IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 2 Ver. II (Mar - Apr. 2015), PP 59-63
www.iosrjournals.org

Special Double Sampling Plan for Truncated Life Tests Based On Generalised Log-Logistic Distribution

D. Malathi [1,] and Dr. S. Muthulakshmi [2,]

[1] *Assistant Professor (Sr.Gr), Department of Mathematics, Velalar College of Engineering & Technology, Erode.*

[2] *Professor, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore.*

Abstract: *The design of special double sampling plan is proposed for the truncated life tests assuming that the lifetime of a product follows generalized log-logistic distribution . The minimum sample sizes of the special double sampling plan are determined to ensure that the median life is longer than the given life at the specified consumer's confidence level. The operating characteristic values are analysed.The minimum median ratios are obtained so as to meet the producer's risk at the specified consumer's confidence level. Numerical illustrations are provided to explain the use of constructed tables. The efficiency of special double sampling plan is analysed with single sampling plan.*

I. Introduction

Acceptance sampling plan is an important tool for ensuring quality on the basis of information yielded by the sample to accept or reject the lot. It is widely used when the testing is destructive and cost of inspection is very high. If the lifetime of a product is high then it might be time consuming to wait until the failure of the product. Therefore, for economy it is usual to terminate a life test by a pre-assigned time t.

Several authors studied the acceptance sampling plans using life tests distributions. Rosaiah and Kantam (2005) introduced acceptance sampling plan based on the inverse Rayleigh distribution using mean life. Aslam and Chi-Hyuck Jun(2010) developed a double acceptance sampling plan for generalized log-logistic distribution using median life. Srinivasa Rao, Kantam et.al (2012) designed acceptance sampling plan for percentiles based on inverse Rayleigh distribution.

The purpose of this paper is to propose a special double sampling plan for the truncated life test assuming that the life time of a product follows generalized log-logistic distribution. This distribution plays a vital role in hydrology to model stream flow and precipitation, estimating mortality rate from cancer following diagonosis and so on.The minimum sample sizes are calculated by incorporating minimum average sample number for various confidence levels, shape parameters and the ratio of experimental time to the specified median life. The operating characteristic values of the designed sampling plans are analysed and the minimum median ratios of the life time are obtained for the specified producer's risk.

Generalised Log-Logistic Distribution

Assume that the lifetime of a product under consideration follows generalized log-logistic distribution. The probability density function and cumulative distribution function of generalized log-logistic distribution are

$$f(t)=\left[\frac{(\beta/\sigma)(t/\sigma)^{\beta-1}}{\{1+(t/\sigma)^{\beta}\}^{2}}\right]^{\theta},\ t>0,\sigma>0,\alpha>0,\beta>0$$

and

$$F(t,\sigma,\beta,\theta)=\left[\frac{(t/\sigma)^{\beta}}{1+(t/\sigma)^{\beta}}\right]^{\theta},\ t\geq 0,\sigma>0,\beta>0,\theta>0$$

where σ is the scale parameter,β and θ are the shape parameters. The median of generalized log-logistic distribution is

$$m=\sigma\left(\frac{0.5^{\frac{1}{\theta}}}{1-0.5^{\frac{1}{\theta}}}\right)^{\frac{1}{\beta}}$$

Here the median is proportional to σ, the scale parameter. If θ =1generalised log-logistic distribution becomes log-logistic distribution and its median is equal to the scale parameter σ.

Design of the Proposed Special Double Sampling Plan

Assume that the quality of a product can be represented by its median lifetime ,m. Design the special double sampling plan for the truncated life test consists of determination of (i) Sample size (ii) the ratio of true median life to the specified median life m/m_0 . The submitted lot will be accepted if the data supports the following null-hypothesis H_0: $m \geq m_0$ against the alternative hypothesis, H_1:$m<m_0$.The significance level for the test is 1-P* , where P* is the consumer's confidence level.The operating procedure of special double sampling plan for the truncated life test consists of the following steps:

Draw a random sample of size n_1 from the lot and put on test for pre-assigned experimental time t_o and observe the number of defectives d_1 . If $d_1 \geq 1$ reject the lot.

If $d_1=0$, draw a second random sample of size n_2 and put on test for time t_0 and observe the number of defectives d_2. If $d_2 \leq 1$ accept the lot, otherwise reject the lot.

In a special double sampling plan the decision of acceptance is made only after inspecting the second sample. It is more convenient to make a termination time as a multiple of the specified median life m_0, by assigning $t_0=am_0$ for the specified multiplier **a**. For a given P*, the proposed sampling plan may be characterized by the parameters (n_1,n_2,β,θ,a) .

