IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 3 Ver. I (May - Jun. 2015), PP 42-50
www.iosrjournals.org

Common Fixed Point Theorems for Four Weakly Compatible Self-Mappings in Fuzzy Metric Space Using (JCLR) Property

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Abstract: *In this paper we prove a common fixed point theorem for four weakly compatible self-mappings in fuzzy metric space by using (JCLR) property. An example is given which shows the validity of main theorem. We also extend the result to two finite families of self-mappings with pairwise commuting.*
Key Words: *Fuzzy Metric Space, Weakly Compatible Mappings, E.A Property, (CLR) Property and (JCLR) Property.*

I. Introduction

In 1965, Zadeh [33] introduced the concept of fuzzy set. In the last two decades tremendous development has taken place in the study of fuzzy set theory. Fuzzy set theory has applications in applied sciences such as neural network, stability theory, mathematical programming, modeling theory, image processing, control theory, communication, engineering sciences, medical sciences etc.

In 1975, Kramosil and Michalek [18] introduced the concept of fuzzy metric space. George and Veeramani [11] modified the concept of fuzzy metric space introduced by Kramosil and Michalek [18], which opened on avenue for further development of analysis in fuzzy metric spaces.

In 2002, Aamri and El-Moutawakil [1] defined the notion of E.A property for self-mappings which contains the class of non-compatible mappings in metric spaces. E.A property replaces the completeness condition by closedness condition of the range and relaxes the completeness of the whole space, continuity of one or more mappings and containment of the range of one mapping into the range of the other.

A number theorems were proved for self-mappings satisfying E.A property [or CLR's property]. Many authors have proved common fixed point theorems in fuzzy metric spaces for different contractive conditions. For details refer to [3, 5-16, 17, 20, 22, 24, 25, 26, 27, 28, 31, 32].

II. Preliminaries

2.1 Fuzzy Metric Space

A fuzzy metric space is a triple (X, M, T) where X is a nonempty set, T is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ and the following conditions are satisfied for all $x, y \in X$ and $t, s > 0$:

(FM-1) $M(x, y, t) > 0$;
(FM-2) $M(x, y, t) = 1 \Leftrightarrow x = y$;
(FM-3) $M(x, y, t) = M(y, x, t)$;
(FM-4) $M(x, y, \cdot): (0, \infty) \to [0, 1]$ is continuous;
(FM-5) $M(x, z, t + s) \geq T(M(x, y, t), M(y, z, s))$. □

2.2 Weakly Compatible Mappings

Two self-mappings f and g of a non-empty set X are said to be weakly compatible (or coincidentally commuting) if they commute at their coincidence points, i.e., if $fz = gz$ for some $z \in X$, then $fgz = gfz$. □

2.3 E.A. Property

Two self-mappings f and g of a fuzzy metric space (X, M, T) are said to satisfy E.A. property, if there exists a sequence $\{x_n\}$ in X such that $\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = u$ for some $u \in X$. □

2.4 (CLR) Property

Let f and g be self-mappings on a fuzzy metric space (X, M, T). Then the pair (f, g) is said to satisfy CLRg property (common limit in the range of g property) if $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gx$ for some $x \in X$. □

2.5 Increasing Function

Let Φ be class of all mappings ϕ :[0, 1] →[0, 1] satisfying the following conditions:

(ϕ1) ϕ is continuous and nondecreasing on [0, 1];

(ϕ 2) $\phi(x) > x$ for all $x\in(0, 1)$.

2.6 Lemma

Let (X, M, T) be a fuzzy metric space. If there exists $k \in (0, 1)$ such that $M(x, y, kt) \geq M(x, y, t)$ for all $x, y \in X$ and $t > 0$, then $x = y$. □

III. Main Result

In this paper, we first introduce the notion of *the joint common limit in the range property* of two pairs of self-mappings.

