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Solving Multi-Objective Fuzzy Solid Transportation Problem Based On Expected Value And The Goal Programming Approach

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Abstract: *In the present paper, the multi-objective solid transportation problem with fuzzy coefficients for the objectives and constraints is modelled and then solved. Fuzzy goal programming is used to the linear multi-objective solid transportation problem, and an optimal compromise solution is obtained. Firstly, expected values of the fuzzy objective functions are considered to derive crisp values. In this method, a defuzzification model, which is an application of fuzzy linear programming and conditions for a solid transportation problem are imposed. Fuzzy programming technique and goal programming approach are applied to derive optimal compromise solutions of multi-objectives. Three numerical examples are presented using the above mentioned methodology and the appropriate comparative study is also included. Obtained concluding remarks are given in the last section.*
Keywords: *Solid transportation problem, Fuzzy sets, Expected value fuzzy programming technique, Goal programming approach.*

I. Introduction

The traditional transportation problem (TP) can be generalized by solid transportation problem (STP), its three-dimensional properties are supply, demand and conveyance capacities.

Schell [1] and Haley [2] introduced the STP. If optimized objective is more than one in the STP, then the problem is called multi-objective solid transportation problem (MOSTP). Zimmermann [3] was the first person to introduce the fuzzy programming approach for handling the multi-objective problems. The MOSTP was solved by several investigators using various methods. Bit et al. [4] used fuzzy programming approach, Ida et al. [5] presented a neural network method. Gao and Liu [6] developed two-phase fuzzy algorithms to solve MOSTP. Kundu et al. [7] presented a multi-objective, multi-item STP. Pramanik et al. [8] have developed a multi-objective STP in a fuzzy random environment. Li et al. [9] presented a genetic algorithm for solving the MOSTP with coefficients of the objective function as fuzzy numbers. Liu and Liu [10] presented the expected value model. Jiménez and Verdegay [11] applied an evolutionary algorithm based on parametric approach to solve fuzzy STP. In addition, Li et al. [12] designed a neural network approach for multi-criteria STP and they also presented an improved genetic algorithm to solve MOSTP with fuzzy numbers. Gen et al. [13] gave a genetic algorithm for solving bicriteria fuzzy STP.

Charnes and Cooper [14] were the first ones to introduce the Goal programming (GP) approach and the program was advanced by Ijiri [15], Lee [16], Ignizio [17, 18] and Narasimhan [19, 20]. GP has appeared as a levelheaded tool for solving multi-objective fuzzy programming problems. The GP approach to fuzzy programming problems introduced by Mohamed [21] is extended to solve fuzzy multi-objective fractional linear programming problems. Using trapezoidal membership function the objectives are transformed into fuzzy goals by means of assigning an aspiration level to each of them.

To the best of the authors' knowledge, it may be noticed that previous studies did not include the goal programming approach for solving multi-objective fuzzy solid transportation problem based on expected value models. We show that the optimal solution of the multi-objective fuzzy solid transportation problem (MOFSTP) can be found simply by solving an equivalent crisp LP problem

The paper is organized as follows: Some preliminaries and notations are given in section 2. In section 3, the problem formulation of MOFSTP is discussed, while the defuzzification process is discussed in section 4. In section 5, the proposed procedure using fuzzy GP is presented. Numerical examples with comparative study are given in section 6. Concluding remarks are presented in the last section 7.

II. Preliminaries and notations

Definition 2.1: A fuzzy number $\widetilde{A} = (a_1, a_2, a_3)$, is called the triangular fuzzy number where $a_1 < a_2 < a_3$, if the membership function of $\widetilde{A}$ is defined by:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & \text{when } a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-b}, & \text{when } a_2 \le x \le a_3 \\ 0, & \text{otherwise} \end{cases}$$

