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New Operators on Ford-Fulkerson Algorithm

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Abstract: *The maximum flow in network involves sending the maximum amount of material from a specified source vertex s to another specified sink vertex t, subject to capacity restrictions on the amount of material that can flow along each path.A closely related we formed a new concept of α_i -closure space and α_i -interior space using subgraphs generated by Ford- Fulkerson algorithm steps, also we induced some new topological properties .*
Keywords: *Graph theory, Rough set, Topology, Fuzzy set and Data mining.*

I. Introduction

Topology originated a branch of mathematics played an important role in the development of the general theory. The concept of topology can be helpful in understanding phenomena in modern materials science, complex system science, networks of life, economics, relativity and quantum theory, which represent 20th century physics and information science from a broader perspective.

Graph theory is a widely applied frame work in information, technology, geography, and computer science. It is primarily Concerned with maximally efficient flow or connectivity in networks(Grossand Yellen,1999). There are a number of interesting theorems, relative to capacitated networks, that give necessary and sufficient conditions for existence of flows satisfying some conditions for various kinds. Typical of these are the supply-demand theorem due to a condition for the existence of a flow satisfying demands at certain nodes from supplies at other nodes ,which states a condition for the existence of circulatory flow in a network in which each path has upper flow values and lower flow values

In this paper we add some new results along this lines which described asfollows. We fist establish a new definitions for closure space and interior space concept on a network with certain of the nodes designated as sources ,other as sinks, and assume that each source is required to send, and each sink to receive , an amount that lies between this prescribed bounds. The new results asserts that if there is a flow that sends out of each source an amount at least as great as the lower bound for the source,and into each sink no mor the upper bound for the sink, and if there is a flow that sends out of each source no more than the upper bound for the source, and into each sink at least as much as the lower bound for the sink, then there is a flow that meets all the requirements simultaneously.

A flow network is a directed graph where each edge has a capacity and each edge receives a flow. A network can be used to model traffic in a road system, fluids in pipes, current in an electrical circuit, web service networks [1,2,8], or anything similar in which something travel through network of nodes, an example from the medical field: it is known that the topology of the networks produced by cancer or other tissue anomalies is different from that of normal tissue.The basic concepts of the networks are presented. Our purpose is explaining Ford-Fulkerson algorithm and evaluate the maximal flow in the networks.

Also, we give new definitions of α_i −closure and α_i − interior operator in the networks flow using the concept of classical closure operator and topological structures generated from from the steps of maximum flow algorithm on a connected graphs, of network.

1.1 Basic Concepts

A network N is a digraph with two distinguished subsets of vertices X and Y, the vertices in X are the source of N and those in Y are the sinks of N . Vertices which neither source nor sink are called intermediate vertices denoted by I .The function C is the capacity function of N and its value on an arc a is the capacity of a [8].A source is a vertex(s) of a directed graph with indegree 0. A Sink is a vertex (t) of a directed graph with out degree 0.A Capacity is the maximum flow that may be sent through an edge or a vertex.A flow in network N is an integer valued function f defined on its arc set A such that :

1- The rate of flow along an arc cannot exceed the capacity of the arc $0 \le f(a) \le c(a)$ for all $a \in A$

2-The rate at which material is transported in to v is equal to the rate at which it is transported out of v
$f^{-}(v) = f^{+}(v) \quad \text{for all } v \in I$.

A flow augmenting path in a network with a given flow f_{ij} on each edge e_{ij} is a path from s to t such that :
- No flow in forward edge is equal to capacity thus $f_{ij} < c_{ij}$.
-No backward edge has flow 0 thus $f_{ij} > 0$. Let N be a network with a single source x and single sink y, a cut in N is a set of arcs of the form $(S,\overline{S})$, where, $x \in S$, $y \in \overline{S}$ [16].

Example 1.1.1

In Fig. (4.2) a network flow N,where the first number is the capacity and the second is a given flow

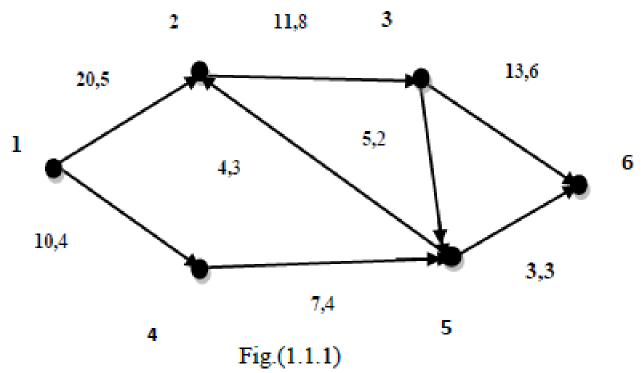
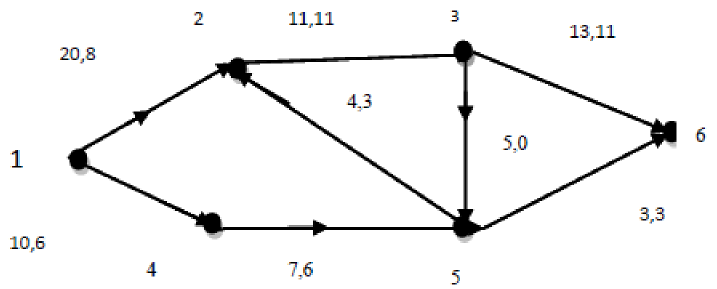


Fig.(1.1.1)

The only augmenting paths are $p_1 : 1 \to 2 \to 3 \to 6$, $p_2 : 1 \to 4 \to 5 \to 3 \to 6$ .If there's a flow augmenting path, we can use it to push through it an additional flow as the following:First compute Δ_{ij} such as $\Delta_{ij} = c_{ij} - f_{ij}$ for forward edge, $\Delta_{ij} = f_{ij}$ for backward edge and $\Delta = \min \Delta_{ij}$. And add Δ to each flow in this path . In above network $\Delta = 3$ in $p_1, \Delta = 2$ in p_2

And the maximum flow is shown in Fig. (1.1.2)



Fig(1.1.2)

We notice that network flow with a cut set (S,T)has capacity of cut is the sum of the capacities of all forward edge in (S,T) cap(S,T)=11+7=18

Any given flow in a network N is the net flow through any cut set (S,T) of N.

