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# Optimization of CASP-CUSUM Schemes Based on Truncated Erlangian Distribution using Lobatto Integration Method

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***Abstract:*** *The quality and reliability are fundamental criteria to accept or reject the manufactured products. Thus the Acceptance Sampling Plans are being introduced to accept or reject the lots of finished products in the present scenario. Such types of methods are involved in the inspection quality of product which is destructive where 100% inspection is not possible such as bullets, batteries, bulbs and so on. This paper study the CASP-CUSUM Schemes based on the assumption that the continuous variable under consideration follows a truncated Erlangian distribution. It is used the Lobatto Integration Method to solve the truncated integral equations. The Erlangian Distribution plays a vital role in Statistical Quality Control, particularly in estimating reliability by considering its distribution. Optimum CASP-CUSUM Schemes are suggested based on numerical results obtained.*
***Keywords****: CASP-CUSUM Schemes, type-C OC Curve, ARL, Truncated Erlangian distribution*

## I. Introduction

Acceptance sampling is concerned with inspection and decision making regarding lots of product by the techniques of quality of assurance in 1930's and 1940's Acceptance Sampling Plans were one of the important statistical tools on the field of Statistical Quality Control. These techniques require specification of a probability model describing the life of the products. In this process the Erlangian distribution has been studied by Shah and Dave **[08].**

Acceptance Sampling Plans are widely used in business, industry, producing engineering products etc. For example a company receives a segment of product from a vender, the product is after a component or raw material used in the manufacturing process. A sample is taken from a lot and examined the quality with respect to certain characteristics of interest. On the basis of information supplied by the sample, an optimal decision is to be made. Often a decision is taken to accept the lot or reject the lot. After the decision, accepted lots are put it into production while rejected lots may be returned to the vendor. In such case, to take optimal decisions with regard to "Acceptance Sampling Plans" are widely used from past two decades. Thus most acceptance sampling plans are working as an alarm in decision making therefore we will develop a procedure for calculating the values relevant characters P (A), ARL's of CASP-CUSUM to make the required decision.

In these lines Sobel and Tischendrof **[09]** studied acceptance sampling plans based on an exponential distributions. Gupta and Groll **[03]** developed sampling plans based on Gamma life test sample data. Goode and Kao **[02]** proposed samples plans based on the Weibull distribution and determined appropriate measures of CUSUM schemes.

In this study, we proposed acceptance sampling method based on the truncated Erlangian distribution under the assumption that the variable under study distributed according to truncated Erlangian distribution under this assumption we determined an appropriate measures for CUSUM schemes by using Lobatto Method of Integration. Thus it is more worthwhile to study some interesting characteristics of this distribution. The characteristics and applications of the Erlangian distribution were discussed.

## II. Erlangian Distribution

A continuous random variable X assuming non-negative values is said to have Erlangian Distribution with parameters λ and M, its probability density function is given by:

$$f(x) = \frac{e^{-M\lambda x}(M\lambda)^M x^{M-1}}{(M-1)!} \qquad 0 < x < \infty, \quad \lambda > 0$$

$$= 0 \text{ otherwise} \qquad (2.1)$$

## III. Truncated Erlangian Distribution

The random variable X is said to follow a Truncated Erlangian Distribution if:

$$f_B(x) = \frac{e^{-M\lambda x}(M\lambda)^M x^{M-1}}{(M-1)!\left[1-(M\lambda)^M e^{-M\lambda B}\left\{\sum_{j=1}^{M}\frac{B^{M-j}}{(M-j)\,!\,(M\lambda)^j}\right\}\right]} \qquad (3.1)$$

= 0 otherwise

Where ,B is the upper truncation point of the Erlangian distribution.

## IV. Description of the Plan and Type of OC curve

Beattie D.W **[01]** has suggested the method for constructing the continuous acceptance sampling plan. The procedure, suggested by him consists of a chosen decision interval namely, "**Return interval**" with the length $h'$, above the decision line is taken. We plot on the chart the sum $S_m$ = $\sum(X_i - k)$, $X_i$'$s(i = 1,2,3....)$ are distributed independently and k is the reference value. If the sum lies in the area of normal chart, the product is accepted and if it lies in the of the return chart, the product is rejected, subject to the following assumptions.

