IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 2 Ver. IV (Mar - Apr. 2015), PP 75-83
www.iosrjournals.org

Generalized Common Fixed Theorem in Sequentially Compact Intuitionistic Fuzzy Metric Spaces

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Abstract: *The aim of this paper is to introduce the notion of sequentially compact intuitionistic fuzzy metric spaces and prove a generalized common fixed point theorem for pairs of weakly compatible self mappings in this newly defined space .*
Keywords: *Intuitionistic fuzzy metric space, sequentially compact intuitionistic fuzzy metric space , compatible mappings , weakly compatible mappings , common fixed point .*

I. Introduction

The concept of Fuzzy sets was initially investigated by Zadeh [17] as a new way to represent vagueness in everyday life . as a generalization of fuzzy sets introduced Zadeh [17], Atanassov [2], introduced the concept of intuitionistic fuzzy sets . Recently , using the idea of intuitionistic fuzzy sets , In 2004 , Park [12] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorms. In 2006 , Turkoglu [16] proved Jungck's [6] , common fixed point theorem in the setting of intuitionistic fuzzy metric spaces for commuting mappings .Recently , in 2006 , Alaca et al. [1] using the idea of Intuitionistic fuzzy sets , defined the notion of intuitionistic fuzzy metric space with the help of continuoun t-norm and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [10] . Jungck and Rhoades [6] gave more generalized concept weak compatibility then compatibility .Recently , many authors have studied fixed point theorem in intuitionistic fuzzy metric spaces ([1] , [12] , [14,16]) . Recently , Rao , K.P.R. , Rao , K.R.K. and Rao , T.Ranga [13] introduced the concept of sequentially compact fuzzy metric space . Using this concept , we introduce the notion sequentially compact intuitionistic fuzzy metric spaces and prove a generalized common fixed point theorem for pairs of weakly compatible self mappings in this newly defined space.

II. Preliminaries :-

Definition :-2[1] :

A binary operation $*:[0,1]\times[0,1]\rightarrow[0,1]$ is a continuous t-norm if it satisfies the following

Conditions:

1. $*$ is associative and commutative ,
2. $*$ is continuous ,
3. $a*1=a$ for all $a\in[0,1]$,
4. $a*b\leq c*d$ whenever $a\leq c$ and $b\leq d$, for each $a,b,c,d\in[0,1]$.

Example :- Two typical examples of continuous t-norm are

$$a*b=ab \quad \text{and}$$
$$a*b=\min(a,b)$$

Definition 2[2] :

A binary operation $\lozenge:[0,1]\times[0,1]\rightarrow[0,1]$ is a continuous t-conorm if satisfies the following

Conditions:

1. $\lozenge$ is associative and commutative,
2. $\lozenge$ is continuous,
3. $a\lozenge 0=a$ for all $a\in[0,1]$,
4. $a\lozenge b\leq c\lozenge d$ whenever $a\leq c$ and $b\leq d$,for each $a,b,c,d\in[0,1]$.

Example :- Two typical examples of continuous t-conorm are

$a\Diamond b=\min(a+b,1)$ **and**

$a\Diamond b=\max(a,b)$

Definition 2[3] : A 5-tuple $(X,M,N,*,\Diamond)$ is called a intuitionistic fuzzy metric space if X is an arbitrary (non-empty) set , $*$ is a continuous t-norm , $\Diamond$ is a continuous t-conorm and M , N are fuzzy sets on $X^2\times[0,\infty)$,satisfying the following conditions : for each $x,y,z\in X$ and $t,s>0,$

i. $M(x,y,\mathrm{t})+N(x,y,t)\le 1,$

ii. $M(x,y,0)=0,$

iii. $M(x,y,t)=1$ **if and only if** $x=y$,

iv. $M(x,y,t)=M(y,x,t,)$,

v. $M(x,y,t)*M(y,z,s)\le M(\mathrm{x},\mathrm{z},t+s),$

vi. $M(x,y,.):[0,\infty)\to[0,1]$ **is left continuous ,**

vii. $\lim_{t\to\infty}M(x,y,t)=1$,

viii. $N(x,y,0)=1$,

ix. $N(x,y,\mathrm{t})=0$ **if and only if** $x=y$,

x. $N(x,y,\mathrm{t})=N(y,x,t),$

xi. $N(x,y,\mathrm{t})\Diamond N(y,z,s)\ge N(x,z,t+s),$

xii. $N(x,y,.):[0,\infty)\to[0,1]$ **is right continuous ,**

xiii. $\lim_{t\to\infty}N(x,y,t)=0$,

Then (M,N) is called intuitionistic fuzzy metric on X. The function $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non –nearness between x and y with respect to t, respectively.