A flow f in a network N cannot exceed the capacity of any cut set (S,T)in N.A flow from S to T in a network N is maximum if and only if there doesn't exist a flow augmenting path from S to T in N.Thus a flow is maximum when doesn't exist a flow augmenting path[16].

Ford-Fulkerson Algorithm

A network flow inspired computer network applications which considered a point-to-point communication network on which a number of information sources be multicast to certain sets of destinations. We present an algorithm for determining a maximum flow in a network [1, 8]. This algorithm starting from the initial flow and it recursively constructs a sequence of flow of increasing value and terminates with a maximum flow.

1. Assign an initial flow f_{ij}
2. Label s by ∅,mark the other vertices " un labeled"

3. Find a label vertex i that has not yet been scanned. Scan ias the follows: for every unlabeled adjacent vertex j if $c_{ij} > f_{ij}$.
4. Compute $\Delta_{ij} = c_{ij} - f_{ij}$, $\Delta_{ij} = 1$ if i=1 and $\Delta_{ij} = \min(\Delta_i, \Delta_{ij})$ if $i > 1$.
5. Label j with "forward label"(i^+, Δ_j)or if $f_{ij} > 0$
6. Compute $\Delta_j = \min(\Delta_i, f_{ji})$ and label j by a "backward label"$(i^-, \Delta_j$. If no such j by a "backward label"$(i^-, \Delta_j$ if no such j exist then output f is the maximum flow (stop), else continue (go to step4)
7. Repeat 3 until t is reached. (This gives a flow augmenting path p) if it is possible to reach t then output f is the maximum flow (stop) else continue (go to step 5).
8. Back track the path using the labels.
9. Increase the existing flow by Δ_i as $f = f + \Delta_i$
10. Remove all labels from vertices 2,…..,n and go to step 3. End Ford- Fulkerson.

Example 1.1.2: In Fig. (4.5) we compute the maximum flow by applying Ford- Fulkerson algorithm.

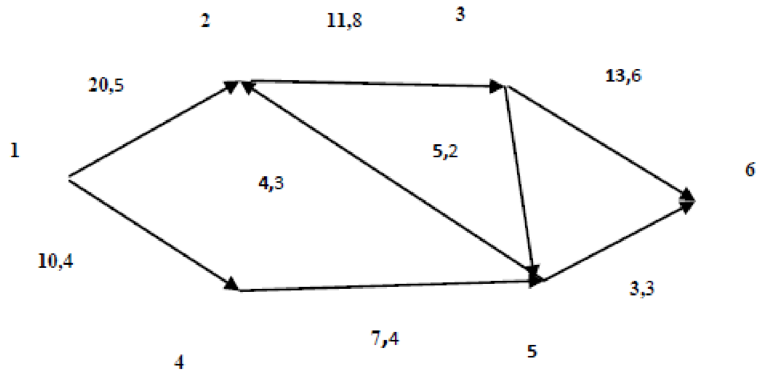


Fig. (1.1.3)

Let s(1) by∅ ,mark {2,3,4,5,6}(un labeled) and initial flow f=9 .
Step1 Compute $\Delta_{12} = 20 - 5 = \Delta_2$ label 2 by (1+,15) , $\Delta_{14} = 10 - 4 = \Delta_4$ label 4 by (1+,6).Compute $\Delta_{23} = 11 - 8, \Delta_3 = \min(\Delta_2, 3) = 3$ label 3 by (2+,3) .Compute $\Delta_5 = \min(\Delta_2, 3) = 3$,label 5 by (2-,3).

Step 2 Compute $\Delta_{36} = 13 - 6, \Delta_6 = \min(\Delta_3, 7) = 3$ label 6 by (3+,3) .
$P: s = 1 \to 2 \to 3 \to 6 = t$ is a flow augmenting path
Step 3 Put $\Delta = 3$ the flow is $f_{12} = 8$, $f_{23} = 11$, $f_{36} = 9$ and other f_{ij} unchanged

Step 4 Remove labels on vertices 2,.., 6 and go to step1
Step 5 Compute $\Delta_{12} = 20 - 8 = \Delta_2$ label 2 by (1+,12) , $\Delta_{14} = 10 - 4 = \Delta_4$ label 4 by (1+,6).Compute $\Delta_{14} = 10 - 4, 4$ label 4 by (1+,6) .Compute $\Delta_5 = \min(\Delta_2, 3) = 3$,label 5 by (2-,3). $\Delta_3 = \min(\Delta_5, 2) = 2$ label 3 by (5-,2).$\Delta_{36} = 13 - 9, \Delta_6 = \min(\Delta_3, 4) = 2$ label 6 by (3+,2).
Step 6 $P: s = 1 \to 2 \to 5 \to 3 \to 6 = t$ is a flow augmenting path
$\Delta_i = 2$ and the flow is $f_{12} = 10$, $f_{52} = 1$ $f_{35} = 0$ $f_{36} = 11$
and other f_{ij} unchanged and the flow is maximum.

II. New Operators On Ford-Fulkerson Algorithm Steps

In several areas of Computer Science, one is interested in using abstract mathematical structures as a basis for modelling certain phenomena of the real world. This interest is particularly strong in the knowledge representation subfield of artificial intelligence so, it must putting some conditions on closure operator to be compatibles with its application.