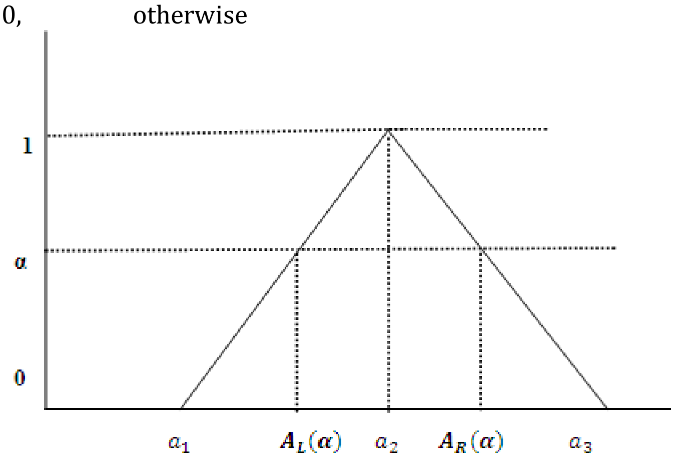

Fig. 1: Triangular Fuzzy Number

Definition 2.2: A fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$, is called the trapezoidal fuzzy number where $a_1 < a_2 < a_3 < a_4$, if the membership function of $\tilde{A}$ is defined by:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & \text{when } a_1 \le x \le a_2 \\ 1, & \text{when } a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3}, & \text{when } a_3 \le x \le a_4 \\ 0, & \text{otherwise.} \end{cases}$$

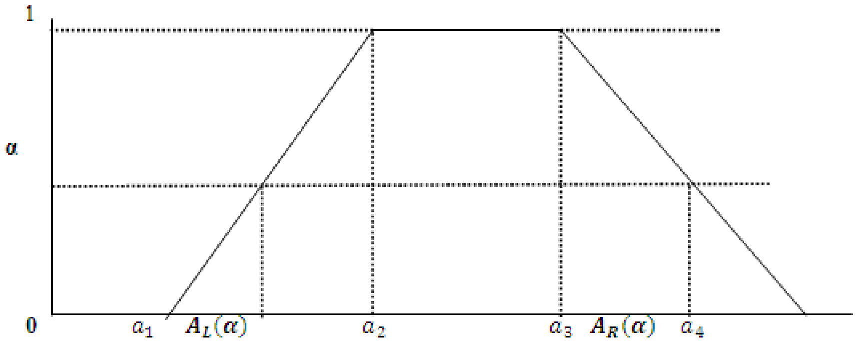

Fig. 2: Trapezoidal Fuzzy Number

Definition 2.3: (Liu et al. [10]) Let $\tilde{\xi}$ be a fuzzy variable. Then the expected value of $\tilde{\xi}$ is defined as

$$E[\xi] = \int_0^{\infty} Cr\{\xi \ge r\}dr - \int_{-\infty}^{0} Cr\{\xi \le r\}dr,$$

provided that at least one of the two integral is finite. If $\tilde{\xi}$ is a triangular fuzzy variable (r_1, r_2, r_3), then the expected value of $\tilde{\xi}$ is $(1/4)(r_1 + 2r_2 + r_3)$. If $\tilde{\xi}$ is a trapezoidal fuzzy variable (r_1, r_2, r_3, r_4), , then the expected value of $\tilde{\xi}$ is $(1/4)(r_1 + r_2 + r_3 + r_4)$.

Defuzzification 2.4:

Kikuchi [22] proposed a defuzzification method to find the most appropriate set of crisp numbers. For each of many possible sets of values that satisfy the relationships the lowest membership grade is checked and the set whose lowest membership grade is the highest is chosen as the best set of values for the problem. This process is performed using the fuzzy linear programming method.

Let the membership function for each value as $\mu_A(x)$, $\mu_B(y)$ and $\mu_C(z)$ where A, B, C are the fuzzy set of approximate numbers, suppose we have corresponding crisp values x, y, z. But each of them satisfies the relationship $R_j(x)$, $j \in N$ among them.

Then the following fuzzy linear programming model is formulated:

$Max\ \ \lambda,$

s.t. $\mu_A(x) \geq \lambda, \quad \mu_B(y) \geq \lambda, \quad \mu_C(z) \geq \lambda,$

and the relationship $R_j(x),\ j \in N.$

$\lambda \geq 0, A, B, C \geq 0$

where λ is the minimum degree of membership that one of the values A, B, C takes, i.e.

$\lambda^* = Max\,\lambda = MaxMin\left[\mu_A(x), \mu_B(y), \mu_C(z)\right].$

Kikuchi [22] applied this method to a traffic volume consistency problem taking all observed values as triangular fuzzy numbers. Dey and Yadav [23] modified this method with trapezoidal fuzzy numbers.