3.1 *Definition*

Let (X, M, T) be a fuzzy metric space and $f, g, p, q : X \to X$ be self-mappings. The pairs (f, q) and (p, g) are said to satisfy the *joint common limit in the range of q and g property* (shortly, (JCLRqg) property) if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} qx_n = \lim_{n\to\infty} py_n = \lim_{n\to\infty} gy_n = qu = gu, \quad \text{-- (1)}$$

for some $u \in X$.

3.2 Remark

If $p = f$, $q = g$ and $\{x_n\} = \{y_n\}$ in (1), then we get the definition of (CLRg) property. □

Throughout this section, Φ denotes the set of all continuous and increasing functions φ :[0, 1]^5 →[0, 1] in any coordinate and $\varphi(t, t, t, t, t) > t$ for all $t \in[0, 1)$.

Following are examples of some functions $\varphi \in\Phi$:

1) $\varphi(x_1, x_2, x_3, x_4, x_5) = (\min\{x_i\})^h$, for some $0 < h <1$.
2) $\varphi(x_1, x_2, x_3, x_4, x_5) = x_1^h$ for some $0 < h <1$.
3) $\varphi(x_1, x_2, x_3, x_4, x_5) = T(T(T(T(x_1, x_2), x_3), x_4), x_5)^h$, for some $0 < h <1$ and for all t-norms T such that $T(t, t) = t$.

Now, we state and prove our main result.

3.3 *Theorem*

Let (X, M, T) be a fuzzy metric space and f, g, p and q be mappings from X into itself. Further, let the pairs (f, q) and (p, g) are weakly compatible and there exists a constant $k \in \left(0, \frac{1}{2}\right)$ such that

$(qx, gy, t), M(fx, qx, t), M(py, gy, t),$

$M(fx, py, kt) \geq \varphi$ -- (2)

M $(fx, gy, \alpha t)$, $M(py,$ $qx, 2t - \alpha t)$

holds for all $x, y \in X$, $\alpha \in (0,$ $2)$, $t > 0$ and $\varphi \in \Phi$. If (f, q) and (p, g)

satisfy the (JCLRqg) property, then f, g, p and q have a unique common

fixed point in X.

***Proof* :**

Since the pairs (f, q) and (p, g) satisfy the (JCLRqg) property, there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} qx_n = \lim_{n\to\infty} py_n = \lim_{n\to\infty} gy_n = qu = gu,$$

for some $u \in X$.

Now, we assert that $gu = pu$. Using (2), with $x = x_n$, $y = u$, $\alpha = 1$,
we get

$M(qx_n, gu, t)$, $M(fx_n, qx_n, t)$, $M(pu, gu, t)$,

$M(fx_n, pu, kt) \geq \varphi$

$M(fx_n, gu, t)$, $M(pu, qx_n, t)$

Taking the limit as $n \to \infty$, we have

$M(gu, gu, t)$, $M(gu, gu, t)$, $M(pu, gu, t)$,

$M(gu, pu, kt) \geq \varphi$.

$M(gu, gu, t)$, $M(pu, gu, t)$

Since φ is increasing in each of its coordinate and $\varphi(t, t, t, t, t) > t$ for all $t \in [0, 1)$, we get $M(gu, pu, kt) \geq M(gu, pu, t)$. By Lemma 2.6, we have $gu = pu$.
Next, we show that $fu = gu$. Using (2), with $x = u$, $y = y_n$, $\alpha = 1$,
we get

M $(fu, py_n, kt) \geq \varphi$ $^{M}(gu, gy_n, t)$, $^{M}(fu, qu, t)$, ^{M} $M(fu, gy_n, t)$, M

Taking the limit as $n \to \infty$, we have

$M(gu, gu, t)$, $M(fu, gu, t)$, M

$M(fu, gu, kt) \geq \varphi$

$M(fu, gu, t)$, M

Since φ is increasing in each of its coordinate and $\varphi(t, t, t \in [0, 1)$, we get $M(fu, gu, kt) \geq M(fu, gu, t)$. By Lemma 2.6, we have $fu = gu$.

