

## Area and side measurement relation of two right angled triangle (Relation All Mathematics)

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**Abstract:** In this research paper ,two right angled triangle relation explained with the help of formulas. This relation explained in two part i.e. **Area relation** and **Sidemeasurement relation**.Right angled triangle can be narrowed in segment and as like right angled triangle called Seg-right angled triangle.Seg-right angled triangle always become in zero area.

Very feamas **Pythagoras theorem** proof with the help of Relation All Mathematics methode. also we are given proof of **DGP theorem** i.e. "In a right angled triangle ,the square of hypoténuse is equal to the subtract of the square of the sidemeasurement and four times the area". We are trying to give a new concept "Relation All Mathematics" to the world .I am sure that this concept will be helpful in Agricultural, Engineering, Mathematical world etc.

**Keywords:** Area, Perimeter, Relation , Seg-right angled triangle , B- Sidemeasurement

### I. Introduction

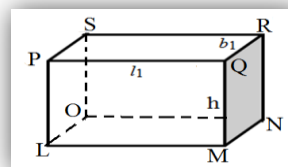
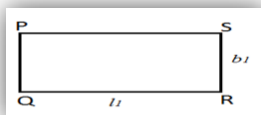
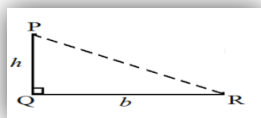
Relation All Mathematics is a new field and the various relations shown in this research, "Area and sidemeasurement relation of two right angled triangle" is a 2<sup>nd</sup> research paper of Relation All Mathematics. and in future ,the research related to this concept, that must be part of " Relation Mathematics " subject. Here ,we have studied and shown new variables ,letters, concepts, relations ,and theorems.. Inside the research paper cleared that relation between two right angled triangle in two parts. i.e. i) Basic theorem of area relation of isosceles right angled triangle and scalene right angled triangle ii) Basic theorem of sidemeasurement relation of isosceles right angled triangle-scalene right angled triangle.. Sidemeasurement is a explained new concept which is very important related to this paper and Relation Mathematics subject.

In this "Relation All Mathematics" we have proved the relation between rectangle-square and two right angled triangles with the help of formula . This relation is explained in two parts **i)Area relation** and **ii) Sidemeasurement relation** . Also we have proved ,the Theorem of right angled triangle whose height is zero i.e. Seg-right angled triangle theorem and along with the theorem of seg-right angled triangles ratio . In this research paper proof that Pythagoras theorem with the help of Relation All Mathematics method. Also we have proved DGP theorem i.e. In a right angled triangle, the square of hypotenuse is equal to the subtract of the squares of the side-measurement and four times the area .This "Relation All Mathematics" research work is near by 300 pages . This research is done considering the Agricultural sector mainly ,but I am sure that it will also be helpful in other sector also.

### II. Basic concept of two right angled triangle

**2.1. Sidemeasurement(B) :-**If sides of any geometrical figure are in right angle with each other , then those sides or considering one of the parallel and equal sides after adding them, the addition is the sidemeasurement .side-measurement indicated with letter 'B'

Sidemeasurement is a one of the most important concept and maximum base of the Relation All Mathematics depend apoun this concept.



**Sidemeasurement of right angled triangle - B (ΔPQR) = b+h**

In ΔPQR ,sides PQ and QR are right angle, performed to each other .

**Sidemeasurement of rectangle-B(□PQRS)= l<sub>1</sub>+ b<sub>1</sub>**

In □PQRS, opposite sides PQ and RS are similar to each other and m∠Q = 90°. here side PQ and QR are right angle performed to each other.

**Sidemeasurement of cuboid**– $E_B(\square PQRS) = l_1 + b_1 + h_1$

In  $E(\square PQRS)$ , opposite sides are parallel to each other and QM are right angle performed to each other. Sidemeasurement of cuboid written as =  $E_B(\square PQRS)$

**2.2) Important points of square-rectangle relation :-**

I) For explanation of square and rectangle relation following variables are used

- i) Area – A
- ii) Perimeter – P
- iii) Sidemeasurement – B

II) For explanation of square and rectangle relation following letters are used

- i) Area of square ABCD – A ( $\square ABCD$ )
- ii) Perimeter of square ABCD – P ( $\square ABCD$ )
- iii) Sidemeasurement of square ABCD – B ( $\square ABCD$ )
- iv) Area of rectangle PQRS – A ( $\square PQRS$ )
- v) Perimeter of rectangle PQRS – P ( $\square PQRS$ )
- vi) Sidemeasurement of rectangle PQRS – B ( $\square PQRS$ )

II) For explanation of two right angled triangle relation, following letters are used