The probability of lot acceptance for a special double sampling plan,

$$P_a = (1-p)^{n_1+n_2}\left[1+\frac{n_2 p}{1-p}\right]$$

where p, the probability that an item fails before t_o, is given by

$$p=\left[\frac{(t_o/\sigma)^{\beta}}{1+(t_o/\sigma)^{\beta}}\right]^{\theta}=\left[\frac{(a\gamma)^{\beta}}{(m/m_0)^{\beta}+(a\gamma)^{\beta}}\right]^{\theta}$$

with $\gamma=\left(\dfrac{(0.5)^{\frac{1}{\theta}}}{1-0.5^{\frac{1}{\theta}}}\right)^{\frac{1}{\beta}}$

and replacing m by m_0, p reduces to

$$p=\left[\frac{(a\gamma)^{\beta}}{1+(a\gamma)^{\beta}}\right]^{\theta}$$

The minimum sample sizes ensuring $m \geq m_0$ at the consumer's confidence level P*may be found as the solution of the inequality

$$P_a \leq 1-P^* \qquad (1)$$

Multiple solutions for the sample sizes n_1 and n_2 exist for equation (1).In order to find the optimal sample sizes the minimum of ASN is incorporated along with specification (1).The determination of the minimum sample sizes for special double sampling plan reduces to

Minimize $ASN = n_1 + n_2(1-p)^{n_1}$

subject to $(1-p)^{n_1+n_2}\left[1+\dfrac{n_2 p}{1-p}\right] \leq 1-P^*$ (2)

n_1 and n_2 are integers with $n_2 \leq n_1$.

Table 1 is constructed to present the minimum sample sizes for the first and second sample with specified P*(=0.75,0.90,0.95,0.99),**a**(=0.3,0.5,0.7,0.9,1.1,1.5,1.9) and shape parameter β (=2,3,4) under log-logistic distribution and generalized log-logistic distribution using equation (2)

Numerical values of Table 1 reveal that

(i) increase in **β** increases sample sizes for any P* with fixed **a.**

(ii) increase in **a** decreases the sample sizes for any P* with fixed **β.**

(iii) increase in θ increases the sample sizes for any P* with fixed **a** and **β.**

Operating Characteristic Values

OC values depict the performance of the sampling plan in discriminating the quality of the submitted product. Tables 2 and 3 give the operating characteristic values for log-logistic distribution when β=4 and generalised log-logistic distribution when (β,θ)= (2,2) respectively.

Numerical values of Table-2 and Table-3 reveal that

(i) increase in P* decreases the OC value for fixed **a** and m/m_0

(ii) increase in **a** decreases the OC value for fixed P* and m/m_0

(iii) increase in m/m_0 increases the OC value for fixed **a** and P*.

Minimum Median Ratio

In order to keep the producer's risk at the specified level and consumer's confidence at the required level,the minimum median ratio m/m_0 may be obtained by solving $P_a \geq 1-\alpha$ and presented in Table 4 by using the sample sizes presented in Table 1.

For Log-logistic and Generalised log-logistic distribution it is seen that

increase in **β** decreases the minimum median ratios for fixed P* and **a**.

increase in **a** increases the minimum median ratios for fixed P* and **β**.

increase in P* increases the minimum median ratios for fixed **a** and **β**.

Further for Generalised log-logistic distribution increase in **θ** decreases the minimum median ratios for fixed P*,**a** and **β**.

Numerical Illustration: To reduce the mortality rate of cancer, there are various types of treatment are available with different protocols like photon protocol, collect-Budwig and cesium chloride. The cesium chloride has the high percentage of reducing mortality rate of cancer when compared with other protocols. The team of doctors who are specialized in giving treatment for cancer patients would like to analyse the mortality rate of the methods and medicines applied. In order to meet this objective a sample of patients who are undergoing a particular type of treatment say cesium chloride protocol are to be chosen and are observed over a specified period of time. This study of efficiency with human beings warrants lesser sample size and shorter duration of observation. Under this situation, among the available acceptance sampling plans, special double sampling plan with truncated life test is advisable. The lifetime distribution of cancer patient with cesium chloride treatment is assumed to follow a generalized log-logistic distribution with $\beta = 3$ and $\theta = 2$. Table 1 yields the sample sizes as n_1=8, n_2=7 for the median lifetime to be longer than say for example 208 days at 90% level of confidence with the truncated lifetest of 146 days (multiplier a=0.7) According to this plan, 8 patients have to be selected randomly from the patients getting treatment with cesium chloride and are to be observed for 146 days. If no failure occurs during the observed period select a second random sample of 7 patients and observe them for 146 days. If atmost one failure occurs the treatment may be continued, otherwise it may be concluded that the procedure with cesium chloride is not achieving the expected result.