(py_n, gy_n, t), (py_n, qu, t)

$($ $gu, gu, t)$,

(gu, gu, t)

$t, t, t) > t$ for all

Now, we assume that $z = fu = gu = pu = qu$. Since the pair (f, q) is weakly compatible, $fqu = qfu$ and then $fz = fqu = qfu = qz$. Again since (p, g) is weakly compatible, it follows $gpu = pgu$ and hence $gz = gpu = pgu = pz$.

We show that $z = fz$. To prove this, using (2), with $x = z$, $y = u$, $\alpha = 1$, we get

$$M(fz, pu, kt) \geq \varphi \begin{pmatrix} M(qz, gu, t), M(fz, qz, t), M(pu, gu, t), \\ M(fz, gu, t), M(pu, qz, t) \end{pmatrix}$$

and so

$$M(fz, z, kt) \geq \varphi \begin{pmatrix} M(fz, z, t), M(fz, fz, t), M(z, z, t), \\ M(fz, z, t), M(z, fz, t) \end{pmatrix}.$$

Since φ is increasing in each of its coordinate and $\varphi(t, t, t, t, t) > t$ for all $t \in [0, 1)$, $M(fz, z, kt) \geq M(fz, z, t)$, which implies that $fz = z$. Hence $z = fz = qz$.

Next, we show that $z = pz$. To prove this, using (2), with $x = u$, $y = z$, $\alpha = 1$, we get

$$M(fu, pz, kt) \geq \varphi \begin{pmatrix} M(qu, gz, t), M(fu, qu, t), M(pz, gz, t), \\ M(fu, gz, t), M(pz, qu, t) \end{pmatrix}$$

and so

$$M(z, pz, kt) \geq \varphi \begin{pmatrix} M(z, pz, t), M(z, z, t), M(pz, pz, t), \\ M(z, pz, t), M(pz, z, t) \end{pmatrix}.$$

Since φ is increasing in each of its coordinate and $\varphi(t, t, t, t, t) > t$ for all $t \in [0, 1)$, $M(z, pz, kt) \geq M(pz, pz, t)$, which implies that $z = pz$. Hence $z = pz = gz$. Therefore, we conclude that $z = fz = gz = pz = qz$. This implies f, g, p and q have a common fixed point z.

For uniqueness of common fixed point, we let w be another common fixed point of the mappings f, g, p and q. On using (2) with $x = z$, $y = w$, $\alpha = 1$, we have

$$M(fz, pw, kt) \geq \varphi \begin{pmatrix} M(qz, gw, t), M(fz, qz, t), M(pw, gw, t), \\ M(fz, gw, t), M(pw, qz, t) \end{pmatrix}$$

and then

$$M(z, w, kt) \geq \varphi \begin{pmatrix} M(z, w, t), M(z, z, t), M(w, w, t), \\ M(z, w, t), M(w, z, t) \end{pmatrix}.$$

Since φ is increasing in each of its coordinate and $\varphi(t, t, t, t, t) > t$ for all $t \in [0, 1)$, $M(z, w, kt) \geq M(z, w, t)$, which implies that $z = w$.

Therefore f, g, p and q have a unique common fixed point. □

3.4 Remark

From the results, it is asserted that (JCLRqg) property never requires conditions closedness of the subspace, continuity of one or moremappings and containment of ranges amongst involved mappings.

3.5 Remark

Theorem 3.3 improves and generalizes the results of Abbas *et al.* ([2], Theorem 2.1) and Kumar ([19], Theorem 2.3) without any requirement of containment amongst range sets of the involved mappings and closedness of the underlying subspace.

3.6 Remark

Since the condition of t-norm with $T(t, t) = t$ for all $t \in [0, 1]$ is replaced by arbitrary continuous t-norm, Theorem 3.3 also improves the result of Cho *et al.* ([4], Theorem 3.1) without any requirement of completeness of the whole space, continuity of one or more mappings and containment of ranges amongst involved mappings.