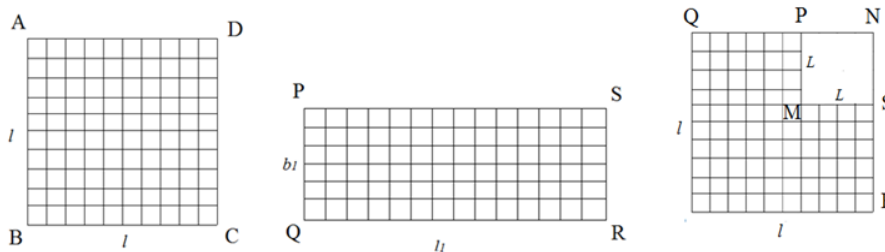
- In isosceles right angled triangle  $\triangle ABC [45^\circ - 45^\circ - 90^\circ]$ , side is assumed as 'l' and hypotenuse as 'X'
- In scalene right angled triangle  $\triangle PQR [\theta_1 - \theta_1' - 90^\circ]$  it's base 'b<sub>1</sub>' height 'h<sub>1</sub>' and hypotenuse assumed as 'Y'
- In scalene right angled triangle  $\triangle LMN [\theta_2 - \theta_2' - 90^\circ]$  it's base 'b<sub>2</sub>' height 'h<sub>2</sub>' an hypotenuse assumed as 'Z'

- i) Area of isosceles right angled triangle ABC – A ( $\triangle ABC$ )
- ii) Side-measurement of isosceles right angled triangle ABC – B ( $\triangle ABC$ )
- iii) Area of scalene right angled triangle PQR – A ( $\triangle PQR$ )
- vi) Sidemeasurement of scalene right angled triangle PQR – B ( $\triangle PQR$ )

**2.3) Important Reference theorem of previous paper which used in this paper:-**

**Theorem :** Basic theorem of area relation of square and rectangle

Perimeter of square and rectangle is same then area of square is more than area of rectangle, at that time area of square is equal to sum of the, area of rectangle and Relation area formula of square-rectangle(K).



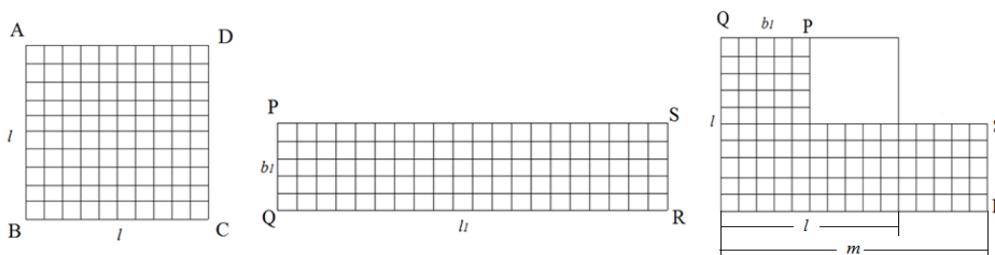
**Figure I : Area relation of square and rectangle**

**Proof formula :-**  $A(\square ABCD) = A(\square PQRS) + \left[ \frac{(l_1 + b_1)}{2} - b_1 \right]^2$

[Note :- The proof of this formula given in previous paper and that available in reference]

**Theorem :-** Basic theorem of perimeter relation of square-rectangle

Area of square and rectangle is same then perimeter of rectangle is more than perimeter of square, at that time perimeter of rectangle is equal to product of the, perimeter of square and Relation perimeter formula of square-rectangle(V).



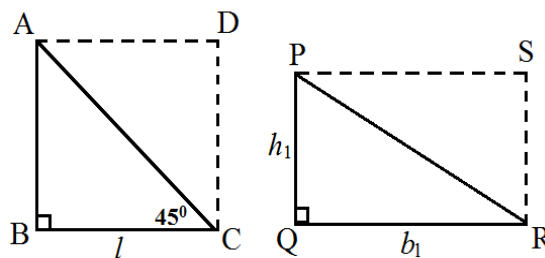
**Figure II : Perimeter relation of square-rectangle**

**Proof formula :-**  $P(\square PQRS) = P(\square ABCD) \times \frac{1}{2} \left[ \frac{(n^2+1)}{n} \right]$

[Note :- The proof of this formula given in previous paper and that available in reference]

**Theorem -1:** Basic theorem of area relation of isosceles right angled triangle and scalene right angled triangle. The Sidemeasurement of isosceles right angled triangle and scalene right angled triangle is same then area of isosceles right angled triangle is more than area of scalene right angled triangle, at that time area of isosceles right angled triangle is equal to sum of the, area of scalene right angled triangle and Relation area formula of isosceles right angled triangle - scalene right angled triangle(  $K'$  ).