In the following Hi denotes the subgraph Hi=(V(Hi), E(Hi)) which is represent the subgraph in step i for Ford-Fulkerson algorithm of a graph G=(V(G), E(G)), He denotes the subgraph He=(V(He), E(He)) which is represent the subgraph in end step for Ford-Fulkerson Algorithm, and C^i will be used for a family of subgrahs of the power set of P(V(Hi)) obtained by closure operators $cl_{\alpha_i}(K)$where $K \subseteq H_i$.

Definition 2.1

Let N=(G,s,t,c) be a network with capacity c_{ij} ,s start vertex , t terminal vertex and flow f_{ij} , α_i –neighborhoods can defined as $N_{v_i}^{\alpha_i} = \{v_j : v_j \in V, e_{ij} \in E , \frac{f_{ij}}{c_{ij}} > \alpha_i\}$ and α_i -closure operators , $cl_{\alpha_i}^1(K) = \{v_j : v_j \in H_i , N_{v_i}^{\alpha_i} \cap V(K) \neq \emptyset\} \subseteq P(V(H_i))$ for $i, j, k = 1, \dots, n , -1 < \alpha_i \leq 1$ defined on a family C^i of subgrahs from G=(V,E) and satisfies axioms.

(1) $cl_{\alpha_i}^1(\emptyset) = \emptyset$ and $cl_{\alpha_i}^1(V(G)) = V(G)$ (2) $H_i \subseteq H_j \subseteq G$ implies $cl_{\alpha_i}^1(H_i) \subseteq cl_{\alpha_i}^1(H_j)$

(3) $cl_{\alpha_i}^1(H_i \cup H_j) = cl_{\alpha_i}^1(H_i) \cup cl_{\alpha_i}^1(H_j)$, for all $H_i , H_j \subseteq G$

The pair (V(G), $cl_{\alpha_i}^{ij}$) is α_i -closure space this notations is close to the classical notation of topology. , α_i -closure space differs from topologies in that they do not require the open subgraphs to be stable with respect to finite intersection. Furthermore, there exists the maximal open subgraph denoted as max(C_j^i) in C_j^i : j = 1, 2.The α_i −interior operator (Dual operator) can defined by $int_{\alpha_i}^{ij}(K) = V(G) - (cl_{\alpha_i}^{ij}((V(G) - K))$ which satisfy axioms: (1) $int_{\alpha_i}^{ij}(\emptyset) = \emptyset$ and $int_{\alpha_i}^{ij}(V(G)) = V(G)$

(2) $H_i \subseteq H_j \subseteq G$ implies $int_{\alpha_i}^{ij}(H_i) \supseteq int_{\alpha_i}^{ij}(H_j)$

(3) $int_{\alpha_i}^{ij}(H_i \cap H_j) = int_{\alpha_i}^{ij}(H_i) \cap int_{\alpha_i}^{ij}(H_j)$, for all $H_i , H_j \subseteq G$

The pair (V(G), $int_{\alpha_i}^{ij}$) is α_i -interior space.

Definition 2.2

Let N=(G,s,t,c) be a network with capacity c_{ij} ,s start vertex , t terminal vertex and flow f_{ij}, a (V(G), $cl_{\alpha_i}^{i1}$)on a subgraph H_i of Ford-Fulkerson algorithm in step i of a graph G is a family where $C_1^i = \{cl_{\alpha_i}^{1i}(K) : K \subseteq H_i\} \subseteq P(V(H_i))$ together V(G). The elements of $C_1^i$are called 1-closed subgraphs of α_i -closure space in step i (V(G), $cl_{\alpha_i}^{i1}$).

Definition 2.3

Let N=(G,s,t,c) be a network with capacity c_{ij} ,s start vertex , t terminal vertex and flow f_{ij}, a (V(G), $cl_{\alpha_i}^{i2}$)on a subgraph Hi of Ford-Fulkerson algorithm in step i of a graph G is a family

$C_2^i = \{cl_{\alpha_i}^{i2}(K) : K \subseteq Hi\} \subseteq P(V(Hi))$ where

$cls_{\alpha_i}^{i2}(K) = V(K) \cup \{v_j : v_j \in H_i , N_{v_i}^{\alpha_i} \cap V(K) \neq \emptyset\} \subseteq P(V(H_i))$

together V(G). The elements of (V(G), $cl_{\alpha_i}^{i2}$) are called α_i-closed subgraphs of the (V(G), $cls_{\alpha_i}^{i2}$)in step i.

The complement of jα_i-open subgraph is called jα_i-closed subgraph. The jα_i-interior of subgraph K is $int_{\alpha_i}^{1i}(K)$ = ∪ {V(O) : O is j-open subgraph, V(O) ⊆ V(K)}, and the jα_i-closure of subgraph K is $cl_{\alpha_i}^{1i}(K)$ = ∩ {V(F) : F is jα_i-closed subgraph,
V(K) ⊆ V(F)} where j = 1, 2.

Example 2.1 :

Given a network flow N=(G,a,d,c)

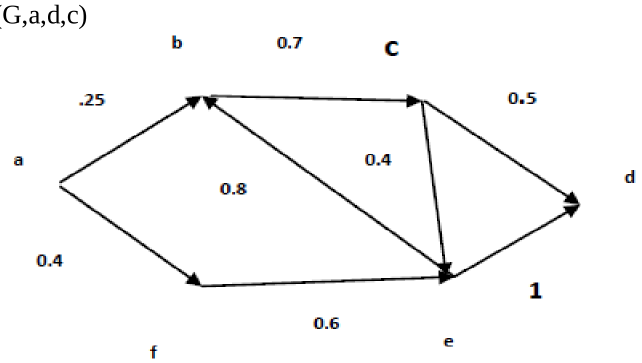


Fig.(2.1)

Here we find a weighted graph, of maxinum weight connecting two specified vertices a and d, evaluate α_i – closure spaces (V, $cl_{\alpha_i}^{ij}$) and α_i – interior spaces (V, $int_{\alpha_i}^{ij}$)corresponding to each step for Ford-Fulkerson algorithm.