3.7 Corollary

Let (X, M, T) be a fuzzy metric space and f, g, p and q be mappings from X into itself. Further, let the pairs (f, q) and (p, g) are weakly compatible and there exists a constant $k \in \left(0, \frac{1}{2}\right)$ such that

$$M(fx, py, kt) \geq \left[a_1(t) M(qx, gy, t) + a_2(t) M(fx, qx, t) + a_3(t) M(py, gy, t) + a_4(t) M(fx, gy, \alpha t) + a_5(t) M(py, qx, 2t - \alpha t) \right]^{\frac{1}{2}} \quad (3)$$

holds for all $x, y \in X$, $\alpha \in (0, 2)$, $t > 0$ and $a_i : \mathbb{R}^+ \to (0, 1]$ such that $\sum_{i=1}^{5} a_i(t) = 1$. If (f, q) and (p, g) satisfy the (JCLRqg) property, then f, g, p and q have a unique common fixed point in X.

Proof **:**

By Theorem 3.3, we define

$$\varphi(x_1, x_2, x_3, x_4, x_5) = \left(a_1(t) x_1 + a_2(t) x_2 + a_3(t) x_3 + a_4(t) x_4 + a_5(t) x_5 \right)^{\frac{1}{2}}$$

then the result follows.

3.8 Remark

Corollary 3.7 improves the result of Cho *et al.* ([4], Theorem 3.1) without any requirement of completeness of the whole space, continuity of one or more mappings and containment of ranges amongst involve mappings while the condition of t-norm $T(t, t) = t$ for all $t \in [0, 1]$ is replaced by an arbitrary continuous t-norm.

3.9 Corollary

Let (X, M, T) be a fuzzy metric space and f, g be mappings from X into itself. Further, let the pair (f, g) be weakly compatible and

there exists a constant $k \in \left(0, \frac{1}{2}\right)$ such that

$$M(fx, fy, kt) \geq \varphi\begin{pmatrix} M(gx, gy, t), M(fx, gx, t), M(fy, gy, t), \\ M(fx, gy, \alpha t), M(fy, gx, 2t - \alpha t) \end{pmatrix} \quad \text{-- (4)}$$

holds for all $x, y \in X$, $\alpha \in (0, 2)$, $t > 0$ and $\varphi \in \Phi$. If (f, g) satisfies the (CLRg) property, then f and g have a unique common fixed point in X.

Proof **:**

Take $p = f$ and $q = g$ in Theorem 3.3, then we get the result. □ Our next theorem is proved for a pair of weakly compatible self- mappings in fuzzy metric space (X, M, T) using E.A property under additional condition of closedness of a subspace.

3.10 *Theorem*

Let (X, M, T) be a fuzzy metric space. Further, let the pair (f, g) of self-mappings be weakly compatible satisfying inequality (4) of Corollary 3.9. If f and g satisfy E.A property and the range of g is a closed subspace of X, then f and g have a unique common fixed point in$X$.

Proof **:**

Since the pair (f, g) satisfies E.A property, there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = z$$

for some $z \in X$. It follows, from $g(X)$ being a closed subspace of X that there exists $u \in X$ such that $z = gu$. Therefore f and g satisfy the (CLRg) property. From Corollary 3.9, the result follows. □

In what follows, we present some illustrative examples which demonstrate the validity of the hypothesis and degree of utility of our results.