III. Problem Formulation

A multi-objective fuzzy solid transportation problem is formulated as follows:

$$Z_r = \min \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} c_{ijk}^{r} x_{ijk}, \qquad r = 1, 2, 3..., R \qquad (1)$$

$$s.t. \sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \leq s_i, \qquad i = 1, 2, ...m,$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \geq d_j, \qquad j = 1, 2, ...n,$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \leq e_k, \qquad k = 1, 2, ...K,$$

$$x_{ijk} \geq 0, \quad \forall i, j, k$$

Consider a product is to be transported from m sources to n destinations in a STP. At each source, let s_i be the amount of a homogeneous product we want to transport to n destinations to satisfy the demand for d_j units of the product. Here e_k, called conveyance, denotes the units of this product that can be carried by k different modes of transportation and also the objectives Z_r $(r = 1,2, \dots R)$ are to be minimized.

$$Max\,\lambda$$

$$s.t. \quad \mu_{s_i}(s_{ic}) \geq \lambda, \mu_{d_j}(d_{jc}) \geq \lambda, \mu_{e_k}(e_{kc}) \geq \lambda,$$

$$\sum_{i=1}^{m} s_{ic} \geq \sum_{j=1}^{n} d_{jc}, \qquad \sum_{k=1}^{K} e_{kc} \geq \sum_{j=1}^{n} d_{jc}.$$

$$\forall i, j, k.$$

A penalty value of the unit shipping cost c_{ijk}^{r} is associated with fuzzy transportation from i^{th} origin to j^{th} destination by the k^{th} conveyance. We need to determine a feasible way of shipping the available amounts to satisfy the demand such that the MOSTP costs is minimized.

IV. Defuzzification

The We consider s_i, d_j and e_k $(\forall i, j, k)$ are any triangular or trapezoidal fuzzy numbers (here triangular fuzzy numbers are denoted by $s_i = (s_i^1, s_i^2, s_i^3)$, $d_j = (d_j^1, d_j^2, d_j^3)$ and $e_k = (e_k^1, e_k^2, e_k^3)$, whereas trapezoidal fuzzy numbers are denoted by $s_i = (s_i^1, s_i^2, s_i^3, s_i^4)$, $d_j = (d_j^1, d_j^2, d_j^3, d_j^4)$ and $e_k = (e_k^1, e_k^2, e_k^3, e_k^4)$) with their membership functions as μ_{s_i}, μ_{d_j} and μ_{e_k} respectively. Now to solve the above problem, we first find corresponding crisp numbers, say, s_{ic}, d_{jc} and e_{kc} $(\forall i, j, k)$ so that for each item, total available resources greater than or equal to the total demands and also total conveyance capacities greater than or equal to the total demands for all items, i.e.

$$\sum_{i=1}^{m} s_{ic} \geq \sum_{j=1}^{n} d_{jc}, \qquad \sum_{k=1}^{K} e_{kc} \geq \sum_{j=1}^{n} d_{jc}. \tag{2}$$

For this purpose we apply the defuzzification method based on fuzzy linear programming.

The method is to introduce an auxiliary variable λ and formulate the following linear programming: where λ is the minimum degree of membership that one of the values of the variables s_{ic}, d_{jc}, e_{kc} takes, i.e. $Max\,\lambda = \lambda^* = MaxMin[\mu_{s_i}(s_{ic}), \mu_{d_j}(d_{jc}), \mu_{e_k}(e_{kc})]$, where

for triangular fuzzy numbers:

$$\mu_{s_i}(s_{ic}) = \begin{cases} \dfrac{s_i - s_i^1}{s_i^2 - s_i^1}, & \text{if } s_i^1 \leq s_{ic} \leq s_i^2; \\ \dfrac{s_i^3 - s_i}{s_i^3 - s_i^2}, & \text{if } s_i^2 \leq s_{ic} \leq s_i^3; \\ 0, & \text{otherwise.} \end{cases}$$

and for trapezoidal fuzzy numbers:

$$\mu_{s_i}(s_{ic}) = \begin{cases} \dfrac{s_i - s_i^1}{s_i^2 - s_i^1}, & if\ s_i^1 \leq s_{ic} \leq s_i^2; \\ 1, & if\ s_i^2 \leq s_i \leq s_i^3; \\ \dfrac{s_i^4 - s_i}{s_i^4 - s_i^3}, & if\ s_i^3 \leq s_{ic} \leq s_i^4. \end{cases}$$

and similarly for $\mu_{d_j}(d_{jc})$ and $\mu_{e_k}(e_{kc})$.