Definition 2[4]: Let $(X,M,N,*,\Diamond)$ be an intuitionistic fuzzy metric space . Then a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t>0$ and $p>0$,

$$\lim_{n\to\infty}M(x_{n+p},x_n,t)=1,\quad \lim_{n\to\infty}N(x_{n+p},x_n,t)=0.$$

Definition2[5]: Let $(X,M,N,*,\Diamond)$ be an intuitionistic fuzzy metric space . Then a sequence $\{x_n\}$ in X is said to be convergent to a point $x\in X$ if for all $t>0$

$$\lim_{n\to\infty}M(x_n,x,t)=1 \quad\text{and}\quad \lim_{n\to\infty}N(x_n,x,t)=1 .$$

Since $*$ and $\Diamond$ are continuous , the limit is uniquely determined from (v) and (xi), respectively.

Definition2[6]: An intuitionistic fuzzy metric space $(X,M,N,*,\Diamond)$ is said to be complete if every Cauchy sequence is convergent .

Definition2[7]: Let A and B be mappings from an intuitionsitic fuzzy metric space $(X,M,N,*,\Diamond)$ in to itself . The mappings A and B are said to be compatible if

$\lim_{n\to\infty}M(ABx_n,BAx_n,t)=1$,

$\lim_{n\to\infty}N(ABx_n,BAx_n,t)=0$ for all $t>0$, whenever $\{x_n\}$ is a sequence in X

Such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some $z \in X$.

Definition2[8]: Self mappings A and B be mappings from an intuitionsitic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be weakly compatible if they commute at their coincidence point , that is $Ax = Bx$ implies that $ABx = BAx$ for some $x \in X$.

It is easy to see that if self mappings A and B of an intuitionsitic fuzzy metric space $(X, M, N, *, \Diamond)$ is compatible then they are weakly compatible.

Definition 2[9] : $(X, M, N, *, \Diamond)$ is said to be sequentially compact intuitionistic fuzzy metric space If every sequence in X has a convergent subsequence in it .

Let Φ be the set of all function ϕ , $\phi' : [0,1]^5 \to [0,1]$ such that

i. ϕ , ϕ' are non decreasing and non increasing in all coordinates respectively ,

ii. $\phi(t_1, t_2, t_3, t_4, t_5), \phi'(t_1, t_2, t_3, t_4, t_5)$ are continuous in t_4 and t_5 and

iii. $\phi(t,t,t,t,t) > t, \phi'(t,t,t,t,t) < t$ for every $t \in [0,1)$.

III. Main Result:

Here afterwards , assume that $(X, M, N, *, \Diamond)$ be a sequentially compact intuitionistic fuzzy metric space with $t * t \geq t$, $s \Diamond s \leq s$ $\forall t, s \in [0,1]$

Theorem 3.1: Let P, Q, A, B, S and T be self mappings on $(X, M, N, *, \Diamond)$ such that :

(i) $P(X) \subset ST(X)$ and $Q(X) \subset AB(X)$,

(ii) P and AB are continuous or Q and ST are continuous ,

(iii) $AB = BA, ST = TS, PB = BP, TQ = QT$,

(iv) The pairs (P, AB) and (Q, ST) are weakly compatible ,

(v) There exists $q \in (0,1)$ such that for every $x, y \in X$ and $t > 0$

$$M(Px, Qy, qt) \geq \min\{M(ABx, STy, t) * M(Px, ABx, t) * M(Qy, STy, t) * M(Px, STy, t)\}$$

$$N(Px, Qy, qt) \leq \max\{N(ABx, STy, t) \Diamond N(Px, ABx, t) \Diamond N(Qy, STy, t) \Diamond N(Px, STy, t)\};$$

(vi) for all $x, y \in X, \lim_{t\to\infty} M(x, y, t) = 1, \lim_{t\to\infty} N(x, y, t) = 0$;

If the pair of maps (P , AB) is reciprocal continuous and semi compatible maps then P , Q , S , T , A and B have a unique common fixed point in X .