3.11 Example

Let $X = [2, 19)$ with the metric d defined by $d(x, y) = |x - y|$ and for each $t \in [0, 1]$ define

$$M(x, y, t) = \begin{cases} \dfrac{t}{t + |x - y|}, & \text{if } t > 0 \\ 0, & \text{if } t = 0 \end{cases}$$

for all $x, y \in X$. Clearly (X, M, T) is a fuzzy metric space with t-norm defined by $T(a, b) = \min\{a, b\}$ for all $a, b \in [0, 1]$. Consider a function $\varphi : [0, 1]^5 \to [0, 1]$ defined by $\varphi(x_1, x_2, x_3, x_4, x_5) = (\min\{x_i\})^{\frac{1}{2}}$. Then we have $M(fx, fy, t) \geq \varphi(x_1, x_2, x_3, x_4, x_5)$. Define the self-mappings f and g on X by

$$fx = \begin{cases} 2, & \text{if } x \in \{2\} \cup (3, 19) \\ 15, & \text{if } x \in (2, 3]. \end{cases} \quad \text{and} \quad gx = \begin{cases} 2, & \text{if } x = 2 \\ 12, & \text{if } x \in (2, 3] \\ \dfrac{x+1}{2}, & \text{if } x \in (3, 19). \end{cases}$$

Taking $\{x_n\} = 3 + \frac{1}{n}$ or $\{x_n\} = \{2\}$, it is clear that the pair (f, g) satisfies the (CLRg) property since

$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = 2 = g(2) \in X.$$

It is noted that $f(X) = \{2, 15\} \subseteq [2, 10) \cup \{12\} = g(X)$. Thus, all the conditions of Corollary 3.9 are satisfied for a fixed constant $k \in \left(0, \frac{1}{2}\right)$ and 2 is a unique common fixed point of the self-mappings f and g. Also, all the involved mappings are even discontinuous at their unique common fixed point 2. Here, it may be pointed out that $g(X)$ is not a closed subspace of X. □

3.12 Example

In the setting of Example 3.11, replace the mapping g by the following, besides retaining the rest:

$$gx = \begin{cases} 2, & \text{if } x = 2 \\ 10, & \text{if } x \in (2, 3] \\ \dfrac{x+1}{2}, & \text{if } x \in (3, 19) \end{cases}$$

Taking $\{x_n\} = 3 + \frac{1}{n}$ or $\{x_n\} = \{2\}$, it is clear that the pair (f, g) satisfies the E.A property since

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = 2 \in X.$$

It is noted that $f(X) = \{2, 15\} \not\subseteq [2, 10] = g(X)$. Thus, all the conditions of Corollary 3.9 are satisfied and 2 is a unique common fixed point of the mappings f and g. Notice that all the involved mappings are even discontinuous at their unique common fixed point 2. Here, it is worth noting that $g(X)$ is a closed subspace of X. □

Our next theorem extends Corollary 3.9 to two finite families of self-mappings which are pairwise commuting.

3.13 *Theorem*

Let $\{f_i\}_{i=1}^{m}$ and $\{g_j\}_{j=1}^{n}$ be two finite families of self-mappings in a fuzzy metric space (X, M, T) such that $f = f_1 f_2 \dots f_m$ and $g = g_1 g_2 \dots g_n$ which satisfy the inequality (4) of Corollary 3.9. If the pair (f, g) satisfies (CLRg) property, then f and g have a unique point of coincidence.

Moreover, $\{f_i\}_{i=1}^{m}$ and $\{g_j\}_{j=1}^{n}$ have a unique common fixed point provided the pair of families $(\{f_i\},\{g_j\})$ commute pairwise, where $i \in \{1, 2, \dots, m\}$ and $j \in \{1, 2, \dots, n\}$.

Proof **:**
The proof of this theorem can be completed on the lines of Theorem 3.1 contained in Imdad *et al*. [17], hence details are avoided. □

Putting $f_1 = f_2 = \dots = f_m = f$ and $g_1 = g_2 = \dots = g_n = g$ in Theorem 3.13, we get followings result:

3.14 Corollary

Let f and g be two self-mappings of a fuzzy metric space (X, M, T). Further, let the pair (f^m, g^n) satisfies (CLRg) property. Then there exists a constant $k \in \left(0, \frac{1}{2}\right)$ such that

$$M(f^m x, f^n y, kt) \geq \varphi \left(\begin{array}{c} M(g^n x, g^n y, t), M(f^m x, g^n x, t), M(f^m y, g^n y, t), \\ M(f^m x, g^n y, \alpha t), M(f^m y, g^n x, 2t - \alpha t) \end{array} \right)$$

holds for all $x, y \in X$, $\alpha \in (0, 2)$, $t > 0$, $\varphi \in \Phi$ and m and n are fixed positive integers, then f and g have a unique common fixed point provided the pairs (f^m, g^n) commute pairwise.