Now if we denote right and left sides of membership functions $\mu_{s_i}(s_{ic})$ by $\mu_{s_i}^l(s_{ic})$ and $\mu_{s_i}^r(s_{ic})$ respectively and so on for $\mu_{d_j}(d_{jc})$ and $\mu_{e_k}(e_{kc})$. Then the above programming becomes

$$\begin{aligned} & Max\,\lambda \\ & s.t. \quad \mu_{s_i}^l(s_{ic}) \geq \lambda, \quad \mu_{s_i}^r(s_{ic}) \geq \lambda, \\ & \mu_{d_j}^l(d_{jc}) \geq \lambda, \quad \mu_{d_j}^r(d_{jc}) \geq \lambda, \\ & \mu_{e_k}^l(e_{kc}) \geq \lambda, \quad \mu_{e_k}^r(e_{kc}) \geq \lambda, \\ & \sum_{i=1}^{m} s_{ic} \geq \sum_{j=1}^{n} d_{jc}, \qquad \sum_{k=1}^{K} e_{kc} \geq \sum_{j=1}^{n} d_{jc}. \\ & \forall\, i, j, k. \end{aligned} \tag{3}$$

V. Solution Methodology

Consider that c_{ijk}^r are all fuzzy numbers. After obtaining the defuzzified values s_{ic}, d_{jc} and e_{kc} $(\forall\, i, j, k)$ by the above procedure, problem (1) becomes,

$$Min\, Z_r = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} c_{ijk}^r x_{ijk}, \qquad r = 1, 2, 3..., R \tag{4}$$

$$s.t. \sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \leq s_{ic}, \qquad i = 1, 2, ...m,$$

$$\sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk} \geq d_{jc}, \qquad j = 1, 2, ...n,$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le e_{kc}, \qquad k = 1, 2, ...K,$$

$$x_{ijk} \ge 0, \quad \forall\, i, j, k\,.$$

Now, we use the following method to solve the problem.

5.1 Using expected value

Here we minimize the expected value of the objective functions and then the above problem becomes

$$Min\, E[Z_r] = E\left[\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} c_{ijk}^{r} x_{ijk}\right], \qquad r = 1, 2, 3..., R \tag{5}$$

$$s.t. \sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \le s_{ic}, \qquad i = 1, 2, ...m, \tag{6}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \ge d_{jc}, \qquad j = 1, 2, ...n, \tag{7}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le e_{kc}, \qquad k = 1, 2, ...K, \tag{8}$$

$$x_{ijk} \ge 0, \quad \forall\, i, j, k\,.$$

which is equivalently written as

$$Min\, E[Z_r] = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} E[c_{ijk}^{r}]\, x_{ijk}, \qquad r = 1, 2, 3..., R \tag{9}$$

s.t. the constraints (6) –(8),

$$x_{ijk} \ge 0, \quad \forall\, i, j, k\,.$$

The expected value model (Liu et al. [10]) can be formulated for the model (1) by using expected value to both the objective functions and the constraints. But here the crisp equivalence form (the deterministic values of supplies $E\left[\tilde{s}_i\right]$, demands $E[\tilde{d}_j]$ and conveyance capacities $E\left[\tilde{e}_k\right]$) may not satisfy the required conditions (2). So this method gives a feasible solution only when the fuzzy supplies, demands and conveyance capacities are so that their respective expected values automatically satisfy those conditions.

5.2 Goal Programming Formulation:

The procedure to solve MOFSTP based on fuzzy goal programming techniques is given below:

Step 1: Solve multi-objective problem as a single objective problem each time using only one objective ($r = 1, 2, \ldots\ldots R$) ignore all other objectives, to obtain the optimal solution $X^{r^*} = x_{ijk}$ of R different single objective problems.