1. When the recently plotted point on the chart touches the decision line, then the next point to be plotted at the maximum, i.e., h + $h'$.
2. When the decision line is reached or crossed from above, the next point on the chart is to be plotted from the baseline.

When the CUSUM falls in the return chart, network or a change of specification may be employed rather than outright rejection.

The procedure in brief is given below:

1. Start plotting the CUSUM at 0.
2. The product is accepted when $S_m = \sum(X_i - k) < h$; when $S_m < 0$, return cumulation to 0.
3. When $h < S_m < h + h'$ the product is rejected: when $S_m$ crosses h, i.e., when $S_m > h + h'$ and continue rejecting product until $S_m > h + h'$ return cumulation to $h + h'$.

The Type – C, OC function which is defined as the probability of acceptance of an item as a function of incoming quality, when sampling rate is same in acceptance and rejection regions. Then the probability of acceptance P (A) is given by

$$P(A) = \frac{L(0)}{L(0) + L'(0)} \qquad (4.1)$$

Where $L(0)$ = Average Run Length in acceptance zone and

$L'(0)$ = Average Run Length in rejection zone.

Page, E.S. **[07]** has introduced the formulae for L (0) and $L'$ (0) as:

$$L(0) = \frac{N(0)}{1 - P(0)} \qquad (4.2)$$

$$L'(0) = \frac{N'(0)}{1 - P'(0)} \qquad (4.3)$$

Where $P(0)$ == Probability for the test starting from zero on the normal chart,

$N(0)$ = ASN for the test starting from zero on the normal chart,

$P'(0)$ = Probability for the test on the return chart and

$N'(0)$ = ASN for the test on the return chart.

He further obtained integral equations for the quantities $P(0)$, $N(0)$, $P'(0)$, $N'(0)$ as follows :

$$P(z) = F(k-z) + \int_0^h p(y) f(y+k-z) dy, \quad \ldots.. \tag{4.4}$$

$$N(z) = 1 + \int_0^h N(y) f(y+k-z) dy, \quad \ldots.. \tag{4.5}$$

$$P'(z) = \int_{k+z}^{B} f(y) dy + \int_0^h P'(y) f(-y+k+z) dy \cdots\cdots \tag{4.6}$$

$$\left.\begin{aligned} N'(z) &= 1 + \int_0^h N'(y) f(-y+k+z) dy, \\ F(x) &= 1 + \int_A^h f(x) dx : \\ F(k-z) &= 1 + \int_A^{k-z} f(y) dy, \end{aligned}\right\} \ldots\ldots. \tag{4.7}$$

and z is the distance of the starting of the test in the normal chart from zero.

## V. Method of Solution

In this section, the procedure of the method of solution studied by Narayana Murthy et al [06] was considered to evaluate the integral equation (4.4) in Narayana Murthy et al [06 ]

The main objective of this paper is to obtain solutions of ARL and Type- C OC curves for various CASP-CUSUM charts by assuming that the $X_i$'s follows truncated Erlangian distribution

There are several methods to solve integral equations given from (4.4) to (4.7). Here we have considered a method suggested by Krishna Murthy, E.B. Sen, S.K. **[05]** to obtain the numerical solution for P (A). The method briefly as follows:

We first express the integral equation as:

$$F(x) = Q(x) + \int_c^d R(x,t) F(t)\, dt \tag{5.1}$$

Where $F(x) = P(z)$

$Q(x) = F(k-z)$

$R(x,t) = f(y+k-z)$

Let the integral I = $\int_c^d f(x)\, dx$ be transformed to

$$I = \frac{d-c}{2} \int_c^d f(y)\, dy = \frac{d-c}{2} \sum a_i\, f(t_i) \tag{5.2}$$

