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Linear Birth, Death and Migration Processes for Portfolio Management Modelling

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Abstract : *This paper has proposed a bivariate stochastic model for finding the share allocations in risky and non-risky portfolios of an investment business. Linear birth, death and migration processes have been considered for getting the bivariate processes of risky assets and non-risky assets within the portfolio. Joint probability function for number of units in the above said two groups are derived using differential difference equations. Several statistical measures are derived from the developed model and sensitivity analysis is carried out with numerical illustrations for better understanding of the model.*
Keywords - *Stochastic Modelling, Portfolio Management, Linear birth and death processes, Differential-difference equations.*

I. Introduction

The prices of stocks and other traded financial assets can be modeled by stochastic processes such as Brownian motion or, more often, Geometric Brownian Motion (GBM). It is a continuous-time stochastic process in which the logarithm of the randomly varying quantity follows a Wiener process with drift term. It is an important example of stochastic processes satisfying a stochastic differential equation (SDE). It is used in mathematical finance to model stock prices in the Black–Scholes model. The Brownian motion assumption is intended to capture these uncertainties in a simplified way. In mathematical finance, the Itō integral(1944) was introduced to estimate the total worth of the stock at any given time, using how much stock we hold as integrand and the movement of the prices as the integrator and the pronounced assessment tactic is to observe the change in the prices.

The sums traded are enormous – much more than the world GDP. Lévy processes (1948) are used in mainstream mathematical finance since the mid-nineties. Before only very few "exotic" research papers used Lévy processes in connection with finance. Nowadays almost all sophisticated models are based on Lévy processes or processes behaving like Lévy processes locally. The general mathematical theory has been developed to a large extent before 1950. The Black-Scholes model (1973) was unrealistic and very simple model, may lead to disaster since market environment and conditions are not static, i.e., laws and trading rules change over time. The basic idea behind this formula is an arbitrage equilibrium among three assets: stock, bond, and European call option. It is a risk-neutral valuation because investors in their model economy were implicitly assumed to be risk neutral and they are concerned only with maximizing profits. A more realistic, discrete-time formulation of option pricing (by using a binomial tree) was later proposed by William Sharpe, and formalized by Cox-Ross-Rubinstein (1979).

The Cox–Ross–Rubinstein introduced a simple discrete-time model for valuing options. The fundamental economic principles of option pricing by arbitrage methods are particularly clear in this setting. Its development requires only elementary mathematics, yet it contains as a special limiting case of the celebrated Black-Scholes model, which has previously been derived only by much more difficult methods. The basic model readily lends itself to generalization in many ways. Moreover, by its very construction, it gives rise to a simple and efficient numerical procedure for valuing options for which premature exercise may be optimal. The model was built on earlier work on Poisson models by Cox and Ross, and the Cox–Ingersoll–Ross model for the term structure of interest rates. It allowed a better understanding of how derivative securities are priced, as it is based on the fact that investor risk preferences played no part in the pricing of derivatives—the concept of risk-neutral pricing.

The Stochastic Alpha Beta Rho (SABR) model was introduced by Hagan et al (2002), for modeling the volatility surface and to capture the empirically observed dynamic behavior. Bruce Bartlett et al (2006) discussed the delta and vega risks within the SABR stochastic volatility model by Hagan et al. These risks can be hedged more precisely by adding new terms to the formulas contained in the original SABR paper. The effect of these new terms is minimized when one hedges both vega risks and delta risks, but are substantial when only delta is hedged. In the SABR model, one usually specifies the CEV (constant elasticity of variance) exponent and then selects the correlation parameter to match the volatility skew. The delta risk (as specified in the original

SABR paper) then depends on the exponent chosen. With the new term, the delta risk is much less sensitive to the particular value of exponent, and depends mainly on the slope of the implied volatility curve.

