IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 3 Ver. I (May - Jun. 2015), PP 97-104
www.iosrjournals.org

The Effect of MHD Flow in Shock Formation in One-Dimensional Gas Flow

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Abstract: *This study investigated the effect of MHD flow in shock formation in one dimensional gas flow. A conducting gas has been considered with the flow occurring in the presence of varying magnetic field intensity H. The shock zone has been assumed to be a thin legume that is stationary. An analysis of the velocity profiles, density profiles and temperature distribution that have been obtained has been done. In addition, an investigation on how the Prandtl number, Hartmann number and Eckert number affect the velocity profiles and temperature distribution has been carried out. Differential equations that have been generated from this study are non-linear. The equations have been solved by finite difference method using the MATLAB software and results presented graphically. It has been noted that an increase in Hartmann number causes an increase in velocity profiles while an increase in Prandtl number leads to an increase in temperature distribution and an increase in Eckert number leads to an increase in temperature distribution.*

Keywords: *Magnetohydrodynamics, compressible flow, shock wave, normal shock, oblique shock, weak shock, strong shock.*

I. Introduction

Effect of Magnetohydrodynamics (MHD) flow in compressible fluid flow has become significant. Further, MHD flow effect on shock formation has very wide range of applications in astrophysical world and in engineering. Astrophysicists discovered that ionized gases and strong magnetic fields exist in the universe and hence the need to explain certain phenomena in the universe by use of MHD. Engineers employ MHD flow in shock formation in the design of heat exchangers, shock absorbers for locomotive machines and electromagnetic pumps, Kinyanjui *et al* (2003).

Studies of magnetohydrodynamic shock waves in gases have been reported in the literature. Rankine (1870) and Hugoniot (1889) came up with shock jumps for flow parameters famously known as the Rankine-Hugoniot conditions. Wu (1990) considered the formation, structure and stability of intermediate shocks in dissipative MHD. A conclusion was that, there are free parameters in the structure of intermediate shocks, and that these parameters are related to the shock stability. Sterk *et al* (1990), carried out a study of two-dimensional numerical of a planar field – aligned ideal MHD bow shock flow in a regime where fast MHD switch – on shocks are possible. Numerical problems encountered when high - resolution numerical MHD schemes derived from common computational fluid dynamics approaches used to stimulate this kind of flows were discussed. It is observed that the MHD shock formation effects of our simulations may occur in processes, in the corona of the sun. Angail (2004), studied the MHD model of boundary layer equations for conducting viscous fluids, the effect of free convection with two relaxation times on the flow of viscous conducting fluid was studied. He adopted the solution of one – dimensional transient problem to a whole space distribution of heat sources. He observed that as the Alfven velocity increases, the velocity of the fluid increased. He also noted that the velocity increased as the Grashof number (Gr) increased while it decreases when Prandtl number (Pr) increases. Hyesang *et al* (2007), considered a study of the statistics and energetics of shocks formed in cosmological simulations of a concordance universe, with special emphasis on the effects of non – gravitational processes such as radiative cooling, photoionization / heating, and galactic super wind feedbacks. They also examined the vorticity generated mostly at curved shocks in cosmological simulations. They found out that the dynamics and energetic of shocks are governed primarily by the gravity of matter; hence other non-gravitational processes do not significantly affect the global energy dissipation and vorticity generation at cosmological shocks Their results reinforce scenarios in which the intra cluster medium and warm – hot intergalactic medium contain energetically significant population of non – thermal particles and turbulent flow motions. Mallick and Schramn (2013), observed that shockwaves constitute discontinuities in matters which are relevant in studying the plasma behavior in astrophysical scenarios and in heavy –ion collision. They can produce conical emissions in relativistic collisions and are also thought to be the mechanism behind the acceleration of energetic particles in active galactic nucleic and gamma ray bursts. The shocks are mostly hydrodynamic shocks. In a magnetic background they become MHD shocks. They carried out a study on the space-like and time-like shock discontinuity in magnetic plasma. They observed that the magnetic field has effectively no effect on time-like shocks. The slight anisotropy in the downstream flow velocities is caused by the boosting that brings the quantities from the fluid frame to normal incidence frame. Recently, Anand (2013), considered the generalized jump relations across the magnetohydrodynamic (MHD) shock front in non-ideal gas. Simplified forms of the MHD shock jump relations are obtained in terms of idealness parameter, simultaneously for both weak and strong shocks; when applied magnetic field is strong and when the field is weak. It was realized that the shock velocity, pressure and particle velocity just behind the MHD shock wave in non-ideal gas flow increase with the strength of magnetic field, and non-idealness parameter. The main aim of this paper is to investigate the effect of MHD flow in shock formation in one–dimensional gas flow in the presence of varying magnetic field. The general non-dimensional forms of the continuity, momentum and equations are derived, assuming that the flow is steady; thermal conductivity, electrical conductivity and coefficient of viscosity are constant. The numerical estimations of flow variables are carried out using MATLAB code. The effects of Hartmann, Prandtl and Eckert numbers on flow variables across the shock region are investigated.