**Given :-** Hypotenuse AC divided square ( $\square ABCD$ ) in two part,  $\triangle ABC = \triangle ADC$   
 and in  $\triangle ABC, m\angle B = 90^\circ$  as well as,  
 Hypotenuse PR divided rectangle ( $\square PQRS$ ) in two part,  $\triangle PQR = \triangle RSP$   
 and in  $\triangle PQR, m\angle Q = 90^\circ$   
 here,  $B(\triangle ABC) = B(\triangle PQR)$   
 $2l = (b_1+h_1)$  ,  $b_1 > l$



**Figure III : Area relation of isosceles right angled triangle and scalene right angled triangle**

**To prove :-**  $A(\triangle ABC) = A(\triangle PQR) + \frac{1}{2} \left[ \frac{(b_1+h_1)}{2} - h_1 \right]^2$

**Proof :-** In  $\square ABCD$  and  $\square PQRS$ ,

$$A(\square ABCD) = A(\square PQRS) + \left[ \frac{(b_1+h_1)}{2} - h_1 \right]^2 \quad \dots (i)$$

(Basic theorem of area relation of square and rectangle)

Divided both sides of eq<sup>n</sup> ( i ) with 2

$$\frac{A(\square ABCD)}{2} = \frac{A(\square PQRS)}{2} + \frac{K}{2} \quad \dots (K\text{-Relation area formula of square and rectangle})$$

$$A(\triangle ABC) = A(\triangle PQR) + \frac{1}{2} \left[ \frac{(b_1+h_1)}{2} - h_1 \right]^2 \quad \dots \text{ here, } K' = \frac{K}{2} = \frac{1}{2} \left[ \frac{(b_1+h_1)}{2} - h_1 \right]^2$$

- (Relation area formula of isosceles right angled triangle - scalene right angled triangle(  $K'$  )

$$A(\triangle ABC) = A(\triangle PQR) + \frac{1}{2} \left[ \frac{(b_1-h_1)}{2} \right]^2$$

Hence Basic theorem of area relation of isosceles right angled triangle and scalene right angled triangle is proved.

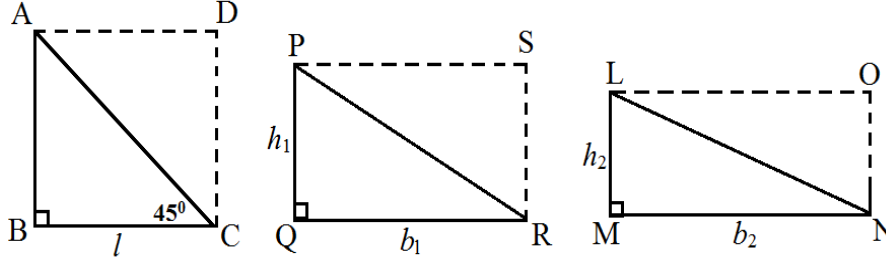
**Example:-**

		$\triangle ABC$	$\triangle PQR$	+ $K'$	REMARKS
<b>Given</b>	<b>Base</b>	10	14		
	<b>Height</b>	10	6		
		<b>LHS</b>	<b>RHS</b>	<b>RHS</b>	
<b>Explanation</b>	<b>Sidemeasurement</b>	20	20		<b>Equal</b>
	<b>area</b>	50	42	8	
	<b>Answer</b>	50	50		<b>LHS=RHS</b>

**Theorem-2 :** Theorem of area relation of two scalene right angled triangles.

If sidemeasurement of two scalene right angled triangle is same then scalene right angled triangle whose base is smaller, its area also is more than another scalene right angled triangle.

**Given :-** In  $\triangle ABC$ ,  $\triangle PQR$  and  $\triangle LMN$ ,  
 $B(\triangle ABC) = B(\triangle PQR) = B(\triangle LMN)$   
 $2l = b_1 + h_1 = b_2 + h_2 \quad \dots \quad b_1 < b_2$



**Figure IV: Area relation of two scalene right angled triangles**

**To prove :-**  $A(\triangle PQR) = A(\triangle LMN) + \frac{1}{2}(h_1 - h_2) \cdot [(b_1 + h_1) - (h_1 + h_2)]$

**Proof :-** In  $\triangle ABC$  and  $\triangle PQR$ ,

$$A(\triangle ABC) = A(\triangle PQR) + \frac{1}{2} \left[ \frac{(b_1 + h_1)}{2} - h_1 \right]^2 \quad \dots (i)$$

... (Basic theorem of area relation of isosceles right angled triangle and scalene right angled triangle)

In  $\triangle ABC$  and  $\triangle LMN$ ,

$$A(\triangle ABC) = A(\triangle LMN) + \frac{1}{2} \left[ \frac{(b_2 + h_2)}{2} - h_2 \right]^2 \quad \dots (ii)$$

