers who tried to build an axiomatic foundation for the defuzzification theory [23] & [24]. It should be noted that with the developments of intelligent technologies, some adaptive and parameterized defuzzification methods that can include human knowledge have been proposed. In [25], they used neural networks for defuzzification. Song, et al. [26], proposed an adaptive learning defuzzification technique. In [27], he proposed a knowledge based defuzzification process become more intelligent. Although so many defuzzification methods have been proposed so far, no one method gives a right effective defuzzified output. The computational results of these methods often conflict, and they don't have a uniform framework in theoretical view. We often face difficulty in selecting appropriate defuzzification methods for some specific application problems. Most of the existing defuzzification methods tried to make the estimation of a fuzzy set in an objective way. However, an important aspect of the fuzzy set application is that it can represent the subjective knowledge of the decision maker. Different decision makers may have different perception for the defuzzification results.

In this paper, we are presented a new approach for identifying critical path based on defuzzification parameters set of fuzzy types activities, by using the concept of the probability density function of each fuzzy activity.. A case study is considered to explain the proposed approach.

II. Basic Concepts

In this section, we are presented basic concepts of fuzzy set theory and fuzzy network problems. Usually the structures embedded in fuzzy set theories are less rich than the Boolean lattice of classical set theory. Moreover, there is also some arbitrariness in the choice of the valuation set for the elements: the real interval [0,1] is most commonly used.

Definition (1) Fuzzy Sets, [27],[28],[29]:

Let U be a universe set. A fuzzy set **A** of **U** is defined by a membership function $\mu_A(x) \to [0,1]$, where $\mu_A(x)$ indicatesthe degree of x in **A** which defined as following:

$$\mu = \begin{cases} 0, & (-\infty, a_1] \\ f_1(x), & [a_1, a_2] \\ 1, & [a_2, a_3] \\ f_2(x), & [a_3, a_4] \\ 0, & [a_4, +\infty) \end{cases} \qquad (1)$$

where a_1, a_2, a_3 and $a_4$are real number, note that $f_1(x)$ and$f_2(x)$ are may be linear or convex nonlinear function.

Definition (2) Fuzzy Number, [30]:

A fuzzy number $\widetilde{A}$is a convex normalized fuzzy set Aof the real line R such that:

1. It exists exactly one $x_0 \in R$ with $\mu_{\widetilde{A}}(x_0) = 1$ ($x_0$is called the mean value of A).
2. $\mu_{\widetilde{A}}(x)$is piecewise continuous.

III. Proposed Approach

Definition (3) Defuzzification, [31]:

Deffuzification is a mapping from space of fuzzy action defined over an output universe into a space of nonfuzzy (crisp) actions.

In this paper we proposed a defuzzification method to obtain a Probability Density Function from Membership Function [32]. If we considered $\widetilde{A}_i = (a_{i1}, a_{i2}, a_{i3}, a_{i4})$ is fuzzy trapezoidal number with membership function defined as (1), where $f_1(x)$ and $f_2(x)$ are linear function, (if they are convex nonlinear functions, we are using any approximated methods to linearized them). Let the function $p_{\widetilde{A_i}}$ defined by $p_{\widetilde{A_i}}(x) = c_i \mu_{\widetilde{A_i}}(x)$ is a probability density function associated with $\widetilde{A}_i$, where c_i can be obtained by the property that $\int_{-\infty}^{\infty} p_i(x)dx = 1$ as following:

$$c_i = \frac{2}{a_{i4} + a_{i3} - a_{i2} - a_{i1}} \qquad (2)$$

Now, we are using the following transformation called Mellin Transform to find the expected value of the activity fuzzy duration times.

Definition (10) Mellin Transform, [33]:
The Mellin transform$\mu_X(s)$ of a probability density function f(x), where x is positive, is defined as
$\mu_X(s) = \int_0^\infty x^{s-1} f(x) dx$(3)
whenever the integral exist.
Now it is possible to think of theMellin transform in terms of expected values recall that the expected value of any function g(x) of the random variable X, whose distribution is f(x), is given by
$E[g(x)] = \int_{-\infty}^{\infty} g(x) f(x) dx$(4)
There for it follows that
$\mu_X(s) = E[x^{s-1}] = \int_0^\infty x^{s-1} f(x) dx$ (5)
Hence
$E[x^s] = M_X(s+1)$ (6)
Thus, the expectation of random variable X is $E[x] = M_X(2)$.
Now, if we let $\widetilde{A} = (a_1, a_2, a_3, a_4)$ an arbitrary trapizoidal fuzzy number, then the density function f(x) corresponding to $\widetilde{A}$ is as follows:

$$p_{\widetilde{A_i}}(x) = \begin{cases} \frac{2(x-a_{i1})}{(a_{i4}+a_{i3}-a_{i2}-a_{i1})(a_{i2}-a_{i1})}, & a_{i1} \le x < a_{i2} \\ \frac{2}{a_{i4}+a_{i3}-a_{i2}-a_{i1}}, & a_{i2} \le x \le a_{i3} \\ \frac{2(a_{i4}-x)}{(a_{i4}+a_{i3}-a_{i2}-a_{i1})(a_{i4}-a_{i3})}, & a_{i3} < x \le a_{i4} \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