II. Basic Equations And Boundary Conditions

The equations governing the one-dimensional gas flow in the presence of magnetic field through a shock region are as follows:

Continuity equation: The flow equation is based on the principle of conservation of mass of the fluid, that is, mass of fluid is conserved and that the flow is continuous. In three dimensions, the continuity equation can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Since there is no flow along the z-axis and y-axis, then the continuity equation reduces to:

$$\frac{\partial v}{\partial t}\rho u=0 \qquad (1)$$

$$\frac{\partial u}{\partial x}=0 \qquad (2)$$

$$\frac{\partial v}{\partial y}=0 \qquad (3)$$

Integrating equation (1) across the shock region we obtain:

$$\rho_1 u_1=\rho_2 u_2=m \qquad (4)$$

Hence, in a steady flow, continuity equation remains unchanged across the shock region.
Similarly, integrating equation (3) across the shock region we obtain:

$$v=-v_0 \qquad (5)$$

Where v_0 is the suction velocity.

Momentum equation: The equation is based on the Newton's second law of linear motion. Also called the Navier-stokes equation. The equation of motion considers the body force due to gravity and electromagnetic force only.

$$\frac{\partial v}{\partial t}=F-\frac{\partial}{\partial x}\int\frac{\partial P}{\rho}+\frac{\gamma}{3}\frac{\partial^2 v}{\partial x^2}+\gamma\frac{\partial^2 v}{\partial x^2} \qquad (6)$$

Where F= the body force= $\vec{J}\, X\, \vec{B}$

$\rho =$ Fluid density

v = Fluid velocity

$\gamma =$ Kinematic viscosity

The Navier-Stokes equation is also written as:

$$\rho(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}) = -\frac{\partial P}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2} - \rho g + \vec{J}\ X\ \vec{B} \qquad (7)$$

Since the flow is steady then, $\frac{\partial u}{\partial t} = 0$ and $u\frac{\partial u}{\partial x} = 0$ because the flow is along the x-axis only. Thus equation (7) reduces to:

$$\rho(\mathrm{v}\frac{\partial u}{\partial y}) = -\frac{\partial P}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2} - \rho g + \vec{J}\ X\ \vec{B} \qquad (8)$$

Differentiating equation (8) with respect to x gives $\frac{\partial^2 P}{\partial x^2} = 0$ which on integrating results into:

$\frac{\partial P}{\partial x} = -P$ (Constant) that is, pressure gradient is constant.

The negative sign shows that fluid pressure decreases across the shock region in the direction of the flow. Hence equation (8) becomes:

$$\rho v\frac{\partial u}{\partial y} = P + \mu\frac{\partial^2 u}{\partial y^2} + \vec{J}\ X\ \vec{B} \qquad (9)$$

Since the flow is horizontal, the effect of force of gravity is insignificant, hence it is ignored.
Further upon substituting equation (5) in equation (9), it reduces to:

$$\rho(-v\frac{\partial u}{\partial y}) = P + \mu\frac{\partial^2 u}{\partial y^2} + \vec{J}\ X\ \vec{B} \qquad (10)$$

But $\vec{J}\ X\ \vec{B} = \sigma\mu_e^2 H^2 U$. Substituting in equation (10) gives:

$$\rho(-\mathrm{v}_0\frac{\partial u}{\partial y}) = \mu\frac{\partial^2 u}{\partial y^2} + P - \sigma\mu_e^2 H^2 U \qquad (11)$$

Where σ is the electrical conductivity.