... (Basic theorem of area relation of isosceles right angled triangle and scalene right angled triangle)

$$A(\triangle PQR) + \frac{1}{2} \left[ \frac{(b_1 + h_1)}{2} - h_1 \right]^2 = A(\triangle LMN) + \frac{1}{2} \left[ \frac{(b_2 + h_2)}{2} - h_2 \right]^2 \quad \dots \text{From equation no. (i) and (ii)}$$

$$\begin{aligned} A(\triangle PQR) &= A(\triangle LMN) + \frac{1}{2} \left[ \frac{(b_2 + h_2)}{2} - h_2 \right]^2 - \frac{1}{2} \left[ \frac{(b_1 + h_1)}{2} - h_1 \right]^2 \\ &= A(\triangle LMN) + \frac{1}{2} \left[ \frac{(b_2 + h_2)}{2} - h_2 + \frac{(b_1 + h_1)}{2} - h_1 \right] \times \left[ \frac{(b_2 + h_2)}{2} - h_2 - \frac{(b_1 + h_1)}{2} + h_1 \right] \\ &\quad \dots (a^2 - b^2) = (a+b) \cdot (a-b), \quad b_1 + h_1 = b_2 + h_2 \quad \dots (\text{Given}) \end{aligned}$$

$$A(\triangle PQR) = A(\triangle LMN) + \frac{1}{2} (h_1 - h_2) \cdot [(b_1 + h_1) - (h_1 + h_2)]$$

.Hence , Theorem of area relation of two scalene right angled triangles is proved.

**Theorem-3:** Basic theorem of sidemeasurement relation of isosceles right angled triangle and scalene right angled triangle.

Area of isosceles right angled triangle and scalene right angled triangle is same then sidemeasurement of scalene right angled triangle is more than sidemeasurement of isosceles right angled triangle , at that time sidemeasurement of scalene right angled triangle is equal to product of the, sidemeasurement of isosceles right angled triangle and Relation sidemeasurement formula of isosceles right angled triangle-scalene right angled triangle(V').

**Given :-** Hypotenuse AC divided square ( $\square ABCD$ ) in two part,  $\triangle ABC = \triangle ADC$

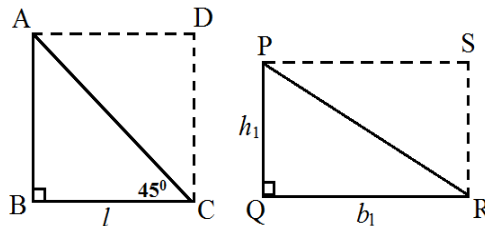
and in  $\triangle ABC, m\angle B = 90^\circ$  as will as,

Hypotenuse PR divided rectangle ( $\square PQRS$ ) in two part,  $\triangle PQR = \triangle RSP$

and in  $\triangle PQR, m\angle Q = 90^\circ$

here,  $A(\triangle ABC) = A(\triangle PQR)$

$$l^2 = b_1 \times h_1, \quad b_1 > l$$



**Figure V: Sidemeasurement relation of isosceles right angled triangle-scalene right angled triangle**

**To prove :-**  $B(\Delta PQR) = B(\Delta ABC) \times \frac{1}{2} \left[ \frac{(n^2+1)}{n} \right]$

**Proof :-** In  $\square ABCD$  and  $\square PQRS$ ,

$$P(\square PQRS) = \frac{1}{2} P(\square ABCD) \times \left[ \frac{(n^2+1)}{n} \right] \dots \text{(i) Basic theorem of perimeter relation of square-rectangle}$$

Divided both sides of eq<sup>n</sup> (i) with 2

$$\frac{P(\square PQRS)}{2} = \frac{P(\square ABCD)}{2} \times \frac{V}{2} \dots V - \text{(relation perimeter formula of square and rectangle)}$$

$$B(\Delta PQR) = B(\Delta ABC) \times \frac{V}{2}$$

$$B(\Delta PQR) = B(\Delta ABC) \times V^2$$

$$B(\Delta PQR) = \frac{1}{2} B(\Delta ABC) \times \left[ \frac{(n^2+1)}{n} \right]$$

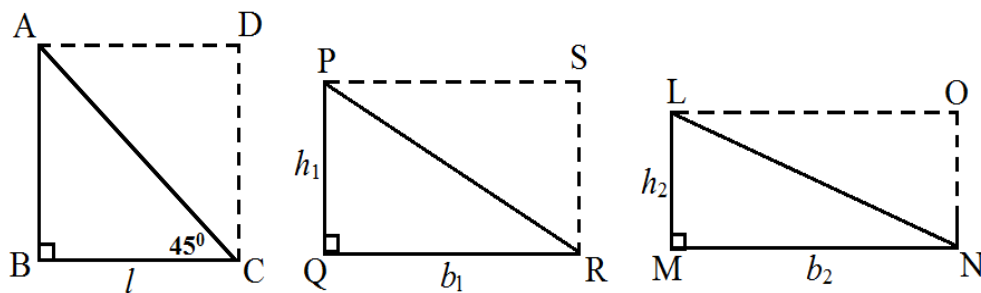
Hence, Basic theorem of sidemeasurement relation of isosceles right angled triangle-scalene right angled triangle is proved.

