IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 2 Ver. III (Mar - Apr. 2015), PP 15-21
www.iosrjournals.org

Plane Gravitational Waves with Cosmic Stringscoupled with Maxwell's Field in Bimetric Relativity

S. D. Deo[1,] Sulbha R. Suple[2,]
Department of MathematicsN.S.Science and Arts College Bhadrawati,
Dist-Chandrapur, (M.S.) 442902, India
Department of Mathematics,KarmavirDadasahebKannamwar College of Engineering,
Nagpur 440009,India

Abstract: *In this paper,* Z = $\left(\frac{\sqrt{3}\,t}{x+y+z}\right)$ *type plane gravitational waves is studied with source Cosmic cloud strings coupled with Electromagnetic fields in Rosen'sbimetric theory of relativity.It is shown that there is nil contribution either from Cosmic cloud orfrom Maxwell'sfield andalso for cosmic cloud strings coupled with Maxwell's field in this theory.Only vacuum model can be constructed.*
Keywords: - *Plane gravitational waves, Cosmic cloud strings, Maxwell's field, Bimetric Relativity.*

I. Introduction

Rosen [12-13] proposed the bimetric theory of gravitation to remove some of the unsatisfactory features of the Einstein's general theory of relativity, by assuming two metric tensors. In this theory he has proposed a new formulation of the general relativity by introducing a background Euclidean metric tensor γ_{ij} in addition to the usual Riemannian metric tensor g_{ij} at each point of the four dimensional space-time. With the flat background metric γ_{ij} the physical content of the theory is the same as that of the general relativity.
Thus, now the corresponding two line elements in a coordinate system x^i are –

$$ds^2 = g_{ij}dx^i dx^j \quad (1.1)$$

And $$d\sigma^2 = \gamma_{ij}dx^i dx^j \quad (1.2)$$

Where ds is the interval between two neighboring events as measured by means of a clockand a measuring rod. The interval $d\sigma$ is an abstract or geometrical quantity not directly measurable. One can regard it as describing the geometry that would exist if no matter were present.H Takeno [16] propounded a rigorous discussion of plane gravitational waves, definedvarious terms by formulating a meaningful mathematical version and obtained numerous results.

Using definition of plane wave, we will use here,
Z = $\left(\frac{\sqrt{3}\,t}{x+y+z}\right)$ type plane gravitational waves by using the line elements,

$$ds^2 = -A\left(dx^2+dy^2\right) - C\left(dz^2-dt^2\right) \quad (1.3)$$

LalK.B. ;Ali,N.[9] have studied wave solutions of the field equations of general relativity in a generalized Takeno's space-time, MitskievicN.V.andPandey S.N. [10], analyzed the motion of test particles in plane gravitational waves.The theory of plane gravitational waves have been studied by many investigators,
H Takeno [17]; Pandey [11]; Goldman I.[5];Gowdy,R.H. [6];Bondi, H. et.al.[1],Torre,C.G.[18]; Hogan, P.A.[8];Deo and Ronghe[14],[15], Deo and Suple [2],[3],[4] and they obtained various solutions .
In continuation of this, we will study Z = $\left(\frac{\sqrt{3}\,t}{x+y+z}\right)$ type plane gravitational wave with Cosmic cloud string coupled with Maxwell's field and will observe the result in the context of Bimetrictheory of relativity.

II. Field Equations In Bimetric Relativity

Rosen N. [12, 13] has proposed the field equations of Bimetric Relativity from Variation

Principle as

$$K_i^j = N_i^j - \frac{1}{2} N g_i^j = -8\pi\kappa T_i^j \quad (2.1)$$

Where $$N_i^j = \frac{1}{2}\gamma^{\alpha\beta}\left[g^{hj} g_{hi|\alpha}\right]_{|\beta} \quad (2.2)$$

$$N = N_\alpha^\alpha \, , \; \kappa = \sqrt{\frac{g}{\gamma}} \quad (2.3)$$

and $g = \left|g_{ij}\right|$, $\gamma = \left|\gamma_{ij}\right|$ (2.4)

Where a vertical bar (|) denotes a covariant differentiation with respect to γ_{ij}