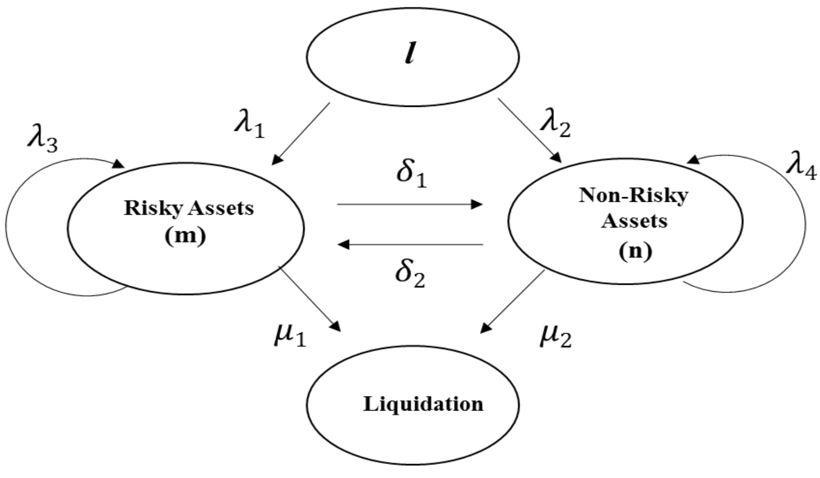
McNeil et al (2007), proposed the Bayesian inference for generalized linear mixed models of portfolio credit risk. Hui Zhao (2012) studied the optimal investment problem for utility maximization with multiple risky assets under the constant elasticity of variance (CEV) model. By applying stochastic optimal control approach and variable change technique, they derived explicit optimal strategy for an investor with logarithmic utility function and analyzed the properties of the optimal strategy. Vasicek (1977) derived a general form of the term structure of interest rates and later studied another popular stochastic processes include mean-reversion. Quantitative Risk Management is highly recommended for financial regulators. The statistical and mathematical tools facilitate a better understanding of the strengths and weaknesses of a useful range of advanced risk-management concepts and models. It consists three main categories of risk in financial markets, namely market risk, credit risk and operational risk. Andersen et al (2009) and Christoffersen et al (2009) developed some models for application to problems in Quantitative Finance on risk management.

The reported literature has given much emphasis on making use of Weiner Processes and Itō-levy processes for understanding the dynamics of financial volatility and risk assessment. However, in the context of financial marketing study, the growth/ loss processes of market behavior over period of time can be studied by observing the behaviour in infinitesimal time interval Δt. The change derivatives have to properly modeled with suitable mathematical tools. Making use of the notion of point processes is more appropriate for studying the price and value dynamics over a period of observation time. The stochastic calculus with differential difference equation approach is a suitable option for formulating the fluctuations of time dependent share values. The investor has to assess the loss and profit on his investment over a period of time‘t’ by analyzing the stochastic processes during ‘Δt’. This thought of approach has motivated the researchers for developing the stochastic models in portfolio management of Risky and non-risky assets. The approach initiated with formulation of differential difference equations and development of differential equations, probability generating functions and derivation of first and second order moments of the stochastic processes.

II. Stochastic Model For Portfolio Theory

In this paper we have proposed a bivariate stochastic model for efficient performance of a portfolio. The developed model is categorized as stochastic model rather than mathematical model. The study has several assumptions and postulates for model construction, development of differential-difference equations, derivation of differential equations as well as statistical measures using bivariate Poisson processes.

The optimum portfolio’s position is always a desire of corresponding investor i.e., it consists of how many shares of which kind at a particular time point the developed model is very advantageous. Here we have considered the investor at any time he/she can invest into the market and at any time he/she can liquefy into cash from the market. And from the liquefied form i.e., cash, if he/she again wants to invest in the share market. Then a Bivariate Poisson process notion we can apply to the Portfolio. The above three transactions, we have considered as the birth and death processes and transmission processes respectively. And whenever the Company issues the bonus shares to the investors, considered as a birth in the process. The following Schematic diagram will explain the model in more detailed way.



Schematic Diagram of the Model
(Portfolio Diversification)

1. Assumptions and Postulates of the model:

Let the events occurred in non-overlapping intervals of time are statistically independent. Let Δt be an infinitesimal interval of time. Initially, let there be 'm' number of shares in the group of Risky shares and 'n' number of shares in the group of Non-Risky shares at time't'. Let λ_1 and λ_2 be the rates of investment of shares from external sources to the groups of Risky Assets and Non-Risky Assets respectively. Let λ_3 and λ_4 be the rates of growth in investment within the groups of Risky Assets and Non-Risky Assets respectively. Let δ_1, δ_2 be the rates of transformation in investments from Risky Assets to Non-Risky Assets and from Non-Risky Assets to Risky Assets respectively. Let μ_1 and μ_2 be the rate of disinvestment of shares per unit time from the groups of Risky Assets and Non-Risky Assets respectively. Further it is assumed that all the above parameters follow Poisson process.