Step 1

v_i	a	b	f	c	e	d	$N^0_{v_i}$	$N^{0.5}_{v_i}$	$N^{0.8}_{v_i}$
φ	--	--	--	--	--	--	φ	∅	∅
a	--	0.25	0.4	∞	∞	∞	{b,f}	∅	∅
b	-0.25	--	∞	0.7	-0.8	∞	{ c}	{ c}	∅
f	-0.4	∞	--	∞	0.6	∞	{ e}	{e}	∅
c	∞	-0.7	∞	--	0.4	0.5	{e, d}	{d}	∅
e	∞	-0.8	-0.6	-0.4	--	1	{d}	{d}	{d}
d	∞	∞	∞	-0.5	-1	--	∅	∅	∅

Table (2.1)

For $\alpha_i = 0$

If V(H0)= {a} ,then (V, $cl^{01}_{\alpha_i}$)= {V(G), ∅, {b,f }}and
(V(G), $int^{01}_{\alpha_i}$)= {V(G), ∅, {a,c,e,d }}

If V(H1)= {a,b} , then (V, $cl^{11}_{\alpha_i}$)= {V(G), ∅, {c},{b,f},{b,f, c}}
(V(G), $int^{21}_{\alpha_i}$)={V(G), ∅, {a,b,f,e,d},{a,c,e,d},{a,e,d}}

If V(H_2)= {a, b, f}, then
(V(G), $cl^{21}_{\alpha_i}$)= {V(G), ∅, {c},{e},{b,f},{b,f, c},{c,e},{b,f,e},{b,f,c,e}
(V(G), $int^{21}_{\alpha_i}$)= {V(G), ∅, {a,b,f,e,d },{a,b,c,f,d},{a,c,e,d},{a,e,d},{a,b, f,d},{a,c,d},{a,d}}

If V(H_3)= {a, b, f,c}, then
(V, $cl^{31}_{\alpha_i}$) = {V(G), ∅,{c},{e},{b,f},{e,d},{b,f,c},{c,e},{b,f,e},{b,f,c,e}{e,c,d},
{b,f,e,d},{b,f,c,e,d}
(V(G), $int^{31}_{\alpha_i}$)= {V(G), ∅, {a,b,f,e,d },{a,b,c,f,d},{a,c,e,d},{a,e,d},
{a,b, f,d},{a,c,d},{a,d},{a,b,f,c},{a,b,f},{a,c},{a}}

If V(H_4) = {a, b, f,c,e}, then
(V(G), $cl^{41}_{\alpha_i}$)= {V(G), ∅,{c},{e},{d},{b,f},{e,d},{b,f,c},{c,e},
{b,f,e},{b,f,c,e}{e,c,d}, {b,f,e,d},{b,f,c,e,d},{c,d},{b,f,d},{b,f,c,d},{c,e,d}}
(V(G), $int^{41}_{\alpha_i}$)= {V(G), ∅, {a,b,f,e,d },{a,b,c,f,d},{a,c,e,d},{a,e,d},
{a,b, f,d},{a,c,d},{a,d},{a,b,f,c},{a,b,f},{a,c},{a},{a,b,f,e},{a,c,e},{a,e},{a,b,f}}

If V(H_5)= {a, b, f,c,e,d}, then
(V, $cl^{51}_{\alpha_i}$)= {V(G), ∅,{c},{e},{d},{b,f},{e,d},{b,f,c},{c,e},{b,f,e},{b,f,c,e}{e,c,d},
{b,f,e,d},{b,f,c,e,d},{c,d},{b,f,d},{b,f,c,d},{c,e,d}}
(V(G), $int^{51}_{\alpha_i}$)= {V(G), ∅, {a,b,f,e,d },{a,b,c,f,d},{a,c,e,d},{a,e,d},
{a,b, f,d},{a,c,d},{a,d},{a,b,f,c},{a,b,f},{a,c},{a},{a,b,f,e},{a,c,e},{a,e},{a,b,f}}

For $\alpha_i = 0.5$

If V(H_0) = {a} , then (V, $cl^{01}_{\alpha_i}$)= {V(G), ∅ }
(V(G), $int^{01}_{\alpha_i}$)= {V(G), ∅}

If V(H_1) = {a, b} , then (V, $cl^{11}_{\alpha_i}$)= {V(G), ∅, { c }}
(V(G), $int^{11}_{\alpha_i}$)= {V(G), ∅,{a,b,f,e,d}

If V(H_2) = {a, b, f} , then (V, $cl^{21}_{\alpha_i}$)= {V(G), ∅, { c },{e},{c,e}}
(V(G), $int^{21}_{\alpha_i}$)= {V(G), ∅,{a,b,f,e,d},{a,b,f,c,d},{a,b,f,d}}

If V(H_3) = {a, b, f,c} ,then
(V, $cl^{31}_{\alpha_i}$)= {V(G), ∅, { c },{e},{d},{c,e},{c,d},{e,d},{c,e,d}}
(V(G), $int^{31}_{\alpha_i}$)= {V(G), ∅,{a,b,f,e,d},{a,b,f,c,d},{a,b,f,d},{a,b,f,c,e},{a,b,f,e},
{a,b,f,c },{a,b,f}}

If H4 = {a, b, f,c,e} , then
(V(G), $cl^{41}_{\alpha_i}$)= {V(G), ∅, { c },{e},{d},{c,e},{c,d},{e,d},{c,e,d}}
(V(G), $int^{41}_{\alpha_i}$)= {V(G), ∅,{a,b,f,e,d},{a,b,f,c,d},{a,b,f,d},{a,b,f,c,e},{a,b,f,e},
{a,b,f,c },{a,b,f}}