Where y = $\dfrac{2x-(c-d)}{d-c}$ where ai's and $t_i$ 's respectively the weight factor and abscissa for the Gauss-Chebyshev polynomial given M.K. Jain and et al **[04]** using (4.1) and (4.2), (4.4) can be written as

$$F(x) = Q(x) \quad \frac{d-c}{2} \sum a_i\, R(x,t_i) F(t_i) \tag{5.3}$$

Since equation (5.3) should be valid for all values of x in the interval (c,d), it must be true for x = $t_i$ . i = 0 (1) n then obtain

$$F(t_i) = Q(t_i) + \frac{d-c}{2} \sum a_i\, R(t_j, t_i) F(t_i) \tag{5.4}$$

J = 0 (1) n

Substituting

$F(t_i) = F_i$, $Q(t_i) = Q_i$, i – 0 (1) n, in (2.4.4), we get

$$F_o = Q_o + \frac{d-c}{2}\left[a_o R(t_o,t_o) F_o + a_1 R(t_0,t_1) F_1 + \ldots. a_n R(t_0,t_n) F_n\right]$$

$$F_1 = Q_1 + \frac{d-c}{2}\left[a_o R(t_1,t_o)F_o + a_1 R(t_1,t_1)F_1 + ....a_n R(t_1,t_n)F_n)\right]$$

…… …… …… ……
…… …… …… ……

$$F_n = Q_n + \frac{d-c}{2}\left[a_o R(t_n,t_o)F_o + a_1 R(t_n,t_1)F_1 + ....a_n R(t_n,t_n)F_n)\right] \qquad (5.5)$$

In the system of equations except $F_i$, i = 0, 1,…………… n are known and hence can be solved for $F_i$. We solve the system of equations by the method of iteration. For this we write the system (4.5) as

$$\left[1 - Ta_0 R(t_o,t_0)\right]F_0 = Q_0 + T\left[a_o R(t_0,t_o)F_o + a_1 R(t_0,t_1)F_1 + ....a_n R(t_0,t_n)F_n)\right]$$

$$\left[1 - Ta_1 R(t_1,t_1)\right]F_1 = Q_1 + T\left[a_0 R(t_1,t_o)F_o + a_1 R(t_1,t_1)F_1 + ....a_n R(t_1,t_n)F_n)\right]$$

…… …… …… ……
…… …… …… ……

$$\left[1 - Ta_n R(t_n,t_n)\right]F_n = Q_n + T\left[a_o R(t_n,t_o)F_o + a_1 R(t_n,t_1)F_1 + ....a_n R(t_n,t_n)F_n)\right] \qquad (5.6)$$

Where $T = \frac{d-c}{2}$.

To start the Iteration process, let us put $F1 = F2 = ....Fn = 0$ in the first equation of (5.1), we then obtain a rough value of $Fo$. Putting this value of $Fo$ and $F2 = F_3 ....Fn = 0$ in the second equation, we get a rough value $F_1$ and so on. This gives the first set of values $F_i$, i = 0,1,2,………., n which are just the refined values of $Fi$, i = 0,1,2,….., n . The process is continued until two consecutive sets of values are obtained up to a certain degree of accuracy. In the similar way solutions $P'(0), N(0), N'(0)$ can be obtained.

Now we proceed to obtain the results of P (A) for various parametric values in the following section.

## VI. Computation OF P (A) And ARL's

In this Section CASP-CUSUM schemes are obtained when the variable considered follows a Truncated Erlangian distribution and hence (3.1) is to be substituted in equation (2.1) from (4.4) through (4.7). There are several methods to solve integral equations (4.4) to (4.7) when the variable under study distributed an appropriate truncated probability distribution. Therefore in this paper we consider method – I developed by Krishna Murthy and Sen S.K.,**[05]**to obtain the numerical solutions for P (A) and ARL's for CASP-CUSUM Schemes when f (.) is truncated Erlangian probability density function (see equation (3.1)) for various parameters like

i) h = Size of the acceptance zone
ii) h' = Size of the rejection zone
iii) λ, M Parameters of the distribution
iv) B = Truncated point of the random variable X and
v) k = The reference value, are calculated through the computer programmes.