Comparitive Study: The OC values according to the ratio m/m_0 (=2,4,6,8) for special double sampling plan and single sampling plan with the ASN value 9 are shown as follows:

SP	C=0	0.6179	0.9581	0.9782	0.9970
	C=1	0.5154	0.9234	0.9434	0.9127
SDSP		0.9560	0.9992	0.9999	0.9999

The table value indicates that special double sampling plan has higher OC values than the single sampling plan for the same quality level.

II. Conclusion

Designing of special double sampling plan with truncated life test when the lifetime of the products follow a generalized log-logistic distribution is proposed ,which is useful in system reliability analysis with economy.

References

[1]. Rao G.S., Kantam R.R.L., Rosaiah K. and Pratapa Reddy J.(2012) Acceptance Sampling Plans for Percentiles based on the Inverse Rayleigh distribution, Electron. J. App. Stat. Anal, Vol.5,No.2 ,164-177.

[2]. Rosaiah K. and Kantam R.R.L.(2005) Acceptance Sampling based on the Inverse Rayleigh Distribution,Economic Quality Control,Vol 20, No.2, 277-286.

[3]. Aslam M.and Jun C.H.(2010) A Double Acceptance Sampling plan for generalized log-logistic distributions with known shape parameters ,Journal of Applied Statistics,Vol.37,No.3,405-414.

[4]. Kantam R.R.L., Rosaiah K. and Rao G.S.(2001) Acceptance sampling based on life tests: log-logistic models, Journal of Applied Statistics,Vol.28,121-128.

[5]. Rao G.S., Kantam R.R.L.(2010) Acceptance sampling plans from truncated life tests based on the log-logistic distribution for percentiles, Economic Quality Control,Vol.25,No.2,153-167.

[6]. Tsai T.R.,Wu S.J.(2006) Acceptance sampling based on truncated life tests for generalised Rayleigh distribution, Journal of Applied Statistics,Vol.33,595-600.
[7]. Duncan A.J.(1986) Quality Control and Industrial Statistics, 5th edition,Richard D.Irwin, Homewood, Illinois.

Table-1 Minimum sample sizes (n_1,n_2)

Log-logistic distribution								
β	P*	**a**						
		0.3	0.5	0.7	0.9	1.1	1.5	1.9
2	0.75	13,12	5,5	3,3	2,2	2,1	2,1	1,1
	0.9	20,18	8,7	5,4	3,3	3,1	2,1	2,1
	0.95	25,23	10,9	6,5	4,4	3,3	2,2	2,1
	0.99	35,35	14,14	8,8	6,5	5,3	3,3	3,2
3	0.75	40,40	10,8	4,4	3,1	2,1	1,1	1,1
	0.9	62,61	15,13	6,6	4,2	3,1	2,1	2,1
	0.95	78,77	18,18	8,6	4,4	3,3	2,2	2,1
	0.99	113,113	26,26	11,10	6,6	4,4	3,2	2,2
4	0.75	131,131	18,17	6,3	3,1	2,1	1,1	1,1
	0.9	203,203	28,26	8,8	4,3	3,1	2,1	1,1
	0.95	256,254	35,33	10,10	5,4	3,2	2,1	2,1
	0.99	372,371	50,50	15,13	7,6	4,3	2,2	2,1
Generalised log-logistic distribution								
(β,θ)	P*	**a**						
		0.3	0.5	0.7	0.9	1.1	1.5	1.9
(2,2)	0.75	33,33	8,6	4,1	3,1	2,1	2,1	1,1
	0.9	51,51	11,11	5,5	4,1	3,1	2,1	2,1
	0.95	64,64	14,14	7,5	4,4	3,3	2,2	2,1
	0.99	93,93	20,20	9,9	6,5	4,4	3,3	3,1
(2,3)	0.75	62,61	9,8	4,2	3,1	2,1	2,1	1,1
	0.9	96,95	14,12	6,4	4,2	3,1	2,1	2,1
	0.95	121,119	17,17	7,6	4,4	3,3	2,2	2,1
	0.99	175,175	25,23	10,9	6,5	4,4	3,3	3,1
(3,2)	0.75	281,281	20,18	5,5	3,1	2,1	1,1	1,1
	0.9	437,435	30,30	8,7	4,3	3,1	2,1	2,1
	0.95	549,548	38,37	10,8	5,3	3,2	2,1	2,1
	0.99	799,798	55,54	14,13	6,6	4,4	3,1	2,2
(3,3)	0.75	12,631,263	31,30	6,4	3,1	2,1	1,1	1,1
	0.9	19,631,962	48,46	9,7	4,3	3,1	2,1	2,1
	0.95	24,682,466	60,59	11,10	5,4	3,2	2,1	2,1
	0.99	35,933,593	87,86	15,15	7,5	4,4	3,1	2,2