If V(H5)= {a, b, f,c,e,d} , then
(V(G), $cl^{51}_{\alpha_i}$)= {V(G), ∅, { c },{e},{d},{c,e},{c,d},{e,d},{c,e,d}}
(V(G), $int^{51}_{\alpha_i}$)= {V(G), ∅,{a,b,f,e,d},{a,b,f,c,d},{a,b,f,d},{a,b,f,c,e},{a,b,f,e},
{a,b,f,c },{a,b,f}}

For $\alpha_i = 0.8$

If $H_0 = \{a\}$, then $(V(G), cl^{01}_{\alpha_i}) = (V(G), int^{01}_{\alpha_i}) = \{V(G), \emptyset\}$

If $H_1 = \{a, b\}$, then $(V(G), cl^{11}_{\alpha_i}) = \{(V(G), int^{11}_{\alpha_i}) = V(G), \emptyset\}$

If $H_2 = \{a, b, f\}$, then $(V(G), cl^{21}_{\alpha_i}) = (V(G), int^{21}_{\alpha_i}) = \{V(G), \emptyset\}$

If $H_3 = \{a, b, f,c\}$,then $(V(G), cl^{31}_{\alpha_i}) = (V(G), int^{31}_{\alpha_i}) = \{V(G), \emptyset\}$

If $H_4 = \{a, b, f,c,e\}$, then $(V(G), cl^{41}_{\alpha_i}) = \{V(G), \emptyset, \{d\}\}$
$(V(G), int^{41}_{\alpha_i}) = \{V(G), \emptyset, \{a,b,f,c,e\}\}$

If $H_5 = \{a, b, f,c,e,d\}$, then $(V(G), cl^{51}_{\alpha_i}) = \{V(G), \emptyset, \{d\}\}$
$(V(G), int^{41}_{\alpha_i}) = \{V(G), \emptyset, \{a,b,f,c,e\}\}$

Step 2: Add 3 to augumented path and obtain α_i – closure spaces $(V, cl^{i2}_{\alpha_i})$

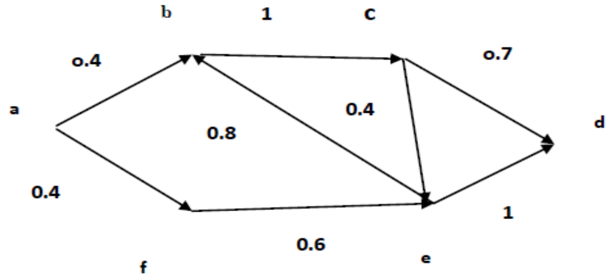


Fig.(4.7)

v_i	a	b	f	c	e	d	$N^0_{v_i}$	$N^{0.5}_{v_i}$	$N^{0.8}_{v_i}$
φ	--	--	--	--	--	--	∅	∅	∅
a	--	0.4	0.4	∞	∞	∞	{b,f}	∅	∅
b	-0.4	--	∞	1	-0.8	∞	{ c}	{ c}	{c}
f	-0.4	∞	--	∞	0.6	∞	{ e}	{e}	∅
c	∞	-1	∞	--	0.4	0.7	{e, d}	{d}	∅
e	∞	-0.8	-0.6	-0.4	--	1	{d}	{d}	{d}
d	∞	∞	∞	-0.7	-1	--	∅	∅	∅

Table (4.1)

We notice that $(V, cl^{i2}_{\alpha_i})$in step 2 are the same as $(V, cl^{i1}_{\alpha_i})$, $i = 1, \ldots ,5$ in step 1 for the same values of α_i except for $\alpha_k = 0.8$ the values of $(V, cl^{i2}_{\alpha_i})$ are

If $H_0 = \{a\}$, then $(V, cl^{02}_{\alpha_i}) = (V(G), int^{02}_{\alpha_i}) = \{V(G), \emptyset\}$

If $H_1 = \{a, b\}$, then $(V, cl^{12}_{\alpha_i}) = \{V(G), \emptyset ,\{c\}\}$
$(V(G), int^{12}_{\alpha_i}) = \{V(G), \emptyset ,\{a,b,f,e,d\}\}$

If $H_2 = \{a, b, f\}$, then $(V, cl^{22}_{\alpha_i}) = \{V(G), \emptyset ,\{c\}\}$
$(V(G), int^{22}_{\alpha_i}) = \{V(G), \emptyset ,\{a,b,f,e,d\}\}$

If $H_3 = \{a, b, f,c\}$,then $(V, cl^{32}_{\alpha_i}) = \{V(G), \emptyset ,\{c\}\}$
$(V(G), int^{32}_{\alpha_i}) = \{V(G), \emptyset ,\{a,b,f,e,d\}\}$

If $H_4 = \{a, b, f,c,e\}$, then $(V, cl^{42}_{\alpha_i}) = \{V(G), \emptyset,\{c\},\{d\},\{c,d\}\}$
$(V(G), int^{42}_{\alpha_i}) = \{V(G), \emptyset ,\{a,b,f,e,d\},\{a,b,c,f,e\},\{a,b,f,e\}\}$

If $H_5 = \{a, b, f,c,e,d\}$, then $(V, cl^{52}_{\alpha_i}) = \{V(G), \emptyset, \{c\},\{d\},\{c,d\}\}$
$(V(G), int^{52}_{\alpha_i}) = \{V(G), \emptyset ,\{a,b,f,e,d\},\{a,b,c,f,e\},\{a,b,f,e\}\}$

In similar manures we can add the vertices of Hi of every step to calculate $(V(G), cls^{i2}_{\alpha_i})$in every step i.