Keeping the above assumptions, the postulates of the model are, the probability of (i) arrival of a share to the group of Risky Assets during Δt through immigration from external sources provided there exits 'l' number of shares at time 't' is $l\lambda_1\Delta t + o(\Delta t)$; (ii) arrival of share to the group of Non-Risky Assets during Δt through immigration from external sources provided there exits 'l' number of shares at time 't' is $l\lambda_2\Delta t + o(\Delta t)$; (iii) growth in investment within the group of Risky Assets, during Δt provided there exits 'm' number of share already in the said group at time 't' is $m\lambda_3\Delta t + o(\Delta t)$; (iv) growth in investment within the group of Non-Risky Assets, during Δt provided there exits 'n' number of share already in the said group at time 't' is $n\lambda_4\Delta t + o(\Delta t)$; (v) transformation of share from the group of Risky Assets to the group of Non-Risky Assets during Δt provided there exits 'm' number of shares are in the Risky Assets group at time 't' is $m\delta_1\Delta t + o(\Delta t)$; (vi) transformation of share from the group of Non-Risky Assets to the group Risky Assets of during Δt provided there exits 'n' number of shares in Non-Risky Assets group at time 't' is $n\delta_2\Delta t + o(\Delta t)$; (vii) withdrawal of share (Liquidation) from the group of Risky Assets during Δt provided there exits 'm' number of shares in the said group at time 't' is $m\mu_1\Delta t + o(\Delta t)$; (viii) withdrawal of share (Liquidation) from the group of Non-Risky Assets during Δt provided there exits 'n' number of shares in the said group at time 't' is $n\mu_2\Delta t + o(\Delta t)$; (ix) no growth in number of shares to the groups of Risky Assets and Non-Risky Assets from external sources, no growth in investment within the group of Risky Assets and Non-Risky Assets, no transformation of shares one group to the other group, no withdrawal of shares from the groups of Risky Assets and Non-Risky Assets during Δt is $1-\{l(\lambda_1+\lambda_2)+m(\lambda_3+\delta_1+\mu_1)+n(\lambda_4+\delta_2+\mu_2)\}.\Delta t + o(\Delta t)$; (x) occurrence of other than the above events during an infinitesimal interval of time is $O(\Delta t)^2$.

III. Differential Equations Of The Model

Let $P_{m,n}(t)$be the probability that there exists 'n' shares in Risky Assets group and 'm' shares in Non-Risky Assets at time 't'. Further let $P_{m,n}(t+\Delta t)$ be the probability that there exists 'n' shares in Risky Assets group and 'm' shares in Non-Risky Assets at time 't+Δt', it may happened as the processes of

$$\begin{aligned} p_{m,n}(t+\Delta t) &= p_{m,n}(t)p_{0,0}(\Delta t) + p_{m,n-1}(t)p_{0,1}(\Delta t) + p_{m-1,n}(t)p_{1,0}(\Delta t) + p_{m,n+1}(t)p_{0,-1}(\Delta t) \\ &\quad + p_{m+1,n}(t)p_{-1,0}(\Delta t) + p_{m+1,n-1}(t)p_{-1,+1}(\Delta t) + p_{m-1,n+1}(t)p_{+1,-1}(\Delta t) + o(\Delta t)^2 \end{aligned}$$

$$\begin{aligned} P_{m,n}(t+\Delta t) &= P_{m,n}(t)[1-\{l(\lambda_1+\lambda_2)+m(\lambda_3+\delta_1+\mu_1)+n(\lambda_4+\delta_2+\mu_2)\}.\Delta t + O(\Delta t)] \\ &\quad + P_{m,n-1}(t)\left\{[l\lambda_2.\Delta t + O(\Delta t)] + [(n-1)\lambda_4.\Delta t + O(\Delta t)]\right\} \\ &\quad + P_{m-1,n}(t)\left\{[l\lambda_1.\Delta t + O(\Delta t)] + [(m-1)\lambda_3.\Delta t + O(\Delta t)]\right\} \\ &\quad + P_{m,n+1}(t)[(n+1)\mu_2.\Delta t + O(\Delta t)] + P_{m+1,n}(t)[(m+1)\mu_1.\Delta t + O(\Delta t)] \\ &\quad + P_{m-1,n+1}(t)[(n+1)\delta_2.\Delta t + O(\Delta t)] + P_{m+1,n-1}(t)[(m+1)\delta_1.\Delta t + O(\Delta t)] \\ &\quad + P_{m\pm i,n\pm i}(t)[O(\Delta t)^2] \quad for\ i \ge 2 \end{aligned}$$

$$\begin{aligned} P'_{m,n} &= -\{l(\lambda_1+\lambda_2)+m(\lambda_3+\delta_1+\mu_1)+n(\lambda_4+\delta_2+\mu_2)\}P_{m,n}(t) + [l\lambda_2+(n-1)\lambda_4]P_{m,n-1}(t) \\ &\quad + [l\lambda_1+(m-1)\lambda_3]P_{m-1,n}(t) + [(n+1)\mu_2]P_{m,n+1}(t) + [(m+1)\mu_1]P_{m+1,n}(t) \\ &\quad + [(n+1)\delta_2]P_{m-1,n+1}(t) + [(m+1)\delta_1]P_{m+1,n-1}(t), \quad for\ m,n \ge 1 \qquad ...(3.1.1) \end{aligned}$$