2.1 Z = $\left(\dfrac{\sqrt{3}\, t}{x+y+z}\right)$ type plane gravitational wave with Cosmic Cloud strings:

For Z = $\left(\dfrac{\sqrt{3}\, t}{x+y+z}\right)$ plane gravitational wave, we have the line element as

$$ds^2 = -A\left(dx^2 + dy^2\right) - C\left(dz^2 - dt^2\right) \quad (3.1)$$

where $A = A(Z)$, C = C(Z) and Z = $\left(\dfrac{\sqrt{3}\, t}{x+y+z}\right)$

Corresponding to the equation (3.1), we consider the line element for background metric γ_{ij} as

$$d\sigma^2 = -\left(dx^2 + dy^2 + dz^2\right) + dt^2 \quad (3.2)$$

and , T_i^j the energy momentum tensor for Cosmic cloud strings is given by

$$T_i^j = T_{i\;strings}^j = \rho v_i v^j - \lambda x_i x^j \quad (3.3)$$

together with $v_4 v^4 = 1 \;\; and \;\; x_1 x^1 = -1$ where v_i is the four-velocity of the cloud of particles, x^i is the four vector representing the direction of anisotropy(x-axis)and ρ is the rest energy density for a cloud of strings with particles attached along the extension .Thus $\rho = \rho_p + \lambda$ where ρ_p is the particle energy density and λ is the tension density of the strings. In co-moving coordinate system we have $T_{1\;strings}^1 = \lambda$, $T_{4\;strings}^4 = \rho$ and $T_{i\;strings}^j = 0$ for i, j=2, 3 and for i $\neq$ j

Using equations (2.1) to (2.4) with (3.1) to (3.3),

We get the field equations as

$$D\left(\frac{\bar{C}^2}{C^2} - \frac{\bar{\bar{C}}}{C}\right) = \frac{16}{3}\pi\kappa\lambda \quad (3.4)$$

$$D\left(\frac{\bar{C}^2}{C^2} - \frac{\bar{\bar{C}}}{C}\right) = 0 \quad (3.5)$$

$$D\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)=0 \quad (3.6)$$

$$D\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)=\frac{16}{3}\pi\kappa\rho \quad (3.7)$$

where $D=\left[\frac{3t^2-(x+y+z)^2}{(x+y+z)^4}\right]$

and $\bar{A}=\frac{\partial A}{\partial Z}$, $\bar{\bar{A}}=\frac{\partial^2 A}{\partial Z^2}$, $\bar{C}=\frac{\partial C}{\partial Z}$, $\bar{\bar{C}}=\frac{\partial^2 C}{\partial Z^2}$

Using equation (3.4) to (3.7), we get

$\lambda=\rho=0$ (3.8)

This equation of state is known as false vacuum.

Equation (3.8) immediately implies that cosmic cloud strings does not exist in Z = $\left(\frac{\sqrt{3}\ t}{x+y+z}\right)$ plane gravitational wave in Rosen's Bimetric theory of relativity.

Hence for vacuum case $\lambda=0=\rho$, the field equation reduced to

$$\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)D=0$$

i.e. $$\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)=0 \quad (3.9)$$

and

$$\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)D=0$$

i.e. $$\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)=0 \quad (3.10)$$

Solving equations (3.9) and (3.10), we have

$A=R_1\, e^{S_1 Z}$ (3.11)

and

$C=R_2\, e^{S_2 Z}$ (3.12)

where R_1 , S_1 and R_2 , S_2 are the constants of integration.

Thus substituting the value of (3.11) and (3.12) in (3.1), we get the vacuum line element as

$$ds^2=-R_1 e^{S_1 Z}\left(dx^2+dy^2\right)-R_2\, e^{S_2 Z}\left(dz^2-dt^2\right) \quad (3.13)$$

Thus, it is found that in plane gravitational wave Z = $\left(\frac{\sqrt{3}\ t}{x+y+z}\right)$, the Cosmic cloud strings does not survive in Bimetric theory of relativity and only vacuum model can be constructed.
By proper choice of co-ordinates the metric (3.13) can be transform to

$$ds^2 = -e^{\alpha Z}\left[dx^2+dy^2+dz^2-dt^2\right] \quad (3.14)$$

Which is free from singularity at t = 0 and the spatial volume of the model is given by

$$V^3 = (-g)^{\frac{1}{2}} = e^{2\alpha Z} \quad (3.15)$$

This study can further be extended with the introduction of cosmological constant λ in the field equation which is defined as $N_i^j = \lambda g_i^j$.

Thus we get
$$\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)D=\lambda \quad (3.16)$$

And
$$\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)D=\lambda \quad (3.17)$$

On solving equation (3.16) we have

$$A = \mathbf{exp}\left[\frac{D'\lambda Z^2}{2}+EZ+F\right] \quad (3.18)$$

Where E and F are constants of integration and $D' = \frac{1}{D} = \left[\frac{(x+y+z)^4}{3t^2-(x+y+z)^2}\right]$

On solving (3.17) we obtain

$$C = \mathbf{exp}\left[\frac{D'\lambda Z^2}{2}+GZ+H\right] \quad (3.19)$$

Where G and H are constants of integration.