Step 3

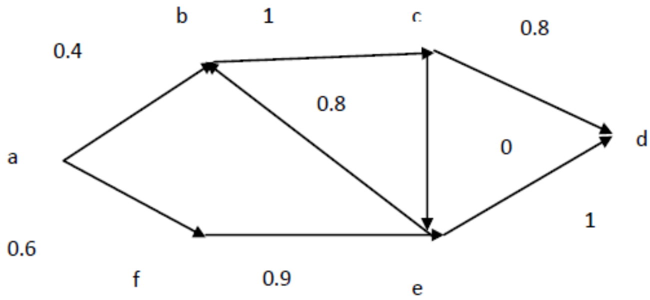


Fig.(2.2)

v_i	a	b	f	c	e	d	$N^0_{v_i}$	$N^{0.5}_{v_i}$	$N^{0.8}_{v_i}$
φ	--	--	--	--	--	--	∅	∅	∅
a	--	0.4	0.6	∞	∞	∞	{b,f}	{f}	∅
b	-0.4	--	∞	1	-0.8	∞	{ c}	{ c}	∅
f	-0.6	∞	--	∞	0.9	∞	{ e}	{e}	∅
c	∞	-1	∞	--	0	0.8	{d}	{d}	∅
e	∞	-0.8	-0.9	0	--	1	{d}	{d}	{d}
d	∞	∞	∞	-0.8	-1	--	∅	∅	∅

Table(2.2)

For $\alpha_i = 0$

If H_0 = {a} , then (V, $cl^{03}_{\alpha_i}$)= {V(G), φ, {b,f }}
(V(G), $int^{01}_{\alpha_i}$)= {V(G), ∅, {a,c,e,d }}

If H_1 = {a, b} , then (V, $cl^{13}_{\alpha_i}$)= {V(G), ∅, {c},{b,f},{b,f, c}}
(V(G), $int^{21}_{\alpha_i}$)={V(G), ∅, {a,b,f,e,d},{a,c,e,d},{a,e,d}}

If H_2 = {a, b,f}, then
(V, $cl^{23}_{\alpha_i}$)= {V(G), ∅, {c},{e},{b,f},{b,f, c},{c,e},{b,f,e},{b,f,c,e}}
(V(G), $int^{21}_{\alpha_i}$)= {V(G), ∅, {a,b,f,e,d },{a,b,c,f,d},{a,c,e,d},{a,e,d},
{a,b, f,d},{a,c,d},{a,d}}

If H_3 = {a, b, f,c}, then
(V, $cl^{31}_{\alpha_i}$)= {V(G), ∅,{c},{e},{d},{b,f},{b,f,c},{c,e},{c,d},{e,d},{b,f,e},{b,f,c,e},
{b,f,d},{b,f,c,d},{b,f,e,d},{c,e,d},{b,f,c,e}}
(V(G),$int^{31}_{\alpha_i}$)={V,∅,{a,b,f,e,d},{a,b,c,f,d},{a,c,e,d},{a,e,d},{a,b,f,d},{a,c,d},{a,d},
{a,b,f,c},{a,b,f},{a,c},{a},{a,b,f,e},{a,c,e},{a,e},{a,b,f},{a,b,f,c}}

If H_4 = {a, b, f,c,e}, then
(V, $cl^{43}_{\alpha_i}$)= {V(G), ∅,{c},{e},{d},{b,f},{b,f,c},{c,e},{c,d},{e,d},{b,f,e},{b,f,c,e},
{b,f,d},{b,f,c,d},{b,f,e,d},{c,e,d},{b,f,c,e}}
(V(G), $int^{41}_{\alpha_i}$)= {V(G), ∅, {a,b,f,e,d },{a,b,c,f,d},{a,c,e,d},{a,e,d},
{a,b, f,d},{a,c,d},{a,d},{a,b,f,c},{a,b,f},{a,c},{a},{a,b,f,e},{a,c,e},{a,e},{a,b,f}}

If H_5 = {a, b, f,c,e,d}, then
(V, $cl^{53}_{\alpha_i}$)= {V(G), ∅,{c},{e},{d},{b,f},{b,f,c},{c,e},{c,d},{e,d},{b,f,e},{b,f,c,e},
{b,f,d},{b,f,c,d},{b,f,e,d},{c,e,d},{b,f,c,e}}
(V(G), $int^{51}_{\alpha_i}$)= {V(G), ∅, {a,b,f,e,d },{a,b,c,f,d},{a,c,e,d},{a,e,d},
{a,b, f,d},{a,c,d},{a,d},{a,b,f,c},{a,b,f},{a,c},{a},{a,b,f,e},{a,c,e},{a,e},{a,b,f}}

For $\alpha_i = 0.5$

If H_0 = {a} , then (V, $cl^{03}_{\alpha_i}$)= {V(G), ∅, {f}}

If H_1 = {a, b} , then (V, $cl^{13}_{\alpha_i}$)= {V(G), ∅, { c },{f},{c,f}}

If H_2 = {a, b, f} , then
(V, $cl^{23}_{\alpha_i}$)= {V(G), ∅, { c },{f},{e},{c,f},{f,e},{c,e},{c,f,e}}

If H_3 = {a, b, f,c} ,then
(V, $cl^{33}_{\alpha_i}$)= {V(G), ∅, { c },{f},{e},{d},{c,d},{f,d},{e,d},{d,c,f},{d,f,e},{d,c,e},
{d,c,f,e}{c,f},{f,e},{c,e},{c,f,e}}

If H_4 = {a, b, f,c,e} , then
(V, $cl^{43}_{\alpha_i}$)= {V(G), ∅, { c },{f},{e},{d},{c,d},{f,d},{e,d},{d,c,f},{d,f,e},{d,c,e},
{d,c,f,e}{c,f},{f,e},{c,e},{c,f,e}}

If H_5 = {a, b, f,c,e,d} , then
(V, $cl^{53}_{\alpha_i}$)= {V(G), ∅, { c },{f},{e},{d},{c,d},{f,d},{e,d},{d,c,f},{d,f,e},{d,c,e},
{d,c,f,e}{c,f},{f,e},{c,e},{c,f,e}}

Thus we arrived to the maximum flow

In above example we notice that $(V, cl^{ki}_{\alpha_i}) \subseteq (V, cl^{(k+1)i}_{\alpha_i})$ for k=0,…,5 and i=1,2,3 (i.e $(V, cl^{(k+1)i}_{\alpha_i})$ is finar than $(V, cl^{ki}_{\alpha_i})$). From abve we must generalized the inclusion relation to formulate topological properties which appeared in Ford-Fulkerson algorithm .