Expressing equation (11) in terms of non-dimensional variables results in the following equation:

$$\frac{\partial^2 u}{\partial y^2} + V_0\frac{\partial u}{\partial y} - M^2 u + C = 0 \qquad (12)$$

Energy equation: The equation is based on conservation of energy which states that energy is neither created nor destroyed but can be transformed from one form to another. It is derived from the first law of thermodynamics which states that the amount of energy added to a system dQ equals to change of internal energy dE plus work done dW, that is,

$$dQ = dE + dW \qquad (13)$$

The first law of thermodynamics requires that

$$\rho\frac{D_e}{D_t} + e(\frac{D_\rho}{D_t} + \rho\frac{\partial q}{\partial t}) = -\frac{\partial Q'}{\partial t} + Q'' - p\frac{\partial q}{\partial t} + \mu\varphi \qquad (14)$$

Where: $Q'' =$ the internal heat generation

$\frac{D}{D_t} =$ the material derivative

$\varphi =$ the dissipative heat

The term in brackets in equation (13) above represent the continuity equation and hence should be equated to zero.

Since the enthalpy is given as

$$h = e + (\frac{1}{\rho})\,\mathrm{p} \tag{15}$$

Then, the substantive enthalpy derivative of the enthalpy term in (14) is given as:

$$\frac{D_h}{D_t} = \frac{D_e}{D_t} + \frac{1}{\rho}\frac{D_p}{D_t} - \frac{p}{\rho^2}\frac{D_\rho}{D_t} \tag{16}$$

The equation can be expressed in terms of temperature by making the following replacements:

$$dh = Tds + \frac{1}{\rho}dp \tag{17}$$

Where: $T =$ absolute temperature and

$ds =$ specific entropy change.

$$ds = \left(\frac{\partial s}{\partial T}\right)dT + \left(\frac{\partial s}{\partial p}\right)dp \tag{18}$$

Thus

$$\left(\frac{\partial s}{\partial p}\right) = -\left(\frac{\partial \frac{1}{\rho}}{\partial T}\right) = \frac{1}{\rho^2}\left(\frac{\partial s}{\partial p}\right) = -\frac{\beta}{\rho} \tag{19}$$

Where: $\beta = -\frac{1}{\rho}\left(\frac{\partial s}{\partial p}\right)$. Represents the coefficient of thermal expansion and $\left(\frac{\partial s}{\partial T}\right) = \frac{C_p}{T}$

Therefore equation (16) can be re-written as:

$$dh = C_p dT + \frac{1}{\rho}(1 - \beta T)dp \tag{20}$$

However, according to Nyabuto (2013),

$$\rho C_p\left(\frac{\partial T}{\partial t}\right) = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma B_0^2 U^2 \tag{21}$$

Where: $k =$ constant of fluid conductivity

$Q" = \sigma B_0^2 U^2 =$ Internal heat generation

$\left(\frac{\partial u}{\partial y}\right)^2 =$ the dissipative heat

Effect of shock conditions

Introducing the shock element taking into consideration the upstream conditions and then expressing equations (12) and (21) in terms of non-dimensional variables and parameters, the equations take the form:

$$\frac{\partial^2 u}{\partial y^2}+V_0\frac{\partial u}{\partial y}-M_1^2 u+\frac{P_1}{\rho_1}=0 \tag{22}$$

Where $M_1^2=\frac{\sigma\mu_1^2 H_a^2}{\rho U^2}$, $\frac{p_1}{\rho_1}=c$ (constant) and p_1,ρ_1, are initial flow variables.

$$t\le 0: u=0,\ t>0, y=-1: u=0, t>0,\ y=1: u=1, \tag{23}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+E_c\left(\frac{\partial u}{\partial y}\right)^2+M_1^2u^2;\theta=T \tag{24}$$

For $t\le 0$: $\theta=0$:t > 0, y= −1: $\theta=0$,t > 0, y = 1: $\theta=1$. (25)

Discretizing the momentum equation (22) i.e. $\frac{\partial^2 u}{\partial y^2}+v_0\frac{\partial u}{\partial y}-M_1^2u+c=0$ gives:

$$\frac{U_{i,j+1}-2U_{i,j}+U_{i,j-1}}{(\Delta y)^2}+\frac{U_{i,j-1}-U_{i,j-1}}{\Delta y}-\frac{M_1^2(U_{i,j+1}-U_{i,j-1})}{2}+0.75=0 \tag{26}$$

And discretizing the energy equation (24); i.e. $\frac{\partial\theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+Ec(\frac{\partial u}{\partial y})^2+EcM_1^2u^2$ gives

$$\frac{\theta_{i,j+1}-\theta_{i,j-1}}{2k}=\frac{1}{\Pr}\frac{\theta_{i,j+1}-2\theta_{i,j}+\theta_{i,j-1}}{k^2}+Ec(\frac{U_{i,j+1}-U_{i,j-1}}{2k})^2+EcM_1^2(\frac{U_{i,j+1}+U_{i,j}}{2}) \tag{27}$$

when discretized using the finite difference scheme.