$$P_{0,0}(t+\Delta t)=P_{0,0}(t)[1-\{l(\lambda_1+\lambda_2)+0(\lambda_3+\delta_1+\mu_1)+0(\lambda_4+\delta_2+\mu_2)\}\Delta t+O(\Delta t)]$$
$$+P_{0,1}(t)\{1.\mu_2.\Delta t+O(\Delta t)\}+P_{1,0}(t)\{1.\mu_1.\Delta t+O(\Delta t)\}$$

$$P'_{0,0}(t)=\left[-\{l(\lambda_1+\lambda_2)\}\right]P_{0,0}(t)+\mu_2.P_{0,1}(t)+\mu_1.P_{1,0}(t) \qquad ...(3.1.2)$$

$$P_{1,0}(t+\Delta t)=P_{1,0}(t)[1-\{l(\lambda_1+\lambda_2)+1.(\lambda_3+\delta_1+\mu_1)+0.(\lambda_4+\delta_2+\mu_2)\}.\Delta t+O(\Delta t)]$$
$$+P_{1,0}(t)\{1.\mu_2.\Delta t+O(\Delta t)\}+P_{2,0}(t)\{2.\mu_1.\Delta t+O(\Delta t)\}+P_{1,0}(t)\{1.\delta_2.\Delta t+O(\Delta t)\}$$

$$P'_{1,0}(t)=\left[-\{l(\lambda_1+\lambda_2)+(\lambda_3+\delta_1+\mu_1)\}\right]P_{1,0}(t)+\mu_2P_{1,1}(t)+2\mu_1P_{2,0}(t)+\delta_2P_{0,1}(t) \quad ...(3.1.3)$$

$$P_{0,1}(t+\Delta t)=P_{0,1}(t)[1-\{l(\lambda_1+\lambda_2)+0(\lambda_3+\delta_1+\mu_1)+1(\lambda_4+\delta_2+\mu_2)\}\Delta t+O(\Delta t)]$$
$$+P_{1,1}(t)\{1\mu_1\Delta t+O(\Delta t)\}+P_{1,0}(t)\{1\delta_1.\Delta t+O(\Delta t)\}+P_{0,2}(t)\{2\mu_2\Delta t+O(\Delta t)\}$$

$$P'_{0,1}(t)=\{-l(\lambda_1+\lambda_2)+(\lambda_4+\delta_2+\mu_2)\}P_{0,1}(t)+\mu_1P_{1,1}(t)+\delta_1P_{1,0}(t)+2(\mu_2P_{0,2}(t) \quad ...(3.1.4)$$

With the initial condition

$P_{M_oN_o}(t)=1,\ P_{i,j}(0)=0\ \forall\, i\neq M_o,\ j\neq N_o$

Where M_o Risky shares and N_o Non-Risky Shares in the Portfolio.

Let $P(x,y;t);$ be the joint probability generating function of

$P_{m,n}(t);$ Where $P(x,y;t)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}x^m y^n P_{m,n}(t)$

Multiplying the equations (3.1) to (3.4) with $x^m y^n$ and summing overall m and n, we obtain

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}x^m y^n P'_{m,n}(t)$$
$$=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}-\left[l(\lambda_1+\lambda_2)\right]x^m y^n P_{m,n}(t)+\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}-\left[m(\lambda_3+\delta_1+\mu_1)\right]x^m y^n P_{m,n}(t)$$
$$+\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}-\left[n\lambda_4+\delta_2+\mu_2)\right]x^m y^n P_{m,n}(t)+\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[l\lambda_2+(n-1)\lambda_2\right]x^m y^n P_{m,n-1}(t)]$$
$$+\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[l\lambda_1+(m-1)\lambda_3\right]x^m y^n P_{m-1,n}(t)+\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[(n+1)\mu_2\right]x^m y^n P_{m,n+1}(t)$$
$$+\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[(m+1)\mu_1\right]x^m y^n P_{m+1,n}(t)+\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[(n+1)\delta_2\right]x^m y^n P_{m-1,n+1}(t)$$
$$+\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[(m+1)\delta_1\right]x^m y^n P_{m+1,n-1}(t)$$
$$=\left[-l(\lambda_1+\lambda_2)\right]P(x,y;t)+\left[-(\lambda_3+\delta_1+\mu_1)\right]x\frac{\partial P}{\partial x}+\left[-(\lambda_4+\delta_2+\mu_2)\right]y\frac{\partial P}{\partial y}$$
$$+\left[l\lambda_2\right]yP(x,y;t)+\left[\lambda_4\right]y^2\frac{\partial P}{\partial y}+\left[l\lambda_1\right]xP(x,y;t)+\left[\lambda_3\right]x^2\frac{\partial P}{\partial x}+\left[\mu_2\right]\frac{\partial P}{\partial y}$$
$$+\left[\mu_1\right]\frac{\partial P}{\partial x}+\left[\delta_2\right]x\frac{\partial P}{\partial y}+\left[\delta_1\right]y\frac{\partial P}{\partial x}$$