Table-2 Operating characteristic values when β =4 for log-logistic distribution

P^*	a	n_1	n_2	m/m_0					
				2	4	6	8	10	12
0.75	0.3	131	131	0.9339	0.9959	0.9991	0.9997	0.9999	0.9999
	0.5	18	17	0.9304	0.9956	0.9991	0.9997	0.9999	0.9999
	0.7	6	3	0.9139	0.9944	0.9989	0.9996	0.9999	0.9999
	0.9	3	1	0.8864	0.9923	0.9985	0.9995	0.9998	0.9999
	1.1	2	1	0.8394	0.9887	0.9977	0.9993	0.9997	0.9999
	1.5	1	1	0.7596	0.9806	0.9961	0.9988	0.9995	0.9998
	1.9	1	1	0.5511	0.9516	0.9900	0.9968	0.9987	0.9994
0.90	0.3	203	203	0.8979	0.9936	0.9987	0.9996	0.9998	0.9999
	0.5	28	26	0.8924	0.9932	0.9987	0.9996	0.9998	0.9999
	0.7	8	7	0.8825	0.9925	0.9985	0.9995	0.9998	0.9999
	0.9	4	3	0.8476	0.9899	0.9979	0.9994	0.9997	0.9998
	1.1	3	1	0.7689	0.9830	0.9966	0.9989	0.9996	0.9998
	1.5	2	1	0.5771	0.9616	0.9922	0.9975	0.9989	0.9995
	1.9	1	1	0.5511	0.9516	0.9900	0.9968	0.9987	0.9994
0.95	0.3	256	254	0.8718	0.9919	0.9984	0.9995	0.9998	0.9999
	0.5	35	33	0.8660	0.9915	0.9983	0.9995	0.9998	0.9999
	0.7	10	10	0.8538	0.9906	0.9981	0.9994	0.9998	0.9999
	0.9	5	4	0.8107	0.9872	0.9975	0.9992	0.9997	0.9998
	1.1	3	2	0.7636	0.9830	0.9966	0.9989	0.9996	0.9998
	1.5	2	1	0.5771	0.9616	0.9922	0.9975	0.9989	0.9995
	1.9	2	1	0.3037	0.9055	0.9802	0.9937	0.9974	0.9987
0.99	0.3	372	371	0.8155	0.9882	0.9977	0.9993	0.9997	0.9999
	0.5	50	50	0.8094	0.9878	0.9976	0.9992	0.9997	0.9998
	0.7	15	13	0.7875	0.9859	0.9972	0.9991	0.9996	0.9998
	0.9	7	6	0.7389	0.9821	0.9965	0.9989	0.9995	0.9998
	1.1	4	3	0.6905	0.9773	0.9955	0.9986	0.9994	0.9997
	1.5	2	2	0.5437	0.9612	0.9922	0.9975	0.9989	0.9995
	1.9	2	1	0.3037	0.9055	0.9802	0.9937	0.9974	0.9987

Table-3 Operating Characteristic values for Generalised log-logistic distribution when (β,θ)=(2,2)