III. Results And Discussions

In this chapter, the non-linear differential equations (22) and (24) together with boundary conditions have been expressed in finite difference form and then solved using MATLAB software. The results on how the Hartmann number, Prandtl number and Eckert number affect the velocity profiles, density profiles and temperature distribution have been presented in graphical form.

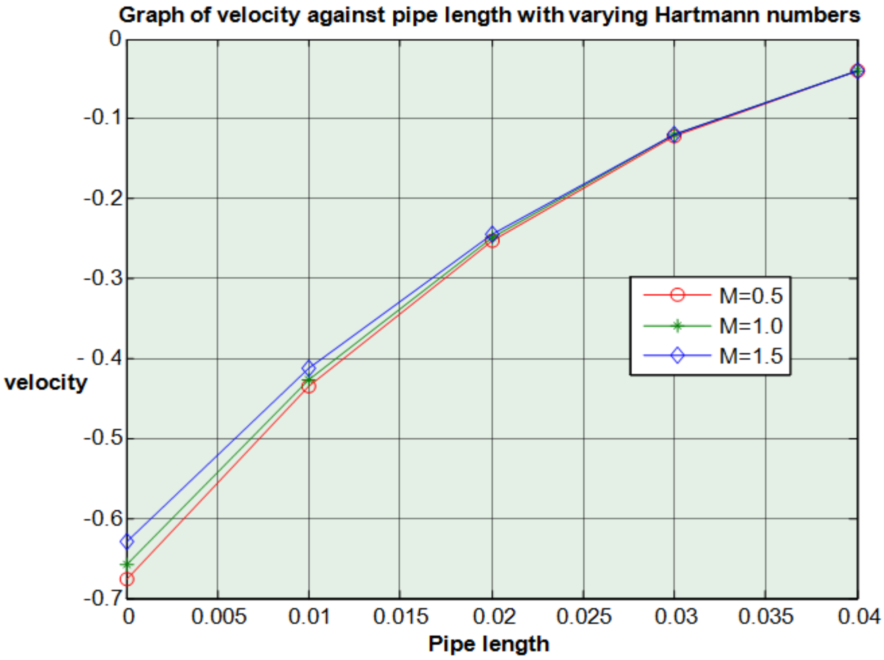

Figure 1: **Variation of velocity against pipe length**

Figure 1 above shows how the velocity of the fluid changes with distance along the shock region. It is noted that values of velocity increase with increase in Hartmann number. The increase in Hartmann number results due to an increase in magnetic force as viscous force decreases. This is so because of the presence of magnetic field in the shock region. Further, as the distance from the upstream region (A) increases, induced magnetic force decreases implying increased fluid flow velocity. This implies that an increase in Hartmann number leads to an increase in velocity profiles across the shock region within a finite distance d. However, as the distance along the pipe increases the velocity profiles for the different Hartmann numbers tend to converge at about $d = 0.03$.