**Numerical Results and Conclusions**

The calculated values are given in the following tables.

| Table - 6.1 :Values Of Arls And Type –C Oc Curves When λ= 0.1 M= 3 k = 0.4 h= 0.25 h'= 0.25 | | | | TABLE - 6.2 VALUES OF ARLs AND TYPE –C OC CURVES When λ=0.1 M= 3 k = 0.4 h= 0 .5 h'= 0.5 | | | |
|---|---|---|---|---|---|---|---|
| B | L(0) | L'(0) | P(A) | B | L(0) | L'(0) | P(A) |
| 2.7 | 1.62076 | 1.14170 | 0.5867097 | 3.3 | 1.88411 | 1.19045 | 0.6128056 |
| 2.6 | 1.71747 | 1.16061 | 0.6087410 | 3.2 | 2.00615 | 1.21102 | 0.6235765 |
| 2.5 | 1.84270 | 1.18387 | 0.6088414 | 3.1 | 2.16127 | 1.23546 | 0.6362801 |
| 2.4 | 2.01042 | 1.21297 | 0.6236067 | 3.0 | 2.36441 | 1.26486 | 0.6514828 |
| 2.3 | 2.24530 | 1.25016 | 0.6423493 | 2.9 | 2.64097 | 1.30075 | 0.6700051 |
| 2.2 | 2.50544 | 1.20886 | 0.6664715 | 2.8 | 3.03799 | 1.34527 | 0.6930893 |
| 2.1 | 3.16874 | 1.36476 | 0.6880612 | 2.7 | 3.65328 | 1.40165 | 0.7227168 |
| 2.0 | 4.26729 | 1.45769 | 0.7453805 | 2.6 | 4.72919 | 1.47483 | 0.7622787 |
| 1.9 | 7.18119 | 1.50642 | 0.8181257 | 2.5 | 7.07538 | 1.57285 | 0.8181306 |
| **1.8** | **35.10541** | **1.82116** | **0.9506816** | **2.4** | **16.03335** | **1.70957** | **0.9036477** |

| TABLE - 6.3 VALUES OF ARLs AND TYPE –C OC CURVES When λ= 0.1 M= 3 k = 0.4 h= 0.75 h'= 0.75 | | | | TABLE - 6.4 VALUES OF ARLs AND TYPE –C OC CURVES when λ= 0.1 M= 3 k = 0.4 h= 1 h'= 1 | | | |
|---|---|---|---|---|---|---|---|
| B | L(0) | L'(0) | P(A) | B | L(0) | L'(0) | P(A) |
| 3.7 | 2.16322 | 1.23512 | 0.6365517 | 4.0 | 2.49342 | 1.28049 | 0.6606987 |
| 3.6 | 2.32063 | 1.25791 | 0.6484851 | 3.9 | 2.70259 | 1.30602 | 0.6741968 |
| 3.5 | 2.52118 | 1.28460 | 0.6624609 | 3.8 | 2.97297 | 1.33571 | 0.6899953 |
| 3.4 | 2.78782 | 1.31620 | 0.6790560 | 3.7 | 3.33526 | 1.37058 | 0.7087488 |
| 3.3 | 3.14594 | 1.35406 | 0.6990982 | 3.6 | 3.84470 | 1.41100 | 0.7212057 |
| 3.2 | 3.66924 | 1.40005 | 0.7238166 | 3.5 | 4.61224 | 1.46181 | 0.7593349 |
| 3.1 | 4.49281 | 1.45688 | 0.7551338 | 3.4 | 5.89587 | 1.52266 | 0.7944797 |
| 3.0 | 5.97286 | 1.52850 | 0.7962375 | 3.3 | 8.46994 | 1.59831 | 0.8412620 |
| 2.9 | 9.39356 | 1.62099 | 0.8528315 | 3.2 | 16.17385 | 1.69447 | 0.9949842 |
| **2.8** | **25.39276** | **1.74419** | **0.9357266** | **3.1** | **180.42842** | **1.81999** | **0.9900137** |