Thus substituting the value of A and C [using (3.18)-(3.19)] the line element (3.1) reduces to

$$ds^2 = -\mathbf{exp}\left[\frac{D'\lambda Z^2}{2}+EZ+F\right]\left(dx^2+dy^2\right)-\mathbf{exp}\left[\frac{D'\lambda Z^2}{2}+GZ+H\right]\left(dz^2-dt^2\right) \quad (3.20)$$

Thus Z = $\left(\frac{\sqrt{3}\ t}{x+y+z}\right)$ plane gravitational wave exists in Bimetric relativity with or without cosmological constant λ respectively.

Z = $\left(\frac{\sqrt{3}\ t}{x+y+z}\right)$ type plane gravitational wave with Maxwell's Field:

In this section, we consider the region of the space-time filled with electromagnetic field whose energy momentum tensor is given by

$$E_{i\ mag}^{j} = -F_{ir}F^{jr}+\frac{1}{4}F_{ab}F^{ab}g_i^j \quad (4.1)$$

where $E_{i\ mag}^{j}$ is the electromagnetic energy tensor, F_i^j is the electromagnetic field tensor.

As the electromagnetic field is moving along the x-direction alone,$F_{23}$is the only nonzero component of Maxwell'stensor F_{ij}.

Maxwell's equation is given by $F_{ij,\,k} + F_{jk,\,i} + F_{ki,\,j} = 0$

gives rise to $F_{23} = - F_{32} = F$ (constant)

Using equations (2.1) to (2.4) and (3.1) - (3.2) with energy momentum tensor (4.1), the field equationsare

$$D\left(\frac{\overline{C}^2}{C^2} - \frac{\overline{\overline{C}}}{C}\right) = \frac{16}{3}\pi\kappa\eta \quad (4.2)$$

$$D\left(\frac{\overline{C}^2}{C^2} - \frac{\overline{\overline{C}}}{C}\right) = -\frac{16}{3}\pi\kappa\eta \quad (4.3)$$

$$D\left(\frac{\overline{A}^2}{A^2} - \frac{\overline{\overline{A}}}{A}\right) = -\frac{16}{3}\pi\kappa\eta \quad (4.4)$$

$$D\left(\frac{\overline{A}^2}{A^2} - \frac{\overline{\overline{A}}}{A}\right) = \frac{16}{3}\pi\kappa\eta \quad (4.5)$$

Where $\eta = \frac{1}{2}\frac{F^2}{AC}$

Solving (4.4) and (4.5), we get

$$\eta = 0 \;\; i.e. F = F_{23} = 0 \quad (4.6)$$

Thus for the space-time (3.1) Maxwell's field does not survive in Bimetric theory of relativity and only vacuum model exists and it is same as defined in equation (3.20).

2.2 Coupling of Cosmic cloud strings with Electromagnetic Field

The energy momentum tensor for a mixture of cosmic cloud string and Electromagnetic field together is given by

$$T_i^j = T_i^j{}_{strings} + E_i^j{}_{mag} \quad (5.1)$$

Where $T_i^j{}_{strings}$ and $E_i^j{}_{mag}$ are already defined.

By the use of co-moving co-ordinate system,the field equation (2.1) to (2.4) for the metric (3.1) and (3.2) corresponding to the energy momentum tensor (5.1) can be written as

$$D\left(\frac{\overline{C}^2}{C^2} - \frac{\overline{\overline{C}}}{C}\right) = \frac{16}{3}\pi\kappa\{\lambda + \eta\} \quad (5.2)$$

$$D\left(\frac{\overline{C}^2}{C^2} - \frac{\overline{\overline{C}}}{C}\right) = -\frac{16}{3}\pi\kappa\eta \quad (5.3)$$

$$D\left(\frac{\overline{A}^2}{A^2} - \frac{\overline{\overline{A}}}{A}\right) = -\frac{16}{3}\pi\kappa\eta \quad (5.4)$$

$$D\left(\frac{\overline{A}^2}{A^2}-\frac{\overline{\overline{A}}}{A}\right)=\frac{16}{3}\pi\kappa\left\{\rho+\eta\right\} \quad (5.5)$$

Using (5.3) and (5.4), we obtain

$$C = A\beta e^{\alpha Z} \quad (5.6)$$

where α *and* β are constants of integration.

On solving (5.2) to (5.5) we get

$$\lambda+\rho+4\eta=0 \quad (5.7)$$

In view of the reality conditions (Hawking S.W. and Ellis G.F.R)[7] i.e. $\lambda>0, \rho>0$ *and* $\eta>0$ must hold.

The above conditions (5.7) is satisfied only when

$$\lambda=0, \rho=0 \text{ and } \eta=0 \quad (5.8)$$

This means that thephysical parameters, viz tension density (λ), rest energy density (ρ) andthe magnetic field alongthe x-axis (η) are identically zero.Thus plane gravitational waves with cosmic cloud strings coupled with Maxwell's field does not survive in bimetric relativity and hence only vacuum model is obtained.