**Example:-**

		$\Delta PQR$	$\Delta ABC$	$\times V^2$	REMARKS
<b>Given</b>	<b>Base</b>	20	10		
	<b>Height</b>	5	10		
		<b>LHS</b>	<b>RHS</b>	<b>RHS</b>	
<b>Explanation</b>	<b>Area</b>	50	50		<b>Equal</b>
	<b>Sidemeasurement</b>	25	20	$\frac{2.5}{2}$	
	<b>Answer</b>	25	25		<b>LHS=RHS</b>

**Theorem-4 :** Theorem of sidemeasurement relation between two scalene right angled triangles .  
 If area of two scalene right angled triangle is same then scalene right angled triangle whose length is more, its sidemeasurement also is more than another scalene right angled triangle.

**Given :-** In  $\Delta ABC$ ,  $\Delta PQR$  and  $\Delta LMN$ ,  
 $A(\Delta ABC) = A(\Delta PQR) = A(\Delta LMN)$   
 Here,  $b_1 \times h_1 = b_2 \times h_2 \dots b_1 < b_2$



**Figure VI: Sidemeasurement relation between two scalene right angled triangles**

**To prove :-**  $P(\Delta PQR) = P(\Delta LMN) \times \left[ \frac{n_2}{n_1} \cdot \frac{(n_1^2+1)}{(n_2^2+1)} \right]$

**Proof :-** In  $\Delta ABC$  and  $\Delta PQR$ ,

$$B(\Delta ABC) = B(\Delta PQR) \times 2 \left[ \frac{n_1}{(n_1^2+1)} \right] \dots \text{(i)}$$

...(Basic theorem of sidemeasurement relation of isosceles right angled triangle-scalene right angled triangle)

In  $\Delta ABC$  and  $\Delta PQR$ ,

$$B(\Delta ABC) = B(\Delta LMN) \times 2 \left[ \frac{n_2}{(n_2^2+1)} \right] \dots \text{(ii)}$$

...(Basic theorem of sidemeasurement relation of isosceles right angled triangle-scalene right angled triangle)

$$B(\Delta PQR) \times 2 \left[ \frac{n_1}{(n_1^2+1)} \right] = B(\Delta LMN) \times 2 \left[ \frac{n_2}{(n_2^2+1)} \right] \dots \text{From equation no. (i) and (ii)}$$

$$B(\Delta PQR) = B(\Delta LMN) \times \left[ \frac{(n_1^2+1)}{n_1} \right] \cdot \left[ \frac{n_2}{(n_2^2+1)} \right]$$

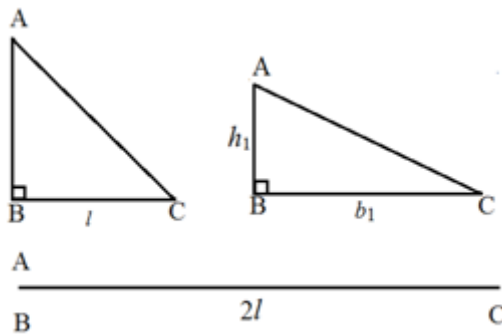
$$B(\Delta PQR) = B(\Delta LMN) \times \left[ \frac{n_2}{n_1} \cdot \frac{(n_1^2+1)}{(n_2^2+1)} \right]$$

Hence , Theorem of sidemeasurement relation between two scalene right angled triangles is proved.

**Theorem-5 :** Seg- right angled triangle theorem.

If sidemeasurement of right angled triangle is kept constant and base is increased till height become zero ,then become Segment is Seg- right angled triangle.

**Given** :- In  $\Delta ABC$   
 $B(\Delta ABC) = (b_1+h_1) = B(\text{Seg } AB\text{-}C)$   
 Now in  $\Delta AB\text{-}C$  ,become a base =  $2l$  ... (height = 0)



**Figure VII: Segment AB-C is Seg- right angled triangle**

**To prove :-** Seg AB-C is Seg- right angled triangle

**Proof** :- In  $\Delta ABC$  ,  
 $A(\Delta ABC) = \frac{1}{2} l^2$  ... (i)  
 In Segment AB-C,  $h_1 = 0$   
 Suppose, Segment AB-C is right angled triangle  
 Now , In Seg AB-C,  
 $A(\text{Seg } AB\text{-}C) = 0$  ... (ii) ( $h_1 = 0$ ) ... Given

$A(\Delta ABC) = A(\text{Seg } AB\text{-}C) + \frac{1}{2} \left[ \frac{(b_1+h_1)}{2} - h_1 \right]^2$   
 ... (Basic theorem of relation of area of isosceles right angled triangle and scalene right angled triangle)

$$= 2l \times 0 + \frac{1}{2} \left[ \frac{(2l+0)}{2} - 0 \right]^2$$

$$= 0 + \frac{1}{2} \left[ \frac{2l}{2} \right]^2$$

$$A(\Delta ABC) = \frac{1}{2} l^2$$

But this equation is satisfied with Basic theorem of area relation of isosceles right angled triangle and scalene right angled triangle.