The Mellin transform is the obtained by:

$$\mathcal{M}_{\widetilde{A_i}}(s) = \int_0^\infty x^{s-1} p_{\widetilde{A_i}}(x) dx = \frac{2}{(a_{i4}+a_{i3}-a_{i2}-a_{i1})(s^2+s)}\left[\frac{(a_{i4}^{s+1}-a_{i3}^{s+1})}{a_{i4}-a_{i3}} - \frac{(a_{i2}^{s+1}-a_{i1}^{s+1})}{a_{i2}-a_{i1}}\right] \quad (8)$$

and

$$E[\widetilde{A}_i] = \mathcal{M}_{\widetilde{A}}(2) = \frac{1}{3}\left[(a_{i1} + a_{2i} + a_{i3} + a_{i4}) + \frac{a_{i1}a_{i2} - a_{i3}a_{i4}}{a_{i4}+a_{i3}-a_{i2}-a_{i1}}\right] \quad (9)$$

Therefore, all the fuzzy durations times are transformed into crisps values as expected values. Now, we implemented the following proposed standard Critical Path Method "CPM. For each activity, an expected duration time tij is calculated using (6).defined, where tij is the time required for the completion of activity (i,j). A critical path is a longest path, and an activity on a critical path is called a critical activity. Let ESi and LFi be the earliest start time event i, and the finish time event i, respectively. Let D_j be a set of events obtained from event j and i< j. We then obtain ESjusing the following equations:
$ES_j = \max_{i \in D_j}[ES_i + t_{ij}]$ and $ES_1 = LS_1 = 0$ (10)
Similarly, let H_i be a set of events obtained from event i and i< j . We obtain LFi using the following equations:
$LF_i = \min_{j \in H_i}[LF_j - t_{ij}]$ and $LF_n = EF_n$ (11)
The interval [ESi, LFj] is the time during which the activity (i,j) must be completed. Now, we can calculate the float time Tij of the activity (i,j) can be computed as follows :
$T_{ij} = LF_j - ES_i - t_{ij}$ (12)
Hence we can obtain the earliest event time, latest event time, and the total float of every activity by using the above last three equations, and the critical events are identified corresponding to their zeros values of total float times.

IV. Case Study

We are considering a project network, figure (1), taken from [34] as a case study.As shown in table (1); the 30 activities are listed with their fuzzy operation times.
In order to solve such problem, the proposed method is implemented to convert the fuzzy time number to crisp time number, explained in the table (2). Performed the standard critical path method CPM, to obtain the following critical path 1⇒2⇒3⇒4⇒5⇒6⇒7⇒9⇒10⇒11⇒12⇒16⇒24 of the fuzzy network project.