Proof :- Let $x_0 \in X$, from (1) there exist $x_1, x_2 \in X$ such that $Px_0 = STx_1$ and $Qx_1 = ABx_2$.

Inductively , we can construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$Px_{2n-2} = STx_{2n-1} = y_{2n-1}$

And

$Qx_{2n+1} = ABx_{2n} = y_{2n}$ for n = 1,2,3, ……….

By using contractive condition (v) ,we obtain

$$M(Px_{2n}, Qx_{2n+1}, qt) \geq \min\{M(ABx_{2n}, STx_{2n+1}, t) * M(Px_{2n}, ABx_{2n}, t) * M(Qx_{2n+1}, STx_{2n+1}, t) * M(Px_{2n}, STx_{2n+1}, t)\}$$

$$N(Px_{2n}, Qx_{2n+1}, qt) \leq \max\{N(ABx_{2n}, STx_{2n+1}, t) \Diamond N(Px_{2n}, ABx_{2n}, t) \Diamond N(Qx_{2n+1}, STx_{2n+1}, t) \Diamond N(Px_{2n}, STx_{2n+1}, t)\}$$

$M(Px_{2n}, Qx_{2n+1}, qt) \geq \min\{M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n}, t) * M(y_{2n+2}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+1}, t)\}$

$N(Px_{2n}, Qx_{2n+1}, qt) \leq \max\{N(y_{2n}, y_{2n+1}, t) \Diamond N(y_{2n+1}, y_{2n}, t) \Diamond N(y_{2n+2}, y_{2n+1}, t) \Diamond N(y_{2n+1}, y_{2n+1}, t)\}$

$M(Px_{2n}, Qx_{2n+1}, qt) \geq \min\{M(y_{2n}, y_{2n+1}, t) * M(y_{2n+2}, y_{2n+1}, t)\}$

$N(Px_{2n}, Qx_{2n+1}, qt) \leq \max\{N(y_{2n}, y_{2n+1}, t) \Diamond N(y_{2n+2}, y_{2n+1}, t)\}$

The only t-norm * satisfying $t * t \geq t$ for all $t \in [0,1]$ is the minimum t-norm , that is
$a * b = \min\{a, b\}$ for all $a, b \in [0,1]$ and
and by the definition 2[2] of continuous t-conorm then
$a \Diamond b = \max\{a, b\}$ for all $a, b \in [0,1]$.
By the conditions ,we have

$M(y_{2n+1}, y_{2n+2}, qt) \geq M(y_{2n}, y_{2n+1}, t)$

$N(y_{2n+1}, y_{2n+2}, qt) \leq N(y_{2n}, y_{2n+1}, t)$

Similarly we have

$M(y_{2n+2}, y_{2n+3}, qt) \geq M(y_{2n+1}, y_{2n+2}, t)$

$N(y_{2n+2}, y_{2n+3}, qt) \leq N(y_{2n+1}, y_{2n+2}, t)$

Thus , we have

$M(y_{n+1}, y_{n+2}, qt) \geq M(y_n, y_{n+1}, t)$ for n = 1,2,3,…….

$N(y_{n+1}, y_{n+2}, qt) \leq N(y_n, y_{n+1}, t)$ for n = 1,2,3,……..

$$M(y_n, y_{n+1}, t) \geq M\left(y_n, y_{n+1}, t/q\right) \geq M\left(y_{n-2}, y_{n-1}, t/q^2\right)$$

…………… …………. ………… ……….

$$\geq M\left(y_1, y_2, t/q^n\right) \to 1 \text{ as } n \to \infty$$

and

$$N(y_n, y_{n+1}, t) \leq N\left(y_n, y_{n+1}, t/q\right) \leq N\left(y_{n-2}, y_{n-1}, t/q^2\right)$$

……… ………. ……….. ……..

$$\leq N\left(y_1, y_2, t/q^n\right) \to 0 \text{ as } n \to \infty ,$$

And hence $M(y_n, y_{n+1}, t) \to 1$ as $n \to \infty$ for $t > 0$
and $N(y_n, y_{n+1}, t) \to 0$ as $n \to \infty$ for $t > 0$
for such $\in > 0$ and $t > 0$,we can choose $n_0 \in N$ such that

