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Comparison of He's variational iteration and Taylor Expansion Methods for the Solutions of Fractional Integro-Differential Equations

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Abstract : *In this paper, we investigate the numerical solution of fractional integro-differential equations by comparison between He's variational iteration method and taylor expansion method. The fractional derivative is described in the Caputo sense. Some numerical examples are presented to illustrate the methods.*
Keywords *- Fractional integro-differential equations, He's variational iteration method, Taylor expansion method, Caputo fractional derivative, Riemann-Liouville.*

I. Introduction

In this paper will be taken the fractional integro-differential equations with a Caputo fractional derivative of the type

$$D_*{}^{\alpha} y(t) = p(t)y(t) + g(t) + \lambda \int_0^t k(t,\tau)F(y(\tau))d\tau \qquad (1.1)$$

For $t \in [0,1]$. With the initial conditions

$$y^{(i)} = \delta_i. \qquad i = 0,1,2,\dots,n-1, \quad n-1 < \alpha \le n, \quad n \in N. \qquad (1.2)$$

Where $g \in L^2([0,1])^2$ are known functions, $y(t)$ is the unknown function, $D_*{}^{\alpha}$ is the Caputo fractional differential operator of order α. Such kinds of equations arise in the mathematical modeling of various physical phenomena, such as heat conduction in materials with memory. Moreover, this equation is encountered in combined conduction, convection and radiation problem [8,22]. So far, fractional calculus as well as fractional differential equations has received increasing attention in recent years. The existence and uniqueness of solutions to fractional differential equations have been investigated in [3,9,11]. In addition, when $\alpha \in N$, equation (1.1) reduced to linear integro-differential equation and the numerical methods for this equation have been extensively studied by many authors [10,15]. There are many methods for seeking approximate solutions such as a Taylor expansion method, Adomian decomposition method, Variational and He's variational iteration method, Collocation method, Wavelet Galerklin, Legendre wavelet, and Lagrange interpolation method,see[2,5,4,12,18].The outline of this paper is as follows: In section 2, we present some definitions. Section 3, contains the application of He's variational iteration method. Section 4, contains the application of Taylor expansion method. Finally, Sec.5 devoted to illustrate some numerical examples.

II. Some Definitions And Notations

Definition 2.1 A real function $f(x),\ x > 0$, is said to be in the space $C_\mu,\ \mu \in R$ if there exists a real number $(p > \mu)$,such that $f(x) = x^p f_1(x)$, where $f_1(x) \in C[0,\infty)$, and it is said to be in the space C_μ^m, iff $f^{(m)} \in C_\mu, m \in N$.

Definition 2.2. [9] D^α (α is real) denotes to the fractional differential operator of order α in the sense of Riemann-Liouville,defined by

$$D^\alpha f(t) = \begin{cases} \dfrac{1}{\Gamma(n-\alpha)} \dfrac{d^n}{dt^n} \displaystyle\int_0^t \dfrac{f(x)}{(x-t)^{\alpha-n+1}} dx, 0 \le n-1 < \alpha \le n \\ \dfrac{d^n f(t)}{dt^n}, \alpha = n \in N \end{cases} \qquad (2.1)$$

Definition 2.3. [9] I^α denotes to the fractional integral operator of order α in the sense of Riemann-Liouville,defined by

$$I^\alpha f(t) = D^{-\alpha} f(t)$$

Such that

$$I^{\alpha}f(t)=\begin{cases}\dfrac{1}{\Gamma(\alpha)}\displaystyle\int_0^t \frac{f(x)}{(x-t)^{1-\alpha}}\,dx\\ f(t),\alpha=0\end{cases} \tag{2.2}$$

Definition 2.4. [9] Let $f\in C_{-1}^{n}, n\in N$. Then the Caputo fractional derivative of $f(t)$, defined by

$$D_{*}^{\alpha}f(t)=\begin{cases}\dfrac{1}{\Gamma(n-\alpha)}\displaystyle\int_0^t \frac{f^{(n)}(x)}{(x-t)^{\alpha-n+1}}dx, 0\le n-1<\alpha\le n\\ \dfrac{d^n f(t)}{dt^n}, \alpha=n\in N\end{cases} \tag{2.3}$$

some basic properties of fractional operator [9] for $f\in C_{\alpha}, \alpha\ge -1, \mu\ge 1, \eta\ge 0, \beta>-1, \delta\ge 0$:

I. $I^{\mu}\in C_0$.
II. $I^{\eta}I^{\delta}f(x)=I^{\eta+\delta}f(x)=I^{\delta}I^{\eta}f(x)$.
III. $D^{\eta}D^{\delta}f(x)=D^{\eta+\delta}f(x)$.
IV. $D^{\delta}I^{\delta}f(x)=f(x)$.
V. $I^{\delta}D_{*}^{\delta}f(x)=f(x)-\sum_{k=0}^{n-1}\sum f^{(k)}(0^{+})\frac{t^k}{k!},\ \ 0\le n-1<\alpha\le n$.
VI. $I^{\delta}x^{\beta}=\frac{\Gamma(\beta+1)}{\Gamma(\delta+\beta+1)}x^{\delta+\beta}, x>0$
VII. $D^{\delta}x^{\beta}=\frac{\Gamma(\beta+1)}{\Gamma(\beta+1-\delta)}x^{\beta-\delta}, x>0$.
VIII. $D^{\delta}1=\frac{t^{-\delta}}{\Gamma(1-\delta)}$.
IX. $D^{\delta}c=\left(\frac{t^{-\delta}}{\Gamma(1-\delta)}\right)c$.
X. $I^{0}f(x)=f(x)$.

III. He's Variational Iteration Metho

In this section, we first present a brief review of he's variational iteration method [10]. then we will propose the reliable modification of the vim [2] for solving fractional integro-differential equations with the nonlocal boundary conditions by constructing an initial trial-function without unknown parameters.

Here we consider the following fractional functional equation

$$Lu+Ru+Nu=g(x) \tag{3.1}$$

where L is the fractional order derivative , R is a linear differential operator, N represents the nonlinear terms, and g is the source term. By applying the inverse operator L_x^{-1} to both sides of (3.1), and using the given conditions, we obtain

$$u=f-L_x^{-1}[Ru]-L_x^{-1}[Nu], \tag{3.2}$$

where the function f represents the terms arising from integrating the source term g and from using the given conditions, all are assumed to be prescribed. The basic character of He's variational iteration method is the construction of a correction functional for (3.1), which reads

$$u_{n+1}(x)=u_n(x)+\int_0^x \lambda(s)\left[Lu_n(s)+R\tilde{u}_n(s)+N\tilde{u}_n(s)-g(s)\right]ds \tag{3.3}$$

where λ is a Lagrange multiplier which can be identified optimally via variational theory [10], u_n is the nth approximate solution, and $\tilde{u}_n$ denotes a restricted variation, i.e. $\delta\tilde{u}_n=0$.

To solve (3.1) by He's VIM, we first determine the Lagrange multiplier λ that will be identified optimally via integration by parts. Then the successive approximations $u_n(x), n\ge 0$, of the solution $u(x)$ will be readily obtained upon using the obtained Lagrange multiplier and by using any selective function u_0. The approximation u_0 may be selected by any function that just satisfies at least the initial and boundary conditions.

With determined λ, then several approximations $u_n(x), n \geq 0$, follow immediately. Consequently, the exact solution may be obtained by using

$$\lim_{n\to\infty} u_n(x) = u(x)\,. \tag{3.4}$$

In summary, we have the following variational iteration formula for (3.2)

$$\begin{cases} u_0(x) \text{ is an arbitrary initial guess,} \\ u_{n+1}(x) = u_n(x) + \int_0^x \lambda(s)[Lu_n(s) + Ru_n(s) + Nu_n(s) - g(s)]ds \end{cases} \tag{3.5}$$

or equivalently, for (3.2):

$$\begin{cases} u_0(x) \text{ is an arbitrary initial guess,} \\ u_{n+1}(x) = f(x) - L_x^{-1}[Ru_n(x)] - L_x^{-1}[\mathrm{N}u_n(x)] \end{cases} \tag{3.6}$$

where the multiplier Lagrange λ, has been identified.