Step 2: Calculate the values of all the R objective functions at all these R optimal solutions X^{r^*} ($r = 1, 2, \ldots\ldots R$) and find the lower bound and upper bound for each objective function given by $L_t = \overline{Z}_r(X^{t*}),\ t = 1, 2, ...R$ and $U_t = Max\{\overline{Z}_r(X^{1*}), \overline{Z}_r(X^{2*}), ...\overline{Z}_r(X^{R*})\}$, respectively,

Step 3: Define a membership function μ_t for the Rth objective function as follows:

$$\mu_t(\overline{Z}_t(x)) = \begin{cases} 1, & \text{if } \overline{Z}_t \le L_t \\ \dfrac{U_t - \overline{Z}_t}{U_t - L_t}, & \text{if } L_t \le \overline{Z}_r(x) \le U_t \\ 0, & \text{if } \overline{Z}_t \ge L_t \end{cases}$$

Then the linear model for MOFSTP can be formulated as:

$$
\begin{aligned}
&\min\ \lambda \\
&\text{s.t. } \frac{U_t-\overline{Z}_t}{U_t-L_t}+d_r^- - d_r^+ =1, \\
&\lambda \ge d_r^-, \qquad r=1,2,....,k, \\
&d_r^- d_r^+ =0, \\
&\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \le s_{ic}, \qquad i=1,2,.....m, \\
&\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \ge d_{jc}, \qquad j=1,2,.....n, \\
&\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le e_{kc}, \qquad k=1,2,.....K, \\
&d_r^-, d_r^+ \ge 0, \\
&\lambda \le 1, \quad \lambda \ge 0, \\
&x_{ijk} \ge 0, \quad \forall i,j,k.
\end{aligned}
$$

Step 4: Solve this crisp model and the obtained solution will be the optimal compromise solution of MOFSTP.

VI. Numerical Example

Example 1: We consider two objective functions with triangular and trapezoidal fuzzy numbers given in Table 1 and Table 2 to illustrate the proposed method.

Table 1: costs c_{ijk}^{1}

i	k=1			k=2		
	j			j		
	1	2	3	1	2	3
1	(8, 9,11)	(4, 6, 9,11)	(10,12, 14,16)	(9, 11,13,15)	(6, 8, 10)	(7, 9,12,14)
2	(8, 10,13,15)	(6, 7, 8,9)	(11, 13,15,17)	(10, 11, 13,15)	(6, 8, 10,12)	(14,16,18,20)

Table 2: costs c_{ijk}^{2}

i	k=1			k=2		
	j			j		
	1	2	3	1	2	3
1	(9,10, 12)	(5, 8, 10,12)	(10, 11,12,13)	(11, 13, 14,15)	(6, 7, 9,11)	(8, 10,11,13)
2	(11,13, 14,16)	(7, 9, 12,14)	(12,14, 16,18)	(14, 16, 20)	(9, 11, 13,14)	(13,14,15,16)

The supplies, demands and conveyance capacities are given as $\tilde{a}_1$=(21, 23, 25), $\tilde{a}_2$ =(28, 32, 35, 37), $\tilde{b}_1$ = (14, 16, 19), $\tilde{b}_2$ = (17, 20, 22, 25), $\tilde{b}_3$ = (12, 15, 18,21), $\tilde{e}_1$ = (21, 24, 26), $\tilde{e}_2$ = (24, 26, 27, 30). Then apply fuzzy programming in (3) and obtained defuzzified values are $\tilde{a}_1$ = 22.27, $\tilde{a}_2$ = 30.54, $\tilde{b}_1$ = 17.09, $\tilde{b}_2$ = 21.82, $\tilde{b}_3$ = 13.90, $\tilde{e}_1$ =24.73, $\tilde{e}_2$ = 28.09. In the following the proposed steps of the previous section is applied and the results are: $\lambda = 0.80985$, x_{111} = 17.09, x_{122} = 4.4951, x_{132} = 0.6849, x_{221} = 7.64, x_{111}^{2} = 22.7, x_{222} = 9.6849, x_{232} = 13.2151, d_1^- = 0.80985, d_2^- = 0.80985, $\overline{Z}_1$ = 574.1754 and $\overline{Z}_2$ = 605.085 .

Example 2: The following numerical example presented by P. Kundu et al. [7] is considered to explain the proposed Method's efficiency. The data are given in Table 3-6.