Definition 2.4

Let $(V, cl_{\alpha_i}^{ki})$is called α_i-subclosure space of a $(V, cl_{\beta_i}^{ki})$if for each α_i-closed subgraph F1 $\in (V, cl_{\alpha_i}^{ki})$, there exists each α_i-closed subgraph F $\in (V, cl_{\beta_i}^{ki})$such that $F1 \subseteq F$ and denoted by $cl_{\alpha_i}^{ki} \subseteq s\, cl_{\beta_i}^{ki}$.

Proposition 2.1

Let N=(G,s,t,c) be a network with capacity c_{ij} ,s start vertex , t terminal vertex and flow f_{ij}, and Hi=(V(Hi), E(Hi)), Hi+1=(V(Hi+1), E(Hi+1)) are subgraphs in G represent the steps i, i+1in Ford-Fulkerson algorithm such that

Hi $\subseteq$ Hi+1, then $cl_{\alpha_i}^{ki}) \subseteq cl_{\alpha_i}^{(k+1)i}$.

Proof.

In a network the neighborhood of x_i is the points that achieves $\frac{f_{ij}}{c_{ij}} > \alpha_i$ and at the first step the ratio $\frac{f_i}{c_i}$ is increase in some points thus$N_{v_i}^{\alpha_i} \subseteq N_{v_i}^{\alpha_j}$ for every $\alpha_j \leq \alpha_i$ Where $N_{v_i}^{\alpha_0}$ is the neighborhood of x_i at step o(before applying the algorithm) and $N_{v_i}^{\alpha_1}$ is the neighborhood of x_i at Step 1Similarly, the next steps $N_{v_i}^{\alpha_o} \subseteq N_{v_i}^{\alpha_1} \subseteq \dots \subseteq N_{v_i}^{\alpha_n}$ so $V, cl_{\alpha_i}^{ki}) \subseteq (V, cl_{\alpha_i}^{(k+1)i})$.

Proposition 2.2

The minimal closed set in any network flow is the sink vertex and it always a singleton.
Proof .From the definition network and α_i-neighborhood .

Proposition 2.3

Let N=(G,s,t,c) be a network with capacity c_{ij} ,s start vertex , t terminal vertex and flow f_{ij}, and Hi=(V(Hi), E(Hi)) subgraph in G represent the steps i, in Ford-Fulkerson algorithm , then the following satisfies:

(a) $int_{\alpha_i}^{ij}(V(H_i)) \subseteq V(H_i) \subseteq cl_{\alpha_i}^{ij}(V(H_i))$ for any subgraph H_i

(b) $cl_{\alpha_i}^{ij}(V(H_i)) = V(H_i)$for any induce subgraph H_i

Proof

(a)If $v_i \in int_{\alpha_i}^{ij}(V(H_i))$ then there exist $N_{v_i}^{\alpha_i}$, $v_j \in V(H_i), e_{ij} \in E(H_i)$ and $\frac{f_{ij}}{c_{ij}} > \alpha_i$ then $v_i \in (V(H_i))$ so $N_{v_i}^{\alpha_i} \subseteq int_{\alpha_i}^{ij}(V(H_i)) \subseteq V(H_i)$. Also, $v_i \in V(H_i)$ then there exists $N_{v_j}^{\alpha_i}$ such that $N_{v_j}^{\alpha_i} \cap V(H_i) \neq \emptyset$ which implies that $v_i \in cl_{\alpha_i}^{ij}(V(H_i))$ so $V(H_i) \subseteq cl_{\alpha_i}^{ij}(V(H_i))$

(b)is obvious .

Proposition 2.4

Let N=(G,s,t,c) be a network with capacity c_{ij} ,s start vertex , t terminal vertex and flow f_{ij}, Hi=(V(Hi), E(Hi)) is subgraph in G represent the steps i with value α_i ,Hp=(V(Hp, E(Hp)) subgraph in G represent the steps p with value α_k in Ford-Fulkerson algorithm and $\alpha_k \geq \alpha_i$ then following satisfies:

(a)If $H_i \subseteq H_{i+1}$ then $int_{\alpha_i}^{ij}(V(H_i)) \subseteq int_{\alpha_i}^{ij}(V(H_{i+1}))$ and $cl_{\alpha_i}^{ij}(V(H_i)) \subseteq cl_{\alpha_i}^{ij}(V(H_{i+1}))$

(b) $cl_{\alpha_k}^{ij}\left(V(H_i) \cap V(H_p)\right) \subseteq cl_{\alpha_i}^{ij}(V(H_i)) \cap cl_{\alpha_i}^{ij}\left(V(H_p)\right)$

(c) $int_{\alpha_i}^{ij}\left(V(H_i) \cup V(H_p)\right) \subseteq int_{\alpha_k}^{ij}(V(H_i)) \cup int_{\alpha_k}^{ij}\left(V(H_p)\right)$

Proof:

If $v_i \in int_{\alpha_i}^{ij}(V(H_i))$then there exist $N_{v_i}^{\alpha_i}$ such that $v_j \in V(H_i), e_{ij} \in E(H_i)$ and $\frac{f_{ij}}{c_{ij}} > \alpha_i$such that $v_i \in (V(H_i))$ so $N_{v_i}^{\alpha_i} \subseteq int_{\alpha_i}^{ij}(V(H_i)) \subseteq V(H_i) \subseteq V(H_{i+1})$ then $v_i \in int_{\alpha_i}^{ij}(V(H_{i+1}))$ so $int_{\alpha_i}^{ij}(V(H_i)) \subseteq int_{\alpha_i}^{ij}(V(H_{i+1}))$.If $v_i \in cl_{\alpha_i}^{ij}(V(H_i))$then there exist $N_{v_i}^{\alpha_i}$ such that $N_{v_i}^{\alpha_i} \cap V(H_i) \neq \emptyset$ then $v_i \in V(H_{i+1})$so,$v_i \in cl_{\alpha_i}^{ij}(V(H_{i+1}))$ which implies $cl_{\alpha_i}^{ij}(V(H_i)) \subseteq cl_{\alpha_i}^{ij}(V(H_{i+1}))$