$$\frac{dP(x,y;t)}{dt}=\left[\mu_1-(\lambda_3+\delta_1+\mu_1)x+\delta_1y+\lambda_3x^2\right]\frac{\partial P}{\partial x}+\left[\mu_2-(\lambda_4+\delta_2+\mu_2)y+\delta_2x+\lambda_4y^2\right]\frac{\partial P}{\partial y}$$
$$+\left[l\lambda_1(x-1)+l\lambda_2(y-1)\right]P(x,y;t)$$

Statistical Measures:

We can obtain the characteristics of the model by using the joint c.g.f. of as below

Expected Number of Risky shares

$$m_{1,0}(t) = e^{-(\mu_1+\delta_1-\lambda_3)t} N_o \qquad \text{... (3.2.1)}$$

No is initial number of shares in the risky group

Expected Number of shares in the Non-Risky Group is

$$m_{0,1}(t) = e^{-(\mu_2+\delta_2-\lambda_4)t} M_o \qquad \text{... (3.2.2)}$$

Mo is initial number of shares in the Non-risky group

Variance of number of Risky shares is

$$m_{2,0}(t) = (\mu_1+\delta_1+\lambda_3) N_o \left\{ \frac{e^{-(\mu_1+\delta_1-\lambda_3)t}}{(\mu_1+\delta_1-\lambda_3)} \right\} + e^{-2(\mu_1+\delta_1-\lambda_3)t} \qquad \text{... (3.2.3)}$$

Variance of Number of Non-Risky Shares is

$$m_{0,2}(t) = (\mu_2+\delta_2+\lambda_4) M_o \left\{ \frac{e^{-(\mu_2+\delta_2-\lambda_4)t}}{(\mu_2+\delta_2-\lambda_4)} \right\} + e^{-2(\mu_2+\delta_2-\lambda_4)t} \qquad \text{... (3.2.4)}$$

Covariance between number of Risky shares and number of Non-Risky shares is

$$m_{1,1}(t) = e^{-(\mu_1+\mu_2+\delta_1+\delta_2-\lambda_3-\lambda_4)t} \qquad \text{... (3.2.5)}$$

IV. Numerical Illustration And Sensitivity Analysis

In order to understand the model behavior on more detailed way, simulated numerical data sets were obtained from equations (3.2.1) to (3.2.5). The values of $m_{1,0}(t)$, $m_{0,1}(t)$, $m_{2,0}(t)$, $m_{0,2}(t)$ and $m_{1,1}(t)$ are computed for different values of λ_3, λ_4, δ_1, δ_2, μ_1, μ_2, N_0 and M_0 when the remaining are constant and presented in Table –3.1.