Using (5.8), the vacuum field equations are

$$D\left(\frac{\overline{A}^2}{A^2}-\frac{\overline{\overline{A}}}{A}\right)=0$$

ie $$\left(\frac{\overline{A}^2}{A^2}-\frac{\overline{\overline{A}}}{A}\right)=0 \quad (5.9)$$

$$D\left(\frac{\overline{C}^2}{C^2}-\frac{\overline{\overline{C}}}{C}\right)=0$$

ie $$\left(\frac{\overline{C}^2}{C^2}-\frac{\overline{\overline{C}}}{C}\right)=0 \quad (5.10)$$

On solving (5.9) and (5.10),we get the same result as defined in equation (3.18) and (3.19)and we get the same vacuum solution which is obtained in (3.20).

III. Conclusion

It is well known that at early stage of universe cosmic strings and magnetic field play a fundamental role in the formation of universe. It is evident, from the literature that Einstein's formalism of general relativity used to establish the existence of cosmic strings. Here it is shown that, in the study of **Z** = $\left(\frac{\sqrt{3}\,t}{x+y+z}\right)$ type plane gravitational wave; there is nil contribution of Cosmic cloud strings coupled with Maxwell's fieldin Bimetric theory of relativity respectively. It is observed that the matter fields either cosmic strings or Electromagnetic fields cannot be a source of gravitational field in the Rosen's biometric theory but only vacuum model exists. Hencebimetric theory doesn't help in any way to study gravitational effects of cosmic strings and Maxwell's field at the early stages of evolution of the universe

Acknowledgments

The authors are thankful to Dr. R. D. Giri, Prof. Emeritus, P.G.T.D. (Mathematics), R. T. M. N. U., Nagpur, India for his constant inspiration.

References

[1]. Bondi H.;Pirani, F.A.E. and Robinson, I.(1959). Gravitational waves in general relativity III.Exact plane waves. Proc. Roy.Soc.Lond.A23, 25, 519-533

[2]. Deo S.D. and Suple S.R. (2013)Plane gravitational waves with cosmic strings in BimetricRelativity.Asian Journal of current Engineering and Maths 2: 2 March – April 131 - 133.

[3]. Deo S.D. and Suple S.R.(2013)Plane gravitational waves with cosmic strings in BimetricRelativity.MathematicaAeterna, Vol. 3, , no. 6, 489 - 496

[4]. Deo S.D. and Suple S.R. (2014)Plane gravitational waves with Macro and Micro Matter fields inBimetric Relativity.International Journal of Mathematics Trends and Technology – Volume 8 Number 1,51-5

[5]. Goldman I. (1978). "Plane waves in bimetric gravitation theory."Gen.Relativity.Grav.,7(11),895-901

[6]. GowdyR.H.(1971). "Gravitational waves in closed universe"Phy.Rev.Lett.27, 826-829

[7]. Hawking S.W. and Ellis G.F.R(1973) The Large scale structure of Space –time,pp.88,Cambridge Uni.Press.

[8]. Hogan P.A.(1999). Gravitational waves and Bertotti-Robinson space- time.Math. Proc. Roy. Irish Acad. 99A, 51-55.

[9]. Lal K.B.; Ali,N.(1970a) Wave solutions of the field equations of General relativity in aGeneralized Takeno's space time.Tensor N.S.21, 134-137

[10]. Mitskievic N.V., Pandey S.N (1980). On a motion of test particles in a plane gravitational wavesClass.Quantum Grav.18 3007-3017

[11]. Pandey, S. N. (1979). Plane wave solutions in Finzi's non-symmetric unified field theory. Theo. Math. Phys. 39, 371-375

[12]. Rosen N. (1940) General Relativity and Flat space I.Phys. Rev.57, 147.

[13]. Rosen N. (1973) A bimetric theory of gravitation I Gen. Rela.Grav.04, 435-47.

[14]. Ronghe A.K. and Deo S.D (2011). - Plane Gravitational Waves In Bimetric Relativity. JVR 6, 1-11

[15]. Ronghe A.K. and Deo S.D. (2011) Plane gravitational waves with wet dark energy.International Journal of Mathematical Archive-2(3) Mar.-391-392

[16]. Takeno H. (1961) The mathematical theory of plane gravitational waves in General Relativity.Scientific report of Research Institute fortheoretical Physics, Hiroshima University, Hiroshima, Ken, Japan (1961)

[17]. Takeno H. (1958)A comparison of plane wave solutions in general relativity with thosein non-symmetric theory. Prog.Theor.Phys.20, 267-276

[18]. Torre C.G. (2006) Gravitational waves – Just plane symmetry. Gen. Rela.Grav. 38,653-662.