Table 3: Penalties/costs c_{ijk}^{11} .

i	k=1			k=2		
	j			j		
	1	2	3	1	2	3
1	(5, 8, 9,11)	(4, 6, 9,11)	(10,12, 14,16)	(9, 11,13,15)	(6, 8, 10,12)	(7, 9,12,14)
2	(8, 10,13,15)	(6, 7, 8,9)	(11, 13,15,17)	(10, 11, 13,15)	(6, 8, 10,12)	(14,16,18,20)

Table 4: Penalties/costs c_{ijk}^{12}.

i	k=1, j=1	k=1, j=2	k=1, j=3	k=2, j=1	k=2, j=2	k=2, j=3
1	(9,10, 12,13)	(5, 8, 10,12)	(10, 11,12,13)	(11, 13, 14,15)	(6, 7, 9,11)	(8, 10,11,13)
2	(11,13, 14,16)	(7, 9, 12,14)	(12,14, 16,18)	(14, 16, 18,20)	(9, 11, 13,14)	(13,14,15,16)

Table 5: Penalties/costs c_{ijk}^{21}.

i	k=1, j=1	k=1, j=2	k=1, j=3	k=2, j=1	k=2, j=2	k=2, j=3
1	(4, 5, 7,8)	(3, 5, 6,8)	(7, 9,10,12)	(6, 7, 8,9)	(4, 6, 7,9)	(5, 7, 9,11)
2	(6, 8, 9,11)	(5, 6, 7,8)	(6, 7, 9,10)	(4, 6, 8,10)	(7, 9,11,13)	(9,10, 11,12)

Table 6: Penalties/costs c_{ijk}^{22}.

i	k=1, j=1	k=1, j=2	k=1, j=3	k=2, j=1	k=2, j=2	k=2, j=3
1	(5, 7, 9,10)	(4, 6, 7,9)	(9, 11, 12,13)	(7, 8, 9,10)	(4, 5, 7,8)	(8,10, 11,12)
2	(10, 11,13,14)	(6, 7, 8,9)	(7, 9,11,12)	(6, 8,10,12)	(5, 7, 9,11)	(9,10, 12,14)

$\tilde{s}_1^1$=(21, 24, 26, 28), $\tilde{s}_2^1$=(28, 32, 35, 37), $\tilde{d}_1^1$ = (14, 16, 19, 22), $\tilde{d}_2^1$ = (17, 20, 22, 25), $\tilde{d}_3^1$ = (12, 15, 18,21), $\tilde{s}_1^2$ = (32, 34, 37, 39), $\tilde{s}_2^2$ = (25, 28, 30, 33), $\tilde{d}_1^2$ = (20, 23, 25, 28), $\tilde{d}_2^2$ = (16,18, 19, 22), $\tilde{d}_3^2$ = (15, 17, 19, 21), $\tilde{e}_1$ = (46, 49, 51, 53), $\tilde{e}_2$ = (51, 53, 56, 59).

Then applying the proposed method, we get the following result $\lambda = 0.1133810$, x_{111}^1 = 15.8, x_{221}^1 = 19.7, x_{231}^1 = 6.8, x_{132}^1 = 7.90, x_{111}^2 = 22.7, x_{122}^2 = 2.41, x_{132}^2 = 8.685, x_{221}^2 = 15.385, x_{231}^2 = 8.1145, d_1^- = 0.1133, d_2^- = 0.1133, $\overline{Z}_1$ = 1110.183 and $\overline{Z}_2$ = 814.396.

The other variables that are not in the above have a zero value.

Table 7 shows the comparison of the results of the objective values of $\overline{Z}_1$ and $\overline{Z}_2$ of the present example with the results obtained by P. Kundu et al. [7]. It is shown that the optimal solution of the proposed problem gives better results by using fuzzy Goal programming approach while compared to Fuzzy linear programming and Global criterion method.

Table. 7: Comparisons of optimal solutions

	P. Kundu et al. [7]		Presented Method
	Fuzzy linear programming	Global criterion method	Goal programming approach
$\overline{Z}_1$	1139.536	1144.894	1110.183
$\overline{Z}_2$	837.4808	832.1250	814.396

Example 3: (Yinzhen Li et al. [12]).
Let us consider a multi-objective solid transportation problem with mixed constraints.