$M(y_n, y_{n+1}, t) > 1 - \in$ for all $n > n_0$

$N(y_n, y_{n+1}, t) < 1 - \in$ for all $n > n_0$

For $m, n \in N$,we suppose $m \geq n$ .then we have

$$M(y_n, y_m, t) \geq M\left(y_n, y_{n+1}, t/(m-n)\right) * M\left(y_{n+1}, y_{n+2}, t/(m-n)\right) * \ldots\ldots\ldots * M\left(y_{m-1}, y_m, t/(m-n)\right)$$

$$\geq (1-\in)*(1-\in)*............*(1-\in)(m-n)times$$
$$\geq (1-\in)$$

and for $m,n \in N$,we suppose $m \leq n$ .then we have

$$N(y_n, y_m, t) \leq N\left(y_n, y_{n+1}, t/_{m-n}\right) \Diamond \mathrm{N}\left(y_{n+1}, y_{n+2}, t/_{m-n}\right) \Diamond ....\Diamond \mathrm{N}\left(y_{m-1}, y_m, t/_{m-n}\right)$$
$$\leq (1-\in) \Diamond (1-\in) \Diamond ....\Diamond (1-\in)(m-n)times$$
$$\leq (1-\in)$$

And hence $\{y_n\}$ is a Cauchy sequence in X.

Since $(X,\mathrm{M},*)$ and $(X,N,\Diamond)$ is complete , $\{y_n\}$ converges to some point $z \in X$. Also its subsequences converges to the same point i.e . $z \in X$.

i.e., $\{Qx_{2n+1}\} \to z$ and $\{STx_{2n+1}\} \to z$ ……………………(1)

$\{Px_{2n}\} \to z$ and $\{ABx_{2n}\} \to z$ ……………………(2)

Since the pair (P , AB) is reciprocally continuous mapping then we have

$$\lim_{n\to\infty} PABx_{2n} = Pz$$

and

$$\lim_{n\to\infty} ABPx_{2n} = ABz$$

and $\lim_{n\to\infty} ABPx_{2n} = ABz$.

And semi compatibility of (P , AB) gives

$$\lim_{n\to\infty} ABPx_{2n} \to ABz$$

Therefore Pz = ABz .

We claim $Pz = ABz = z$.

Step 1 :- Put $x = z$ and $y = x_{2n+1}$ gives in condition (v),

$$M(Pz, Qx_{2n+1}, qt) \geq \min\{M(ABz, STx_{2n+1}, t)*M(Pz, ABz, t)*M(Qx_{2n+1}, STx_{2n+1}, t)*M(Pz, STx_{2n+1}, t)\}$$
$$N(Pz, Qx_{2n+1}, qt) \leq \max\{N(ABz, STx_{2n+1}, t)\Diamond \mathrm{N}(Pz, ABz, t)\Diamond \mathrm{N}(Qx_{2n+1}, STx_{2n+1}, t)\Diamond \mathrm{N}(Pz, STx_{2n+1}, t)\}$$

taking $n \to \infty$ and using equation (i) , we get

$$M(Pz, \mathrm{z}, \mathrm{qt}) \geq \min\{M(z,z,t)*M(Pz,z,t)*M(z,z,t)*\mathrm{M}(Pz,z,t)\}$$
$$N(Pz, \mathrm{z}, \mathrm{qt}) \leq \max\{N(z,z,t)\Diamond \mathrm{N}(Pz,z,t)\Diamond \mathrm{N}(z,z,t)\Diamond \mathrm{N}(Pz,z,t)\}$$

i.e

$$M(Pz, z, qt) \geq M(Pz, z, t)$$

Then we get,

$$Pz = z \ .$$

Therefore ,

$$ABz = Pz = z \ . \qquad …………….(3)$$

and

$$N(Pz, z, qt) \leq N(Pz, z, t)$$

Then we get,

$$Pz = z \ .$$

Therefore ,

$ABz = Pz = z$. ……………(4)

Step 2 :- Putting $x = Bz$ and $y = x_{2n+1}$ in condition (v) , we get

$$M\left(PBz, Qx_{2n+1}, qt\right) \geq \min\left\{M\left(ABBz, STx_{2n+1}, t\right) * M\left(PBz, ABBz, t\right) * M\left(Qx_{2n+1}, STx_{2n+1}, t\right) * M\left(PBz, STx_{2n+1}, t\right)\right\}$$

$$N\left(PBz, Qx_{2n+1}, qt\right) \leq \max\left\{N\left(ABBz, STx_{2n+1}, t\right) \Diamond N\left(PBz, ABBz, t\right) \Diamond N\left(Qx_{2n+1}, STx_{2n+1}, t\right) \Diamond N\left(PBz, STx_{2n+1}, t\right)\right\}$$