(b) ,(c) from Prop. 2.3 and (a)

Proposition 2.5

Let N=(G,s,t,c) be a network with capacity c_{ij} ,s start vertex , t terminal vertex and flow f_{ij} and Hi=(V(Hi), E(Hi)) subgraph in G represent the steps i in Ford-Fulkerson algorithm then following satisfies::

(a) $\mathrm{int}_{\alpha_i}^{ij}\left(V(G)-V(H_i)\right)=V(G)-\mathrm{cl}_{\alpha_i}^{ij}\left(V(H_i)\right)$

(b) $\mathrm{int}_{\alpha_i}^{ij}(\mathrm{int}_{\alpha_i}^{ij}\left(V(H_i)\right))=\mathrm{cl}_{\alpha_i}^{ij}\left(\mathrm{int}_{\alpha_i}^{ij}V(H_i)\right)$

(d) $\mathrm{cl}_{\alpha_i}^{ij}(\mathrm{cl}_{\alpha_i}^{ij}\left(V(H_i)\right))=\mathrm{int}_{\alpha_i}^{ij}(\mathrm{cl}_{\alpha_i}^{ij}(V(H_i)))$

Proof. Obviously

Definition 2.5

Let N=(G,s,t,c) be a network with capacity c_{ij} ,s start vertex , t terminal vertex and flow f_{ij}, a (V(G), $\mathrm{cl}_{\alpha_i}^{ji}$)on a subgraph Hi =(V(Hi), E(Hi)) of Ford-Fulkerson algorithm in step i of a connected graph G=(V,E) .The α_i-independency of a vertices of Hi on G with respect to α_i – closure spaces and defined as follows:

$$\gamma_{\alpha_i}^{ji}(V(H_i))=1-\frac{\left|\mathrm{cl}_{\alpha_i}^{ji}(V(H_i))\right|}{|V(G)|}$$

Example 2.2

From Examples 2.1 we have:

$\gamma_{\alpha_i}^{ij}(V(H_0))=1$, i =1,2,3 j= 0,…,5 and $\alpha_i\in\{0,0.5,0.8\}$

$\gamma_0^{11}(V(H_1))=\frac{3}{6}$, $\gamma_{0.5}^{11}(V(H_1))=1$, $\gamma_{0.8}^{11}(V(H_1))=1$

$\gamma_0^{21}(V(H_1))=1$, $\gamma_{0.5}^{21}(V(H_1))=\frac{3}{6}$, $\gamma_{0.8}^{21}(V(H_1))=1$

$\gamma_0^{31}(V(H_1))=1$, $\gamma_{0.5}^{31}(V(H_1))=\frac{3}{6}$, $\gamma_{0.8}^{31}(V(H_1))=1$

By the same way we can find all α_i-independency for all subgraphs inducd by all step of Ford-Fulkerson algorithm.

Proposition 2.6

Let N=(G,s,t,c) be a network with capacity c_{ij} ,s start vertex , t terminal vertex and flow f_{ij},, G=(V(G), E(G)) be a connected graph, and Hi, Hi+1 be a subgraphs in G represent of Ford-Fulkerson algorithm in step i, i+1, then

$\gamma_{\alpha_i}^{ij}(V(H_i))\geq\gamma_{\alpha_i}^{ij}(V(H_{i+1}))$

for every i = 0, 1, … , k-1 , where k is the unumber of subgraphs in induced by Ford-Fulkerson algorithm in each step i of a connected graph G=(V,E)

Proof.

Let H_i , H_{i+1} be a subgraphs in G represent Ford-Fulkerson algorithm of steps i, i+1:

i = 0, 1, … , k. Since V(Hi) ⊆ V(Hi+1) for every i = 0, 1, … , k, implies

$\mathrm{cl}_{\alpha_i}^{ji}(V(H_i)\subseteq\mathrm{cl}_{\alpha_i}^{ji}(V(H_{i+1})$ then $\frac{\left|\mathrm{cl}_{\alpha_i}^{ji}(V(H_i))\right|}{|V(G)|}\leq\frac{\left|\mathrm{cl}_{\alpha_i}^{ji}(V(H_{i+1}))\right|}{|V(G)|}$ so $\gamma_{\alpha_i}^{ij}(V(H_i))\geq\gamma_{\alpha_i}^{ij}(V(H_{i+1}))$.

Proposition 2.7

Let N=(G,s,t,c) be a network with capacity c_{ij} ,s start vertex , t terminal vertex and flow f_{ij},, G=(V(G), E(G)) be a connected graph, and Hi, be a subgraph in G represent of Ford-Fulkerson algorithm in step i, i+1, then $\gamma_{\alpha_i}^{ij}(V(H_i))\geq\gamma_{\alpha_j}^{ij}(V(H_i))$,$\alpha_i\leq\alpha_j$ for every i = 0, 1, … , k-1 , where k is the unumber of subgraphs in induced by Ford-Fulkerson algorithm in each step i of a connected graph G=(V,E)

Proof. Obviously

III. Conclusion

The basic concepts of the networks are presented. We explain Ford-Fulkerson Algorithm. The aim of this paper is to linked topological directions with graph theory directions through maximal network flow algorithm to evaluate the maximal flow in the networks. Therefore we will try to study applications used this algorithms where a metric important to formulate it and illustrate some hidden topological properties.

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