$$\text{Minimize } Z_r = \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3} c_{ijk}^r x_{ijk}, \qquad r = 1, 2, 3.$$

$$s.t. \sum_{j=1}^{3}\sum_{k=1}^{3} x_{ijk} = 8, \qquad \sum_{j=1}^{3}\sum_{k=1}^{3} x_{2jk} \ge 9, \qquad \sum_{j=1}^{3}\sum_{k=1}^{3} x_{3jk} \le 5,$$

$$\sum_{i=1}^{3}\sum_{k=1}^{3} x_{i1k} = 7, \qquad \sum_{i=1}^{3}\sum_{k=1}^{3} x_{i2k} \ge 6, \qquad \sum_{i=1}^{3}\sum_{k=1}^{3} x_{i3k} \le 5,$$

$$\sum_{i=1}^{3}\sum_{j=1}^{3} x_{ij1} = 10, \qquad \sum_{i=1}^{3}\sum_{j=1}^{3} x_{ij2} \ge 5, \qquad \sum_{i=1}^{3}\sum_{j=1}^{3} x_{ij3} \le 6,$$

$x_{ijk} \ge 0, \quad \forall i, j, k$.

Table 8: Data for Three objective functions

	j	1			2			3		
	k i	1	2	3	1	2	3	1	2	3
C^1	1	(8, 9, 10)	(10,12,14)	(7, 9, 11)	(3, 6, 9)	(8, 9, 10)	(5, 7, 9)	(2, 3, 4)	(6, 7, 8)	(5, 7, 9)
	2	(4, 5, 6)	(5, 6, 7)	(3, 5, 7)	(7,9, 11)	(8, 11, 14)	(1, 3, 5)	(5, 6, 7)	(6, 8, 10)	(5, 6, 7)
	3	(1, 2, 3)	(1, 2, 3)	(1, 1, 1)	(1, 2, 3)	(6, 7, 8)	(6, 7, 8)	(1, 1, 1)	(8, 9, 10)	(1, 3, 5)
C^2	1	(1, 2, 3)	(8, 9, 10)	(6, 8, 10)	(1, 1, 1)	(2, 4, 6)	(1, 1, 1)	(7, 9, 11)	(7, 9, 11)	(4, 5, 6)
	2	(1, 2, 3)	(7, 8, 9)	(1, 1, 1)	(3, 4, 5)	(3, 5, 7)	(1, 2, 3)	(6, 8, 10)	(5, 6, 7)	(8, 9, 10)
	3	(4, 5, 6)	(1, 2, 3)	(5, 7, 9)	(6, 8, 10)	(8, 9, 10)	(6, 7, 8)	(3, 5, 7)	(1, 2, 3)	(3, 5, 7)
C^3	1	(1, 2, 3)	(2, 4, 6)	(5, 6, 7)	(2, 3, 4)	(4, 6, 8)	(3, 4, 5)	(6, 8, 10)	(2, 4, 6)	(7, 9, 11)
	2	(1, 2, 3)	(3, 5, 7)	(1, 3, 5)	(3, 5, 7)	(4, 6, 8)	(4, 6, 8)	(7, 9, 11)	(4, 6, 8)	(1, 3, 5)
	3	(1, 1, 1)	(8, 9, 10)	(1, 1, 1)	(7, 8, 9)	(2, 3, 4)	(7, 9, 11)	(3, 5, 7)	(5, 7, 9)	(10, 11, 12)

By using the proposed method, we get the following optimal compromise solution as $\lambda = 0.3322039$, x_{121} = 7.17, x_{122} = 0.8295, x_{211} = 2.82, x_{212} = 2.77, x_{223} = 3.39, x_{312} = 1.39136, d_1^- = 0.3322, d_2^- = 0.3322, d_3^- = 0.3322 $\overline{Z}_1$ = 94.2678, $\overline{Z}_2$ = 47.9457 and $\overline{Z}_3$ = 78.91.
The other variables that are not in the above have a zero value.

From the above example, it is observed that the objective values $\overline{Z}_1$, $\overline{Z}_2$ and $\overline{Z}_3$ are in good agreement with that of the results obtained by Yinzhen Li et al. [12].

VII. Conclusion

In this paper, we presents a fuzzy Goal programming approach for solving MOFSTP with fuzzy constraints (i.e. Sources, demands and conveyance capacities are fuzzy). In order to solve the model conveniently, we discussed the crisp model with corresponding defuzzified values under the conditions (2) and the expected value models in objective functions for triangular and trapezoidal membership functions. Then multi-objective problems are solved by the fuzzy goal programming approach and three numerical examples are given to illustrate the proposed model. These numerical examples give better optimal results with this approach when results are compared.

On the whole, the proposed fuzzy goal programming approach is more efficient method for the MOFSTP.

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