As $BP = PB, AB = BA,$

so we have

$$P\left(Bz\right) = B\left(Pz\right) = Bz$$

and

$$\left(AB\right)\left(Bz\right) = \left(BA\right)\left(Bz\right) = B\left(ABz\right) = Bz$$

Taking $n \to \infty$ and using (i) , we get

$$M\left(Bz, z, qt\right) \geq \min\left\{M\left(Bz, z, t\right) * M\left(Bz, Bz, t\right) * M\left(z, z, t\right) * M\left(Bz, z, t\right)\right\}$$

$$N\left(Bz, z, qt\right) \leq \max\left\{N\left(Bz, z, t\right) \Diamond N\left(Bz, Bz, t\right) \Diamond N\left(z, z, t\right) \Diamond N\left(Bz, z, t\right)\right\}$$

i.e.

$$M\left(Bz, z, qt\right) \geq M\left(Bz, z, t\right)$$

then we get

$$Bz = z$$

and also we have

$$ABz = z$$

$$\Rightarrow \quad Az = z$$

Therefore , $Az = Bz = Pz = z$ …………(5)

and

$$N\left(Bz, z, qt\right) \leq N\left(Bz, z, t\right)$$

then we get

$$Bz = z$$

and also we have

$$ABz = z$$

$$\Rightarrow \quad Az = z$$

Therefore , $Az = Bz = Pz = z$ ………….(6)

Step -3 :- As $P\left(x\right) \subset ST\left(x\right)$, there exists $u \in X$ such that

$$z = Pz = STu$$

Putting $x = x_{2n}$ and $y = u$ in (v) , we get

$$M\left(Px_{2n}, Qu, qt\right) \geq \min\left\{M\left(ABx_{2n}, STu, t\right) * M\left(Px_{2n}, ABx_{2n}, t\right) * M\left(Qu, STu, t\right) * M\left(Px_{2n}, STu, t\right)\right\}$$

$$N\left(Px_{2n}, Qu, qt\right) \leq \max\left\{N\left(ABx_{2n}, STu, t\right) \Diamond N\left(Px_{2n}, ABx_{2n}, t\right) \Diamond N\left(Qu, STu, t\right) \Diamond N\left(Px_{2n}, STu, t\right)\right\}$$

Taking $n \to \infty$ and using (1) and (2) , we get

$$M\left(z, Qu, qt\right) \geq \min\left\{M\left(z, z, t\right) * M\left(z, z, t\right) * M\left(Qu, z, t\right) * M\left(z, z, t\right)\right\}$$

$$N\left(z, Qu, qt\right) \leq \max\left\{N\left(z, z, t\right) \Diamond N\left(z, z, t\right) \Diamond N\left(Qu, z, t\right) \Diamond N\left(z, z, t\right)\right\}$$

i.e.

$$M\left(z, Qu, qt\right) \geq M\left(z, Qu, t\right)$$

Then we get

$$Qu = z$$

Hence $STu = z = Qu$

$$M\left(STu, STQu, t\right) \geq M\left(STu, Qu, t/r\right) = 1$$

i.e.

$$STu = STQu$$

$$\Rightarrow z = STz$$

and

$$N\left(z, Qu, qt\right) \leq N\left(z, Qu, t\right)$$

Then we get,

$$Qu = z$$

Hence $STu = z = Qu$ …………..(7)

$$N\left(STu, STQu, t\right) \leq N\left(STu, Qu, t/r\right) = 0$$

i.e

$$STu = STQu$$

$\Rightarrow z = STz$. ……………(8)

Step-4 :- Putting $x = x_{2n}$ and $y = z$ in equation (v) , we get

$$M\left(Px_{2n}, Qz, qt\right) \geq \min\left\{M\left(ABx_{2n}, STz, t\right) * M\left(Px_{2n}, ABx_{2n}, t\right) * M\left(Qz, STz, t\right) * M\left(Px_{2n}, STz, t\right)\right\}$$

$$N\left(Px_{2n}, Qz, qt\right) \leq \max\left\{N\left(ABx_{2n}, STz, t\right) \Diamond N\left(Px_{2n}, ABx_{2n}, t\right) \Diamond N\left(Qz, STz, t\right) \Diamond N\left(Px_{2n}, STz, t\right)\right\}$$