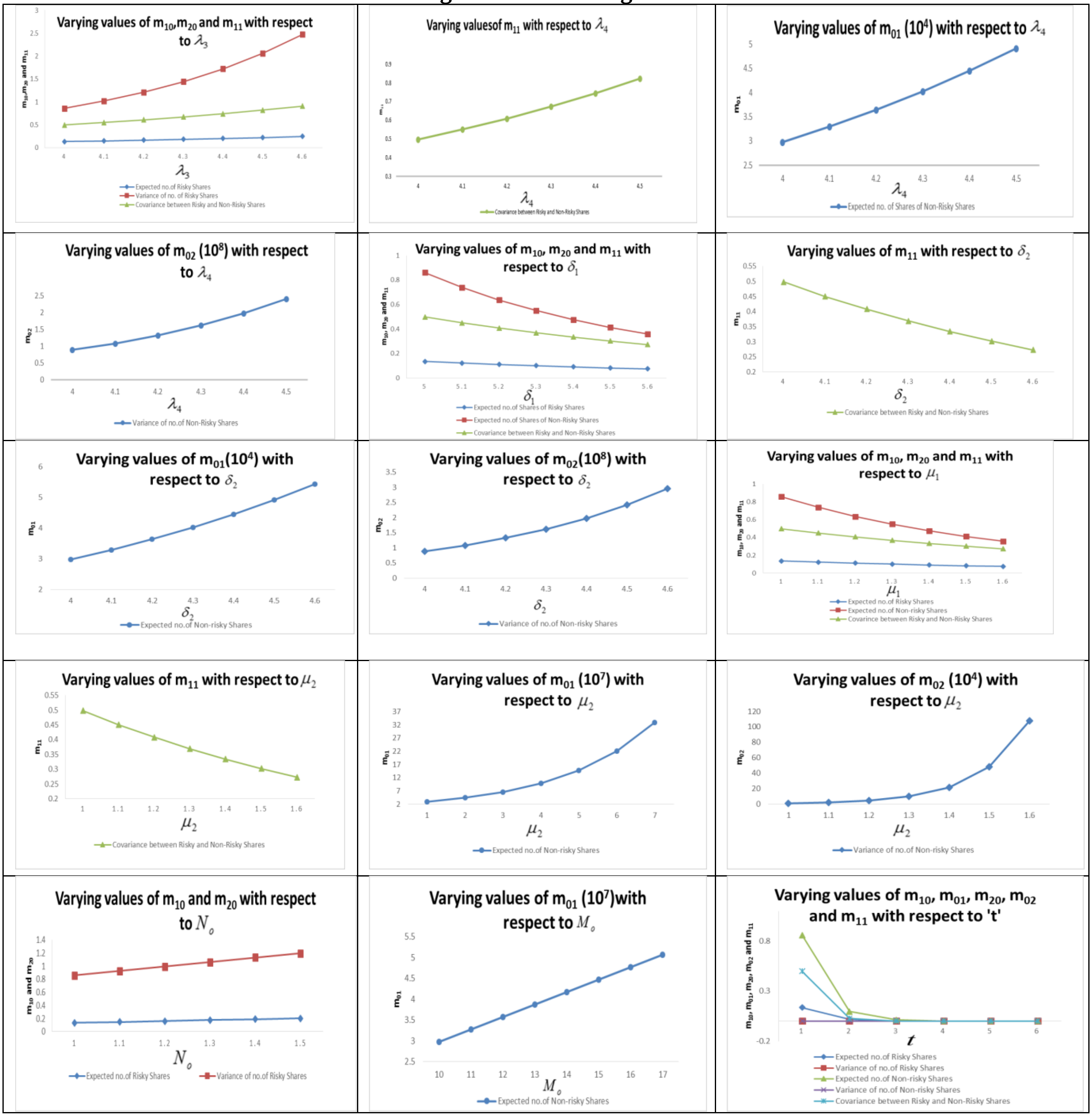
Table –3.1 : Values of m_{10}, m_{01}, m_{20}, m_{02}, and m_{11} for varying values of one among λ_3, λ_4, δ_1, δ_2, μ_1, μ_2, N_0 and M_0 when the remaining are constant.

λ_3	λ_4	δ_1	δ_2	μ_1	μ_2	N_o	M_o	t	m_{10}	m_{01}	m_{20}	m_{02}	m_{11}
4	4	5	4	1	1	1	10	1	0.135	$2.981*10^4$	0.86	$8.883*10^7$	0.49
4.1	4	5	4	1	1	1	10	1	0.15	$2.981*10^4$	1.019	$8.883*10^7$	0.55
4.2	4	5	4	1	1	1	10	1	0.165	$2.981*10^4$	1.21	$8.883*10^7$	0.60
4.3	4	5	4	1	1	1	10	1	0.183	$2.981*10^4$	1.441	$8.883*10^7$	0.67
4.4	4	5	4	1	1	1	10	1	0.202	$2.981*10^4$	1.72	$8.883*10^7$	0.74
4.5	4	5	4	1	1	1	10	1	0.223	$2.981*10^4$	2.06	$8.883*10^7$	0.82
4.6	4	5	4	1	1	1	10	1	0.247	$2.981*10^4$	2.475	$8.883*10^7$	0.90
4	**4**	5	4	1	1	1	10	1	0.135	$2.981*10^4$	0.86	$8.883*10^7$	0.49
4	**4.1**	5	4	1	1	1	10	1	0.135	$3.294*10^4$	0.86	$1.085*10^8$	0.55
4	**4.2**	5	4	1	1	1	10	1	0.135	$3.641*10^4$	0.86	$1.325*10^8$	0.60
4	**4.3**	5	4	1	1	1	10	1	0.135	$4.024*10^4$	0.86	$1.619*10^8$	0.67
4	**4.4**	5	4	1	1	1	10	1	0.135	$4.447*10^4$	0.86	$1.977*10^8$	0.74
4	**4.5**	5	4	1	1	1	10	1	0.135	$4.915*10^4$	0.86	$2.415*10^8$	0.82
4	4	**5**	4	1	1	1	10	1	0.135	$2.981*10^4$	0.86	$8.883*10^7$	0.49
4	4	**5.1**	4	1	1	1	10	1	0.122	$2.981*10^4$	0.739	$8.883*10^7$	0.45
4	4	**5.2**	4	1	1	1	10	1	0.111	$2.981*10^4$	0.636	$8.883*10^7$	0.40
4	4	**5.3**	4	1	1	1	10	1	0.1	$2.981*10^4$	0.55	$8.883*10^7$	0.36
4	4	**5.4**	4	1	1	1	10	1	0.091	$2.981*10^4$	0.475	$8.883*10^7$	0.33
4	4	**5.5**	4	1	1	1	10	1	0.082	$2.981*10^4$	0.412	$8.883*10^7$	0.30
4	4	**5.6**	4	1	1	1	10	1	0.074	$2.981*10^4$	0.358	$8.883*10^7$	0.27
4	4	5	**4**	1	1	1	10	1	0.135	$2.981*10^4$	0.86	$8.883*10^7$	0.49
4	4	5	**4.1**	1	1	1	10	1	0.135	$3.294*10^4$	0.86	$1.085*10^8$	0.45
4	4	5	**4.2**	1	1	1	10	1	0.135	$3.641*10^4$	0.86	$1.325*10^8$	0.40