3.15 Remark

Theorem 3.10, Theorem 3.13 and Corollary 3.14 can also be outlined in respect of Corollary 3.7.

3.16 Remark

Using Example 3.12, we can obtain several fixed point theorems in fuzzy metric spaces in respect of Theorems 3.10 and 3.13 and Corollaries 3.7, 3.9 and 3.14.

References

[1] M. Aamri - D. El Moutawakil, "*Some New Common Fixed Point Theorems under Strict Contractive Conditions*", J. Math. Anal., Appl., 270 (1) (2002), pp. 181-188.

[2] M. Abbas - I. Altun - D. Gopal, "*Common Fixed Point Theorems for Non Compatible Mappings in Fuzzy Metric Spaces*", Bull. Math. Anal. Appl. 1(2) (2009), 47-56.

[3] Y.J. Cho, "*Fixed Points in Fuzzy Metric Spaces*", J. Fuzzy Math. 5 (4) (1997), 949-962.

[4] Y.J. Cho - Sedghi - N. Shobe, "*Generalized Fixed Point Theorems for Compatible Mappings with Some Types in Fuzzy Metric Spaces*", Chaos, Solitons and Fractals, Vol.39, no.5, pp. 2233-2244, 2009.

[5] B. Deshpande, "*Fixed Point and (DS)-Weak Commutativity Condition in Intuitionistic Fuzzy Metric Spaces*", Chaos, Solitons and Fractals, Vol. 42, no. 5, pp. 2722-2728, 2009.

[6] B.C. Dhage, "*Generalised Metric Spaces and Mappings with Fixed Point*", Bull. Calcutta Math. Soc., 84(4) (1992), 329-336.

[7] B.C. Dhage, "*A Common Fixed Point Principle in -Metric Spaces*", Bull. Calcutta Math. Soc., Vol.91 (1999), 475-480.

[8] B.C. Dhage - A.M. Pathan - B.E. Rhoades, "*A General Existence Principle for Fixed Point Theorem in D-Metric Spaces*", Int. J. Math. Sci., Vol.23 (2000), 441-448.

[9] M.A. Erceg. "*Metric Spaces in Fuzzy Set Theory*", J. Math. Anal. Appl., Vol.69 (1979), 205-230.

[10] J.X. Fang, "*On Fixed Point Theorems in Fuzzy Metric Spaces*", Fuzzy Sets and Systems, Vol.46 (1992), 107-113.

[11] A. George - P. Veeramani, "*On Some Results in Fuzzy Metric Space*", Fuzzy Sets and Systems, Vol.64 (1994), 395-399.

[12] A. George - P. Veeramani, "*On Some Results of Analysis for Fuzzy Metric Spaces*," Fuzzy Sets and Systems, Vol. 90, no.3, pp. 365-368, 1997.

[13] H.M. Golshan - H. Naraghi - M. Kazemi, "*T-best Approximation in Fuzzy and Intuitionistic Fuzzy Metric Spaces*", Journal of Nonlinear Analysis and Optimization, Vol. 2, no.1, pp. 75-83, 2011.

[14] M. Grabiec, "*Fixed Point in Fuzzy Metric Spaces*", Fuzzy Sets and Systems, Vo. 27, no.3, (1988), 385-389.

[15] V. Gregori - S. Romaguera, "*Some Properties of Fuzzy Metric Spaces*", Fuzzy Sets and Systems, Vol.115 (2000), 485-489.