Taking $n \to \infty$ and using (2) and step-3 , we get

$$M\left(z, Qz, qt\right) \geq \min\left\{M\left(z, Qz, t\right) * M\left(z, z, t\right) * M\left(Qz, Qz, t\right) * M\left(z, Qz, t\right)\right\}$$

$$N\left(z, Qz, qt\right) \leq \max\left\{N\left(z, Qz, t\right) \Diamond N\left(z, z, t\right) \Diamond N\left(Qz, Qz, t\right) \Diamond N\left(z, Qz, t\right)\right\}$$

i.e.

$$M\left(z, Qz, qt\right) \geq M\left(z, Qz, t\right)$$

Then we get ,

$$Qz = z$$

So $z = Qz = STz$ ………..(9)

and

$$N\left(z, Qz, qt\right) \leq N\left(z, Qz, t\right)$$

Then we get

$$Qz = z$$

So $z = Qz = STz$. ………(10)

Step-5 :- Putting $x = x_{2n}$ and $y = Tz$ in (v) , we get

$$M\left(Px_{2n}, QTz, qt\right) \geq \min\left\{M\left(ABx_{2n}, STTz, t\right) * M\left(Px_{2n}, ABx_{2n}, t\right) * M\left(QTz, STTz, t\right) * M\left(Px_{2n}, STTz, t\right)\right\}$$

$$N\left(Px_{2n}, QTz, qt\right) \leq \max\left\{N\left(ABx_{2n}, STTz, t\right) \Diamond N\left(Px_{2n}, ABx_{2n}, t\right) \Diamond N\left(QTz, STTz, t\right) \Diamond N\left(Px_{2n}, STTz, t\right)\right\}$$

As $AT = TQ$ and $ST = TS$ we have

$QTz = TQz = Tz$ and

$$ST\left(Tz\right) = T\left(STz\right) = TQz = Tz$$

Taking $n \to \infty$ we get,

$M(z,Tz,qt) \geq \min\{M(z,Tz,t)*M(z,z,t)*M(Tz,Tz,t)*M(z,Tz,t)\}$

$N(z,Tz,qt) \leq \max\{N(z,Tz,t)\Diamond N(z,z,t)\Diamond N(Tz,Tz,t)\Diamond N(z,Tz,t)\}$

i.e. $M(z,Tz,qt) \geq M(z,Tz,t)$

then we get,

$Tz = z$

Now $STz = Tz = z$

$\Rightarrow Sz = z$

Hence , $Sz = Tz = Qz = z$ …………(11)

Using (5) and (11) we get

$Az = Bz = Pz = Qz = Tz = Sz = z$

And

$N(z,Tz,qt) \leq N(z,Tz,t)$

then we get,

$Tz = z$

Now $STz = Tz = z$

$\Rightarrow Sz = z$

Hence , $Sz = Tz = Qz = z$ ………….(12)

Combining (5) and (12) , we get

$Az = Bz = Pz = Qz = Tz = Sz = z$

Hence, z is the common fixed point A , B ,S ,T , P and Q .

Uniqueness :-

Let u be another common fixed point of A , B , S , T , P and Q .

Then $Au = Bu = Pu = Qu = Su = Tu = u$.

Put $x = z$ and $y = u$ in (v) , we ge

$M(Pz,Qu,qt) \geq \min\{M(ABz,STu,t)*M(Pz,ABz,t)*M(Qu,STu,t)*M(Pz,STu,t)\}$

$N(Pz,Qu,qt) \leq \max\{N(ABz,STu,t)\Diamond N(Pz,ABz,t)\Diamond N(Qu,STu,t)\Diamond N(Pz,STu,t)\}$

Taking $n \to \infty$, we get

$M(z,u,qt) \geq \min\{M(z,u,t)*M(z,z,t)*M(u,u,t)*M(z,u,t)\}$

$N(z,u,qt) \leq \max\{N(z,u,t)\Diamond N(z,z,t)\Diamond N(u,u,t)\Diamond N(z,u,t)\}$

i.e.

$M(z,u,qt) \geq M(z,u,t)$

then we get,

$z = u$

And

$N(z,u,qt) \leq N(z,u,t)$

then we get,

$z = u$

Therefore z is the unique common fixed point of self maps A , B , S , T , P and Q . Hence proved

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