Hence , Seg- right angled triangle theorem is proved.

(Seg AB-C is Seg-scalene right angled triangle , and it can be written as  $\Delta AB\text{-}C$  )

**Theorem- 6:** Theorem of Seg- right angled triangles ratio

The sidemeasurement of two seg-right angled triangle is equal to the ratio of base of that seg –right angled triangle..

**Given :-** In  $\Delta PQ\text{-}R$  and  $\Delta LM\text{-}N$  ,  
 $h_1 = h_2 = 0$  ,  $[A(\Delta PQ\text{-}R) = A(\Delta LM\text{-}N) = 0]$

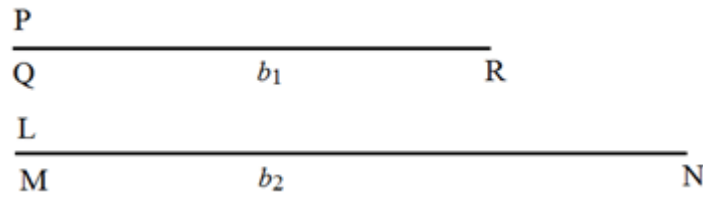


Figure VIII: Seg- right angled triangles ratio

To prove :-  $B(\Delta PQ-R) : B(\Delta LM-N) = b_1 : b_2$

Proof :- In  $\Delta PQ-R$  and  $\Delta LM-N$ ,

$$P(\Delta PQ-R) = P(\Delta LM-N) \times \left[ \frac{n_2}{n_1} \cdot \frac{(n_1^2+1)}{(n_2^2+1)} \right] \dots \text{(Theorem of sidemeasurement relation between two right angled triangles)}$$

$$P(\Delta PQ-R) = P(\Delta LM-N) \times \left[ \frac{b_2}{b_1} \cdot \frac{(b_1^2+l^2)}{(b_2^2+l^2)} \right] \dots l^2 = l_1 \cdot h_1$$

$$P(\Delta PQR) = P(\Delta LMN) \times \left[ \frac{b_2}{b_1} \cdot \frac{(b_1^2)}{(b_2^2)} \right] \dots (h_1 = h_2 = 0, \text{ Given})$$

$$P(\Delta PQR) = P(\Delta LMN) \times \frac{b_1}{b_2}$$

$$\frac{P(\Delta PQR)}{P(\Delta LMN)} = \frac{b_1}{b_2}$$

$$P(\Delta PQR) : P(\Delta LMN) = b_1 : b_2$$

Hence, Theorem of Seg- right angled triangle ratio is proved.

**Theorem- 7:** Proof of **Pythagoras theorem** with the help of Relation All Mathematics Method :

The square of the hypotenuse of a right angled triangle is equal to the sum of the squares of the other two sides.

**Given :-** In  $\Delta PQR$ ,  $\angle PQR=90^\circ$ , base (QR)=  $b_1$ , height (PQ) =  $h_1$ , And hypotenuse (PR) =  $x$