Table –3.1 : Values of m_{10}, m_{01} ,m_{20}, m_{02}, and m_{11} for varying values of one among λ_3, λ_4, δ_1,δ_2, μ_1, μ_2, N_0 and M_0 when the remaining are constant.

λ_3	λ_4	δ_1	δ_2	μ_1	μ_2	N_o	M_o	**t**	m_{10}	m_{01}	m_{20}	m_{02}	m_{11}
4	4	5	**4.3**	1	1	1	10	1	0.135	$4.024*10^{4}$	0.86	$1.619*10^{8}$	0.36
4	4	5	**4.4**	1	1	1	10	1	0.135	$4.447*10^{4}$	0.86	$1.977*10^{8}$	0.33
4	4	5	**4.5**	1	1	1	10	1	0.135	$4.915*10^{4}$	0.86	$2.415*10^{8}$	0.30
4	4	5	**4.6**	1	1	1	10	1	0.135	$5.432*10^{4}$	0.86	$2.95*10^{8}$	0.27
4	4	5	4	**1**	1	1	10	1	0.135	$2.981*10^{4}$	0.86	$8.883*10^{7}$	0.49
4	4	5	4	**1.1**	1	1	10	1	0.122	$2.981*10^{4}$	0.739	$8.883*10^{7}$	0.45
4	4	5	4	**1.2**	1	1	10	1	0.111	$2.981*10^{4}$	0.636	$8.883*10^{7}$	0.40
4	4	5	4	**1.3**	1	1	10	1	0.1	$2.981*10^{4}$	0.55	$8.883*10^{7}$	0.36
4	4	5	4	**1.4**	1	1	10	1	0.091	$2.981*10^{4}$	0.475	$8.883*10^{7}$	0.33
4	4	5	4	**1.5**	1	1	10	1	0.082	$2.981*10^{4}$	0.412	$8.883*10^{7}$	0.30
4	4	5	4	**1.6**	1	1	10	1	0.074	$2.981*10^{4}$	0.358	$8.883*10^{7}$	0.27
4	4	5	4	1	**1**	1	10	1	0.135	$2.981*10^{4}$	0.86	$8.883*10^{7}$	0.49
4	4	5	4	1	**1.1**	1	10	1	0.135	$4.447*10^{4}$	0.86	$1.977*10^{8}$	0.45
4	4	5	4	1	**1.2**	1	10	1	0.135	$6.634*10^{4}$	0.86	$4.401*10^{8}$	0.40
4	4	5	4	1	**1.3**	1	10	1	0.135	$9.897*10^{4}$	0.86	$9.794*10^{4}$	0.36
4	4	5	4	1	**1.4**	1	10	1	0.135	$1.476*10^{5}$	0.86	$2.18*10^{9}$	0.33
4	4	5	4	1	**1.5**	1	10	1	0.135	$2.203*10^{5}$	0.86	$4.851*10^{9}$	0.30
4	4	5	4	1	**1.6**	1	10	1	0.135	$3.286*10^{5}$	0.86	$1.08*10^{10}$	0.27
4	4	5	4	1	1	**1**	10	1	0.135	$2.981*10^{4}$	0.86	$8.883*10^{7}$	0.49
4	4	5	4	1	1	**1.1**	10	1	0.149	$2.981*10^{4}$	0.928	$8.883*10^{7}$	0.49
4	4	5	4	1	1	**1.2**	10	1	0.162	$2.981*10^{4}$	0.995	$8.883*10^{7}$	0.49
4	4	5	4	1	1	**1.3**	10	1	0.176	$2.981*10^{4}$	1.063	$8.883*10^{7}$	0.49
4	4	5	4	1	1	**1.4**	10	1	0.189	$2.981*10^{4}$	1.131	$8.883*10^{7}$	0.49
4	4	5	4	1	1	**1.5**	10	1	0.203	$2.981*10^{4}$	1.198	$8.883*10^{7}$	0.49
4	4	5	4	1	1	1	**10**	1	0.135	$2.981*10^{4}$	0.86	$8.883*10^{7}$	0.49
4	4	5	4	1	1	1	**11**	1	0.135	$3.279*10^{4}$	0.86	$8.883*10^{7}$	0.49
4	4	5	4	1	1	1	**12**	1	0.135	$3.577*10^{4}$	0.86	$8.883*10^{7}$	0.49
4	4	5	4	1	1	1	**13**	1	0.135	$3.875*10^{4}$	0.86	$8.883*10^{7}$	0.49
4	4	5	4	1	1	1	**14**	1	0.135	$4.173*10^{4}$	0.86	$8.883*10^{7}$	0.49
4	4	5	4	1	1	1	**15**	1	0.135	$4.471*10^{4}$	0.86	$8.883*10^{7}$	0.49
4	4	5	4	1	1	1	**16**	1	0.135	$4.77*10^{4}$	0.86	$8.883*10^{7}$	0.49
4	4	5	4	1	1	1	**17**	1	0.135	$5.068*10^{4}$	0.86	$8.883*10^{7}$	0.49
4	4	5	4	1	1	1	10	**1**	0.135	$2.981*10^{4}$	0.86	$8.883*10^{7}$	0.49
4	4	5	4	1	1	1	10	**2**	0.018	$8.886*10^{7}$	0.095	$7.896*10^{1}$	0.02
4	4	5	4	1	1	1	10	**3**	2.479	$2.649*10^{11}$	0.012	$7.017*10^{2}$	1.23
4	4	5	4	1	1	1	10	**4**	3.355	$7.896*10^{14}$	1.678	$6.235*10^{2}$	6.14
4	4	5	4	1	1	1	10	**5**	4.54*	$2.345*10^{18}$	2.27*	$5.541*10^{3}$	3.05
4	4	5	4	1	1	1	10	**6**	6.144	$7.017*10^{21}$	3.072	$4.923*10^{4}$	1.52