P*	a	n_1	n_2	m/m_0					
				2	4	6	8	10	12
0.75	0.3	33	33	0.9978	0.9999	0.9999	0.9999	1	1
	0.5	8	6	0.9895	0.9998	0.9999	0.9999	0.9999	1
	0.7	4	1	0.9653	0.9993	0.9999	0.9999	0.9999	0.9999
	0.9	3	1	0.9056	0.9979	0.9998	0.9999	0.9999	0.9999
	1.1	2	1	0.8425	0.9954	0.9996	0.9999	0.9999	0.9999
	1.5	2	1	0.5556	0.9747	0.9974	0.9995	0.9999	0.9999
	1.9	1	1	0.5454	0.9577	0.9949	0.9991	0.9997	0.9999
0.9	0.3	51	51	0.9967	0.9999	0.9999	0.9999	1	1
	0.5	11	11	0.9855	0.9998	0.9999	0.9999	0.9999	1
	0.7	5	5	0.9561	0.9992	0.9999	0.9999	0.9999	0.9999
	0.9	4	1	0.8761	0.9971	0.9997	0.9999	0.9999	0.9999
	1.1	3	1	0.7733	0.9932	0.9994	0.9999	0.9999	0.9999
	1.5	2	1	0.5556	0.9747	0.9974	0.9995	0.9999	0.9999
	1.9	2	1	0.2974	0.9173	0.9899	0.9981	0.9995	0.9998
0.95	0.3	64	64	0.9958	0.9999	0.9999	0.9999	1	1
	0.5	14	14	0.9815	0.9997	0.9999	0.9999	0.9999	0.9999
	0.7	7	5	0.9393	0.9989	0.9999	0.9999	0.9999	0.9999
	0.9	4	4	0.8708	0.9971	0.9997	0.9999	0.9999	0.9999
	1.1	3	3	0.7585	0.9931	0.9994	0.9999	0.9999	0.9999
	1.5	2	2	0.5196	0.9745	0.9974	0.9995	0.9999	0.9999
	1.9	2	1	0.2974	0.9173	0.9899	0.9981	0.9995	0.9998
0.99	0.3	93	93	0.9939	0.9999	0.9999	0.9999	1	1
	0.5	20	20	0.9736	0.9996	0.9999	0.9999	0.9999	0.9999
	0.7	9	9	0.9211	0.9985	0.9999	0.9999	0.9999	0.9999
	0.9	6	5	0.8119	0.9957	0.9996	0.9999	0.9999	0.9999
	1.1	4	4	0.6841	0.9909	0.9991	0.9998	0.9999	0.9999
	1.5	3	3	0.3473	0.9618	0.9961	0.9993	0.9998	0.9999
	1.9	3	1	0.1622	0.8785	0.9849	0.9971	0.9992	0.9997

Table-4 Minimum median ratios

Log-logistic distribution								
β	P*	a						
		0.3	0.5	0.7	0.9	1.1	1.5	1.9
2	0.75	4.5999	4.7699	5.2013	5.3672	6.5674	8.9868	7.8976
	0.9	5.7542	6.1212	6.6675	6.9678	7.9999	8.9864	11.3452
	0.95	6.9287	7.2643	7.5969	7.6878	8.1021	8.9645	11.2252
	0.99	7.5868	8.2542	8.4532	9.4563	10.4352	10.9856	13.8875
3	0.75	2.6989	2.9843	2.9999	3.4356	3.7356	3.9878	4.9834
	0.9	3.2098	3.3654	3.5467	3.9678	4.3251	4.9789	6.5643
	0.95	3.3674	3.5675	3.8674	3.9856	4.2234	4.9698	6.2314
	0.99	3.9675	4.0215	4.3562	4.4532	4.6523	5.6321	6.4567
4	0.75	2.1211	2.1989	2.3654	2.4276	2.6753	3.1098	4.0456
	0.9	2.3745	2.4657	2.5789	2.7865	2.9999	3.7865	4.0124
	0.95	2.5341	2.6321	2.6399	2.8342	3.0231	3.7651	4.7654
	0.99	2.7765	2.8765	2.9765	3.1314	3.2345	3.6789	4.7865
Generalised log-logistic distribution								
(β,θ)	P*	a						
		0.3	0.5	0.7	0.9	1.1	1.5	1.9
(2,2)	0.75	2.3451	2.6432	2.9871	3.5674	3.8765	5.3421	5.4321
	0.9	2.5432	2.8761	3.2431	3.8765	4.4653	5.3231	6.7543
	0.95	2.6751	3.0123	3.5678	3.9898	4.3212	5.3561	6.9871
	0.99	2.9654	3.4563	3.7654	4.4432	4.7865	5.9898	7.4531
(2,3)	0.75	1.8431	2.1231	2.4534	2.9871	3.3231	4.5621	4.8765
	0.9	1.9821	2.3421	2.6789	3.5431	3.7561	4.5678	5.6753
	0.95	2.1114	2.4421	2.7546	3.2312	3.6512	4.5321	5.8761
	0.99	2.2113	2.5456	2.9876	3.4567	3.9874	4.9898	6.2235
(3,2)	0.75	1.6712	1.8231	2.0231	2.2389	2.5341	2.9858	3.7856
	0.9	1.8712	1.9541	2.1231	2.4356	2.7456	3.4658	4.3567
	0.95	1.8999	1.9873	2.2243	2.4989	2.7435	3.4567	4.4321
	0.99	1.9879	2.1121	2.3214	2.5721	2.8991	3.7821	4.3762
(3,3)	0.75	1.4321	1.5341	1.7321	1.9989	2.3213	2.7981	3.5431
	0.9	1.4987	1.6121	1.8231	2.1231	2.4895	3.1234	3.9899
	0.95	1.5632	1.6578	1.8796	2.1912	2.4887	3.1891	3.9341
	0.99	1.5987	1.7321	1.9689	2.2999	2.5674	3.3241	3.9875