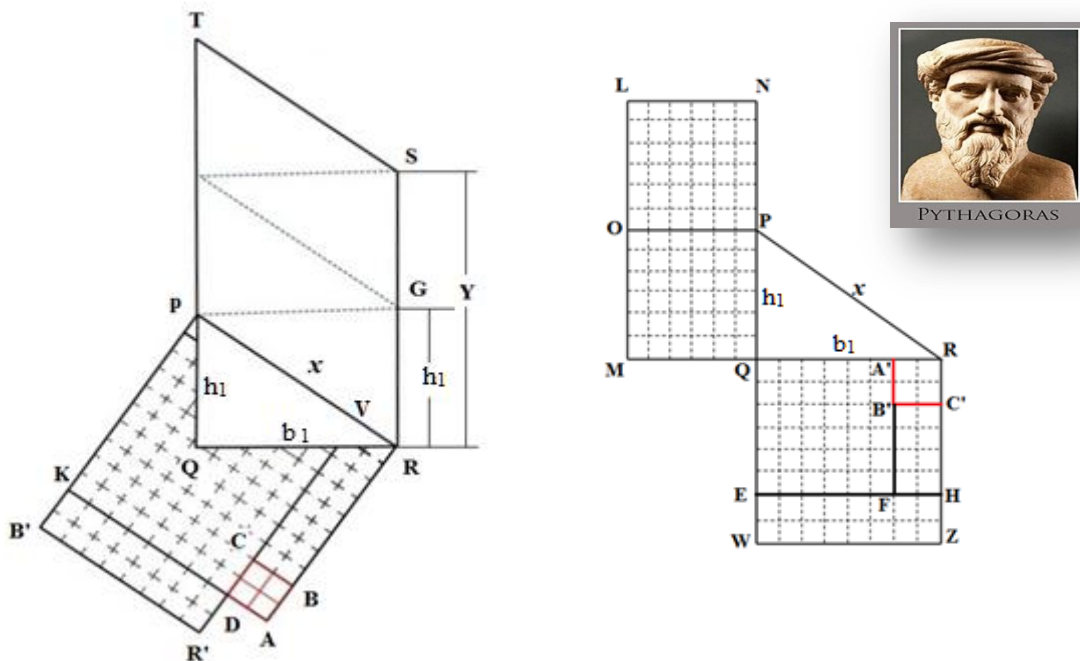


Figure IX: Pythagoras theorem with Relation All Mathematics

**Construction :-** Draw a parallelogram on hypotenuse (PR) as a,

$$y=2 h_1, \text{ Seg } PG \perp \text{ Seg } RS$$

so that,  $A(\square PRST) = 4 \cdot A(\Delta PQR)$

as shown in fig(I)

**To proof :-**  $x^2 = b_1^2 + h_1^2$

**Proof :-** In  $\square PRST$ ,  
 $A(\square PRST) = PG \times RS$   
 $= b_1 \times 2 h_1$   
 $= 2 b_1 h_1$

$A(\square PRST) = 4.A(\triangle PQR) = 2 b_1 h_1 \dots (i)$

Now we are draw square  $\square PKAR$  with side PR, and parallelogram  $\square PRST$  put on square  $\square PKAR$ , which explained as bellow,

In fig –(I)

$A(\square RBCV) = A(\square KDR'B')$

Now in  $\square PRST$  and  $\square PVR'B'$ ,

$PV=PG$  &  $PB'=RS$

$A(\square PRST) = A(\square PVR'B') \dots (ii)$

In  $\square PKAR$ ,

$A(\square PKAR) = x^2$

$A(\square PKAR) = A(\square PVR'B') + A(\square ABCD)$

$x^2 = 2 b_1 h_1 + A(\square ABCD) \dots (iii)$

In fig -2

In  $\triangle PQR$ ,

Draw as ,A square  $\square QWZR$  and  $\square POMQ$  with base (QR) and height (PQ) respectively

New square  $\square POMQ$  put on square  $\square QWZR$  as explained as bellow.

$A(\square POMQ) = A(\square QEFA')$

As will as  $\triangle PQR$  set on square  $\square QWZR$  as bellow

$A(\triangle PQR) = A(\square EWZH) + A(\square FHC'B')$

Now in  $A(\square QWZR)$

$A(\square QWZR) = A(\square QEFA') + A(\square EWZH) + A(\square FHC'B') + A(\square A'B'C'R)$

$b_1^2 = h_1^2 + A(\triangle PQR) + A(\square A'B'C'R)$

$A(\square A'B'C'R) = b_1^2 - h_1^2 - \frac{b_1 h_1}{2} \dots (iv)$

But the fig – 1 and 2

$A(\square ABCD) = A(\square A'B'C'R) \dots (v)$

Now in fig – 2 draw a  $\square LMQN$  as

$A(\square LMQN) = 2 .A(\square POMQ)$   
 $= 2 .h_1^2$

But,

$A(\square LMQN) = 3.A(\triangle PQR)$

$2 h_1^2 = 3 . \frac{b_1 h_1}{2} \dots (vi)$

Now, value of equation –(iv) put on equation no. –(iii)

$x^2 = 2 b_1 h_1 + A(\square ABCD)$

$x^2 = 2 b_1 h_1 + A(\square A'B'C'R) \dots A(\square ABCD) = A(\square A'B'C'R)$

$= 2 b_1 h_1 + b_1^2 - h_1^2 - \frac{b_1 h_1}{2} \dots \text{From eq}^n \text{ no. (iv)}$

$= b_1^2 + h_1^2 - h_1^2 - h_1^2 + 2 b_1 h_1 - \frac{b_1 h_1}{2}$

$= b_1^2 + h_1^2 - 2 h_1^2 + 3 \frac{b_1 h_1}{2}$

$= b_1^2 + h_1^2 - 3 \frac{b_1 h_1}{2} + 3 \frac{b_1 h_1}{2} \dots (vi) [ 2 h_1^2 = 3 . \frac{b_1 h_1}{2} ]$

$x^2 = b_1^2 + h_1^2$

Hence proof, the square of the hypotenuse of a right angled triangle is equal to the sum of the squares of the other two sides.