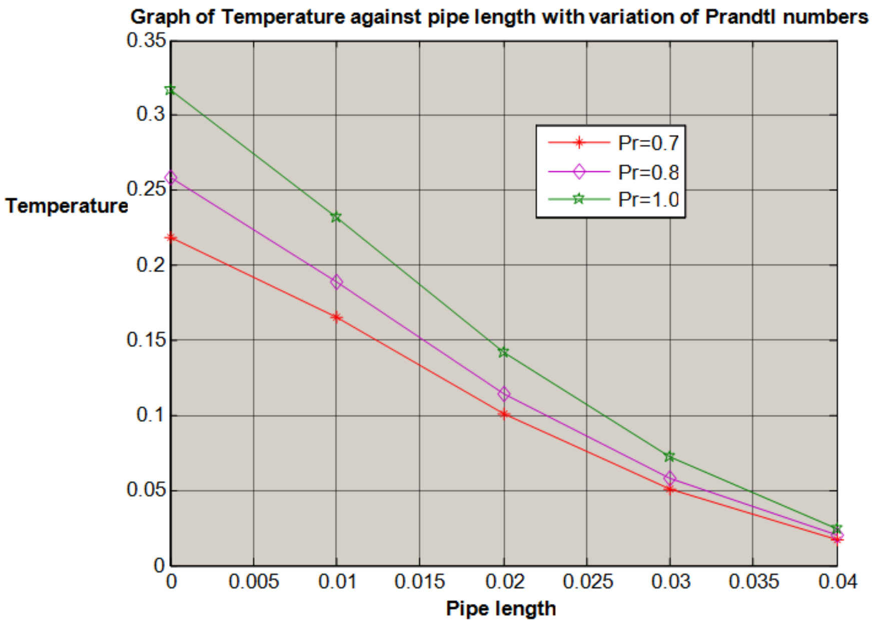

Figure 2: **Change of temperature profile at varying Prandtl number**

Figure 2 above shows how the fluid temperature changes with distance along the shock region at various values of Prandtl number. It is noted that temperature at any value of d increase with increase in Prandtl number. However, values of temperature reduce significantly as the distance along the shock region increases. As the horizontal distance increases, kinetic energy reduces hence slowed rate of thermal diffusion in relation to viscous force.

Variation of temperature profiles with distance at varying Eckert numbers

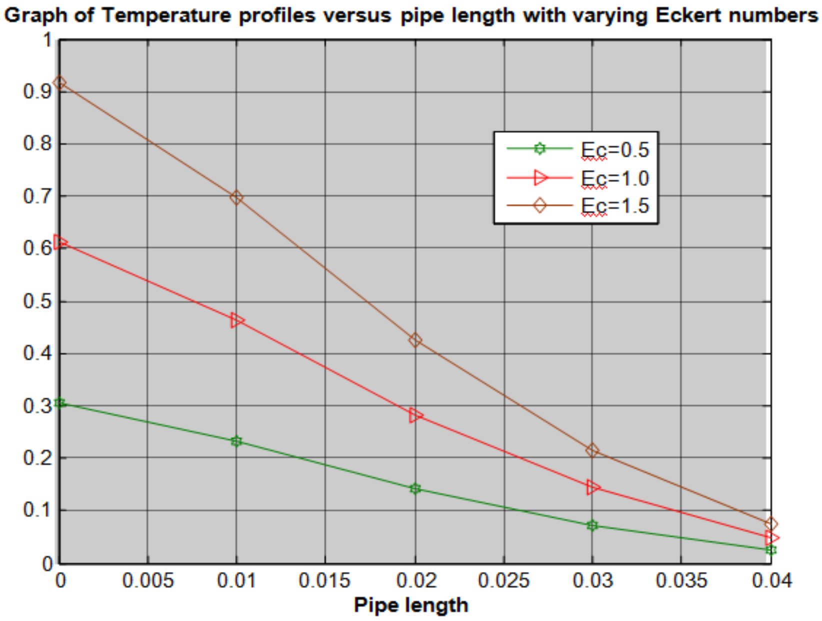

Figure 3: **Variation of temperature profiles at different Eckert numbers.**

Figure 3 reveals that temperature at any value of d increase with increase in Eckert number. However, values of temperature reduce significantly as distance along the shock region increases. This implies that there is more heat generation at lower values of d. However, as the flow progresses across the shock region, the fluid flow is retarded causing reduced collision of particles. This causes a gradual fall in temperature as distance increases along the shock region.

IV. Conclusion

This study focused on the effect of MHD flow in shock formation in one dimensional gas flow. A conducting gas has been considered with the flow occurring in the presence of varying magnetic field intensity H. The fluid flow process is considered within a shock region of distance d. An analysis of the velocity profiles and temperature distribution has been done. The nonlinear differential equations have been solved using the MATLAB software. The effect of Hartmann, Eckert and Prandtl numbers on velocity profiles and temperature distribution has been carried out.

- It has been noted that an increase in Hartmann number causes an increase in velocity profiles across the distance in the shock region.
- Further, an increase in Prandtl number leads to an increase in temperature distribution. However, values of temperature reduce significantly as the distance along the shock region increases. This is due to reduced kinetic energy which results to slowed rate of thermal diffusion in relation to viscous force.
- Similarly, an increase in Eckert number also leads to an increase in temperature distribution across the shock region. However, as the flow distance increases across the shock region, fluid flow is retarded causing a gradual fall in temperature.

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