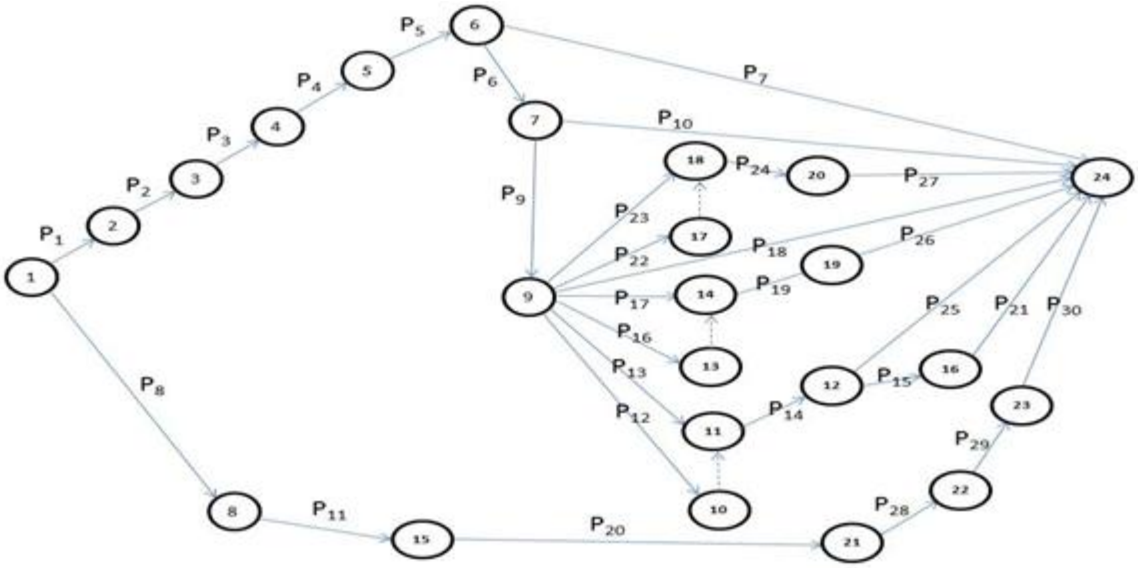

Figure (1): Project Network

Table (1): ProjectConstruction

ActivityItem	Activity Description	PrecedenceItem	Fuzzy Operation Time (per day)
P_1	Concrete works foundation	-	(25,28,30,35)
P_2	Insulation works	P_1	(3,4,4,5)
P_3	Parking area + Roads + Landscape	P_2	(25,29,30,35)
P_4	Back filling works	P_3	(3,7,12,15)
P_5	Sub-base	P_4	(5,6,6,10)
P_6	Steel structure erection	P_5	(26,30,35,40)
P_7	Under ground drainage system	P_5	(7,10,10,13)
P_8	Water tank - civil works	-	(15,21,21,25)
P_9	Steel structure testing	P_6	(2,3,4,5)
P_{10}	Roofing works	P_6	(9,10,12,15)
P_{11}	Water tank – finishing	P_8	(6,7,8,10)
P_{12}	HVAC works - 1st fix	P_9	(12,14,14,16)
P_{13}	Fire fighting works 1st fix	P_9	(7,9,11,12)
P_{14}	Electrical system works - 1st fix	P_{12}, P_{13}	(5,6,7,10)
P_{15}	Flooring	P_{14}	(7,9,11,12)
P_{16}	HVAC work-2nd fix	P_9	(12,14,14,16)
P_{17}	Fire fighting works – 2nd fix	P_9	(7,9,11,12)
P_{18}	Cladding works	P_9	(15,24,25,30)
P_{19}	Electrical system works - 2nd fix	P_{16},P_{17}	(5,6,7,10)
P_{20}	Water tank – MEP	P_{11}	(9,11,12,14)
P_{21}	Finishing works	P_{15}	(15,18,18,20)
P_{22}	HVAC works - 3rd	P_9	(12,14,14,16)
P_{23}	Fire fighting work - 3rd fix	P_9	(7,9,11,12)
P_{24}	Electrical system works -3rd fix	P_{22},P_{23}	(5,6,7,10)
P_{25}	Plumbing works - 1st fix	P_{14}	(5,6,6,8)
P_{26}	Plumbing works – 2nd fix	P_{19}	(5,6,6,8)
P_{27}	Plumbing works - 3rd fix	P_{24}	(5,6,6,8)
P_{28}	Water tank testing	P_{20}	(1,2,2,3)
P_{29}	Testing and commissioning	P_{28}	(1,2,2,3)
P_{30}	Snag list and Initial handling	P_{29}	(5,7,7,9)

Table (2): Corresponding crisps expected duration times

Activity Item	Crisp Duration Time (in days)
P_1	27
P_2	4
P_3	28.625
P_4	11.125
P_5	5.625

P_6	30.625
P_7	10
P_8	22.5
P_9	3.5
P_{10}	10
P_{11}	7.375
P_{12}	14
P_{13}	10.375
P_{14}	6
P_{15}	10.375
P_{16}	14
P_{17}	10.375
P_{18}	26
P_{19}	6
P_{20}	10.75
P_{21}	16.875
P_{22}	14
P_{23}	10.375
P_{24}	6
P_{25}	6.125
P_{26}	6.125
P_{27}	6.125
P_{28}	2
P_{29}	2
P_{30}	7

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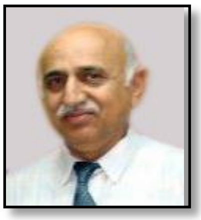
Biography



Prof.Dr.Alaulden N. Ahmed is specialist in different areas of Operation Researches. He received his Ph.D in 1986 from UK. He supervised of more than 50 M.Sc. students and 5 Ph.D students.

Mr.Saad M. Salman,published more than 10 papers in optimization field.