**Theorem- 8: Theorem of proof of hypotenuse through the sidemeasurement and area.**

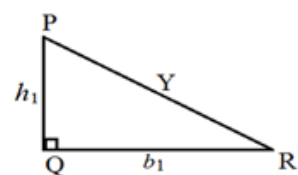
[This theorem is also called DGP THEOREM ]

In a right angled triangle, the square of hypotenuse is equal to the subtract of the squares of the sidemeasurement and four times the area.

**Given :-** In  $\triangle PQR$ ,  $M\angle PQR=90^\circ$

base (QR)=  $b_1$ , height (PQ) =  $h_1$

And hypotenuse (PR) = Y



**Figure (X) : DGP Theorem**



**To proof :-**  $Y^2 = B(\Delta PQR)^2 - 4.A(\Delta PQR)$

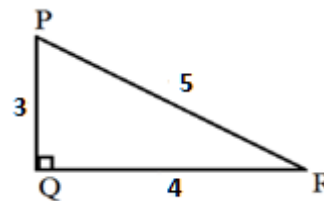
**Proof :-** In  $\Delta PQR$ ,  
 $Y^2 = b_1^2 + h_1^2 \dots$  (Pythagoras theorem)  
 $Y^2 = b_1^2 + h_1^2 + 2 b_1 h_1 - 2 b_1 h_1$   
 $Y^2 = (b_1 + h_1)^2 - 2 b_1 h_1 \dots (a+b)^2 = a^2 + b^2 + 2ab$   
 $Y^2 = (b_1 + h_1)^2 - 4. (\frac{1}{2}. b_1 h_1)$   
 $Y^2 = B(\Delta PQR)^2 - 4. A(\Delta PQR)$

Hence, we are proof that DGP theorem.

[Note-DGP is a name of my Grandfather ,Dhanaji Ganapati Patil]

**Example of DGP theorem:-**

In  $\Delta PQR$  ,  $M\angle PQR=90^\circ$ , Base (QR)= 4 ,Height (PQ) = 3 , And Hypotenuse (PR) = 5 then give proof of DGP theorem ?



**ANS:-**

Given-In  $\Delta PQR$ ,  $M\angle Q=90^\circ$ ,  $b_1$  (QR)= 4 ,  $h_1$  (PQ) = 3  
 And Y (PR) = 5 then proof LHS=RHS.

$$B(\Delta PQR) = b_1 + h_1 \\ = 4 + 3 \\ = 7$$

Sidemeasurement of  $\Delta PQR$  is 7

$$A(\Delta PQR) = \frac{1}{2}. b_1 h_1 \\ = \frac{1}{2}. 4 \times 3 \\ = 6$$

Area of  $\Delta PQR$  is 6

$$Y^2 = B(\Delta PQR)^2 - 4. A(\Delta PQR) \dots (DGP Theorem)$$

$$5^2 = 7^2 - (4 \times 6)$$

$$25 = 49 - 24$$

$$25 = 25$$

**LHS=RHS**

So ,here this example given proof of DGP theorem .

**Theorem- 8-i): Proof of base through the side-measurement and area.**

In  $\Delta PQR$ ,  $M\angle PQR=90^\circ$

base (QR)=  $b_1$  ,height (PQ) =  $h_1$

And hypotenuse (PR) = Y

Now ,in Fig.- (X)

we are know that,

$$Y^2 = B(\Delta PQR)^2 - 4. A(\Delta PQR) \dots (DGP theorem)$$

$$b_1^2 \sec^2 \theta = B(\Delta PQR)^2 - 4. A(\Delta PQR)$$

$$b_1^2 = \frac{[B(\Delta PQR)^2 - 4.A(\Delta PQR)]}{\sec^2 \theta}$$

**Theorem- 8-ii): Proof of height through the side-measurement and area.**

In  $\Delta PQR$ ,  $M\angle PQR=90^\circ$

base (QR)=  $b_1$  ,height (PQ) =  $h_1$

And hypotenuse (PR) = Y

Now ,in Fig. - (X)

We are know that,

$$Y^2 = B(\Delta PQR)^2 - 4. A(\Delta PQR) \dots (DGP theorem)$$

$$h_1^2 \operatorname{cosec}^2 \theta = B(\Delta PQR)^2 - 4.A(\Delta PQR)$$

$$h_1^2 = \frac{[B(\Delta PQR)^2 - 4.A(\Delta PQR)]}{\operatorname{cosec}^2 \theta}$$

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