V. FIgures And Findings



The changing patterns of statistical measures with respect to the study parameters are presented in the Table 3.1 and the following findings are observed.

- $m_{0,1}(t), m_{0,2}(t)\, and\ m_{1,1}(t)$ are invariant and, $m_{1,0}(t)\, and\, m_{2,0}(t)$ increasing functions of initial size of Risky shares N_o.
- $m_{1,0}(t), m_{2,0}(t), m_{0,2}(t)\, and\, m_{1,1}(t)$ is invariant and $m_{0,1}(t)$ is increasing functions of initial size of Non-Risky shares M_o.
- $m_{0,1}(t)\, and\, m_{0,2}(t)$ are invariant and, $m_{1,0}(t), m_{2,0}(t)\, and\ m_{1,1}(t)$ are increasing functions of growth (arrival) rate of Risky Shares λ_3

- $m_{1,0}(t)\, and\, m_{2,0}(t)$ are invariant and $m_{0,1}(t), m_{0,2}(t)\, and\ m_{1,1}(t)$ are increasing functions of growth (arrival) rate of Non-Risky shares λ_4
- $m_{0,1}(t)\, and\, m_{0,2}(t)$ are invariant and $m_{1,0}(t), m_{2,0}(t)\, and\ m_{1,1}(t)$ are decreasing functions of loss rate of Risky shares μ_1 .
- $m_{1,0}(t)\, and\, m_{2,0}(t)$ are invariant, $m_{0,1}(t)\, and\, m_{0,2}(t)$ are increasing functions, and $m_{1,1}(t)$ is decreasing function of loss rate of Non-Risky shares μ_2 .
- $m_{0,1}(t)\, and\, m_{0,2}(t)$ are invariant, $m_{1,0}(t), m_{2,0}(t)\, and\ m_{1,1}(t)$ are decreasing functions of transformation rate of Risky Shares to Non-Risky shares δ_1 .
- $m_{1,0}(t)\, and\, m_{2,0}(t)$ are invariant, $m_{0,2}(t)\, and\ m_{1,1}(t)$ are increasing functions and $m_{1,1}(t)$ is decreasing function of transformation rate of Non-Risky shares to Risky Shares δ_2 .
- $m_{0,1}(t)\, and\, m_{0,2}(t)$ are increasing functions and $m_{1,0}(t), m_{2,0}(t)\, and\ m_{1,1}(t)$ are decreasing functions of time t.

The above findings are describing the dynamics of the measures derived from the developed stochastic model.

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