[16] V. Gregori - A. Sapena, "*On Fixed-point Theorem in Fuzzy Metric Spaces*", Fuzzy Sets and Systems, Vol.125 (2002), 245-252.

[17] M. Imdad - J. Ali, "*Some Common Fixed Point Theorems in Fuzzy Metric spaces*", Math. Commun., Vol.11 (2006), 153-163.

[18] I. Kramosil - J. Michalek, "*Fuzzy Metrics and Statistical Metric Spaces*", Kybernetica, Vol.11 (5) (1975), 336-344.

[19] S. Kumar, "*Fixed Point Theorems for Weakly Compatible Maps under E.A. Property in Fuzzy Metric Spaces*", J. Appl. Math. Inform. Vol.29 (2011), 395-405.

[20] S. Kutukcu - D. Turkoglu - C. Yildiz, "*Common Fixed Points of Compatible Maps of Type (β) on Fuzzy Metric Spaces*", Communications of the Korean Mathematical Society, Vol.21, No.1, 2006, pp. 89-100.

[21] D. Mihet, "*Fixed Point Theorems in Fuzzy Metric Spaces using Property E.A., Non-linear Analysis Theory, Methods & Applications*", Int. J. Math. Math. Sci., Vol.73 (7) (2010), 2184-2188.

[22] P.P. Murthy - S. Kumr - K. Tas, "*Common Fixed Points of Self Maps Satisfying an Integral Type Contractive Condition in Fuzzy Metric Spaces*", Math. Commun. 15(2) (2010), 521-537..

[23] M.S. El Naschie, "*On a Fuzzy Kahler-like Manifold which is Consistent with two-slit Experiment*", Int. J. of Nonlinear Science and Numerical Simulation, 6 (2005), 95-98.

[24] B.D. Pant - S. Chauhan, "*Common Fixed Point Theorems for two pairs of Weakly Compatible Mappings in Menger Spaces and Fuzzy Metric Spaces*", Vasile Alecsandri Univ. Bacau Sci. Stud. Res. Ser. Math. Inform. 21 (2) (2011), 81-96.

[25] D.O. Regan - M. Abbas, "*Necessary and Sufficient Conditions for Common Fixed Point Theorems in Fuzzy Metric Space*", Demonstratio Math. Vol.42 (4) (2009), 887-900.

[26] S. Sedghi, D. Turkoglu - N. Shobe, "*Common Fixed Point of Compatible Maps of Type (r) on Complete Fuzzy Metric Spaces*", Communications of the Korean Mathematical Society, Vol. 24, no. 4, 2009, pp. 581-594.

[27] S.L. Singh - A. Tomar, "*Fixed Point Theorems in FM-Spaces*", J. Fuzzy Math. Vol.12 (4) (2004), 845-859.

[28] W. Sintunavarat - Y.J. Cho - P. Kumam, "*Coupled Coincidence Point Theorems for Contractions without Commutative Condition in Intuitionistic Fuzzy Normed Spaces"*, Fixed Point Theory Appl. Vol. 2011.

[29] W. Sintunavarat - P. Kumam, "*Common Fixed Point Theorems for a Pair of Weakly Compatible Mappings in Fuzzy Metric Spaces*", J. Appl. Math. Vol. 2011, Article ID 637958, 14 pages, 2011.

[30] W. Sintunavarat - P. Kumam "*Common Fixed Points for R-weakly Commuting in Fuzzy Metric Spaces*", Annali dell' Universitâ di Ferr., in Press.

[31] P.V. Subrahmanyam, "*Common Fixed Point Theorems in Fuzzy Metric Spaces*", Inform. Sci., Vol. 83 (1995), 109-112.

[32] R. Vasuki, "*Common Fixed Points for R-weakly Commuting Maps in Fuzzy Metric Spaces*", Indian J. Pure Appl. Math., Vol.30 (4) (1999), 419-423.

[33] L.A. Zadeh, "*Fuzzy Sets*", Inform and Control, Vol.8, No. 3, (1965), 338-353.