| TABLE - 6.5 VALUES OF ARLs AND TYPE –C OC CURVES when λ= 0.1 M= 3 k =0.6 h= 0 .25 h'= 0.25 | | | | TABLE - 6.6 VALUES OF ARLs AND TYPE –C OC CURVES when λ= 0.1 M= 3 k =0.6 h= 0.5 h'= 0.5 | | | |
|---|---|---|---|---|---|---|---|
| B | L(0) | L'(0) | P(A) | B | L(0) | L'(0) | P(A) |
| 2.7 | 1.64056 | 1.14170 | 0.5896508 | 3.3 | 1.89835 | 1.19045 | 0.6145906 |
| 2.6 | 1.74050 | 1.16061 | 0.5000431 | 3.2 | 2.02244 | 1.21102 | 0.6254731 |
| 2.5 | 1.88007 | 1.18387 | 0.6123390 | 3.1 | 2.18019 | 1.23546 | 0.6382946 |
| 2.4 | 2.04341 | 1.21297 | 0.6275088 | 3.0 | 2.38679 | 1.26486 | 0.6536186 |
| 2.3 | 2.28639 | 1.25015 | 0.6465048 | 2.9 | 2.66810 | 1.30075 | 0.6722609 |
| 2.2 | 2.64876 | 1.29886 | 0.6709763 | 2.8 | 3.07199 | 1.34527 | 0.6954511 |
| 2.1 | 3.24236 | 1.36476 | 0.7037719 | 2.7 | 3.69801 | 1.40165 | 0.7251487 |
| 2.0 | 4.38050 | 1.45769 | 0.7503179 | 2.6 | 4.79291 | 1.47483 | 0.7646950 |
| 1.9 | 7.40223 | 1.59642 | 0.8225933 | 2.5 | 7.18093 | 1.57285 | 0.8203236 |
| **1.8** | **36.49874** | **1.62116** | **0.9524748** | **2.4** | **16.30390** | **1.70957** | **0.9050946** |

At the hypothetical values of the parameters λ,M, k, h and h' given at the top of each table, we computed optimum truncated point B at which P (A) the probability of accepting an item is maximum and also attained ARL's values which represents the acceptance zone $L(0)$ and rejection zone $L'(0)$ values. The values of truncated point B of random variable X, $L(0)$, $L'(0)$ and the values for Type – C OC Curve, i.e. P (A) were given in columns I, II, III and IV respectively.

From Tables 6.1 to 6.6 we made the following conclusions:

- The values of h, h' increases then related values of P (A) decreases. So the sizes of accepted and rejected zones and P (A) are inversely related.
- It could be determined that the value of parameter of Erlangian distribution increases then P (A) is also increases. So, the parameter of Erlangian distribution and P (A) are positively related.
- The reference value of 'K' increases then L (0) increases. So, reference value and L (0) are positively related.
- The values of h, h' increases then related values of L (0) decreases. So the sizes of accepted and rejected zones and L (0) are inversely related.
- From Table 6.1 to 6.6 ,it is observed that the size of the acceptance zone is increases as the reference value K increases.
- By observing the above tables the reference value of K increase then P (A) will also increases. So, reference value and P (A) are positively related.

**Table No -6. 7:** Optimum Values of CASP-CUSUM Schemes

| B | λ | M | k | h | $h'$ | L (0) | $L'(0)$ | P (A) |
|---|---|---|---|---|---|---|---|---|
| 1.8 | 0.1 | 3 | 0.4 | 0.25 | 0.25 | 35.10541 | 1.82116 | 0.9506816 |
| 2.4 | 0.1 | 3 | 0.4 | 0.5 | 0.5 | 16.03335 | 1.70957 | 0.9036477 |
| 2.8 | 0.1 | 3 | 0.4 | 0.75 | 0.75 | 25.39276 | 1.74419 | 0.9357266 |
| **3.1** | **0.1** | **3** | **0.4** | **1** | **1** | **180.42842** | **1.81999** | **0.9900137** |
| 1.8 | 0.1 | 3 | 0.6 | 0.25 | 0.25 | 36.49874 | 1.62116 | 0.9524748 |
| 2.4 | 0.1 | 3 | 0.6 | 0.5 | 0.5 | 16.30390 | 1.70957 | 0.9050946 |

By observing the above table the value of P (A) reaches their maximum i.e. **0.9900137**. We obtain optimum CASP-CUSUM Scheme as

$$\begin{bmatrix} \mathrm{B} = 3.1 \\ \lambda = 0.1 \\ M = 3 \\ \mathrm{k} = 0.4 \\ \mathrm{h} = 0.25 \\ \mathrm{h'} = 0.25 \end{bmatrix}$$

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