It is important to note that He's VIM suggests that the u_0 usually defined by a suitable trial-function with some unknown parameters or any other function that satisfies at least the initial and boundary conditions. This assumption made by [2, 11] and others will be slightly varied, as will be seen in the discussion.

The MVIM, that was introduced by Ghorbani et al [2], can be established based on the assumption that the function $f(x)$ of the iterative relation (3.6) can be divided into two parts, namely $f_0(x)$ and $f_1(x)$. Under this assumption, we set

$$f(x) = f_0(x) + f_1(x) \tag{3.7}$$

According to the assumption, (3.7), and by the relationship(3.6), we construct the following variational iteration formula

$$\begin{cases} u_0(x) = f_0(x) \\ u_1(x) = f(x) - L^{-1}{}_x[Rf_0(x)] - L^{-1}{}_x[\mathrm{N}f_0(x)] \\ u_{n+1}(x) = f(x) - L^{-1}{}_x[Ru_n(x)] - L^{-1}{}_x[\mathrm{N}u_n(x)] \end{cases} \tag{3.8}$$

where the multiplier Lagrange λ, has been identified. Here, a proper selection was proposed for the components $u_0(x)$ and $u_1(x)$. The suggestion was that only the part $f_0(x)$ be assigned to the zeroth component i.e. u_0. An important observation that can be made here is that the success of the proposed method depends mainly on the proper choice of the functions f_0 and f_1. As will be seen from the examples below, this selection of u_0 will result in a reduction of the computational work and accelerate the convergence. Furthermore, this proper selection of the components u_0 and u_1 may provide the solution by using one iteration only. To give a clear overview of the content of this study, we have chosen several fractional integro-differential equations with the nonlocal boundary conditions.

IV. Taylor Expansion Method

Consider the equation (1.1) with initial condition (1.2). First, integrate both sides of the equation (1.1) with respect to t for n times. Using Def. (2.3) as follows:

$$\begin{aligned} &\int_0^t \frac{(t-x)^{n-\alpha-1}}{\Gamma(n-\alpha)} y(x)\,dx \\ &\quad = \int_0^t \frac{(t-x)^{n-1} p(x)}{(n-1)!} y(x)\,dx + \int_0^t \frac{(t-x)^{n-1} f(x)}{(n-1)!}\,dx \\ &\quad + \frac{1}{(n-1)!}\int_0^t y(x) \int_x^t k(s,x)\,(s-x)^{n-1} ds\,dx + Q_n(t) \end{aligned} \tag{4.1}$$

Next, Assume that $y \in C^{m+1}[0,1]$ can be represented as Taylor expansion order as follows

$$y(x) = y(t) + y'(t) + y^{(m)}(t)\frac{(x-t)^m}{m!} + y^{(m+1)}(t)\frac{(\eta(x))}{(m+1)!}(x-t)^{(m+1)} \tag{4.2}$$

Where $t < \eta(x) < x$. It is readily shown that the Lagrange reminder $y^{(m+1)}(t)\frac{(\eta(x))}{(m+1)!}(x-t)^{(m+1)}$ is sufficiently small for large enough m provided that $y^{(m+1)}(x)$is uniformly bounded function for any m on the interval $[0,1]$. So, we will neglect the remainder and the truncated Taylor expansion $y\,(x)$ as follows:

$$y(x) \approx \sum_{j=0}^{m} y^{(j)}(t)\frac{(x-t)^j}{j!} \tag{4.3}$$

Substituting the approximate expression (4.3) for $y(x)$ into equation (4.1),

$$\sum_{j=0}^{m}\int_0^t \frac{(t-x)^{n-\alpha-1}}{\Gamma(n-\alpha)} y^{(j)}(t)\frac{(x-t)^j}{j!}dx$$
$$= \sum_{j=0}^{m}\int_0^t \frac{(t-x)^{n-1}p(x)}{(n-1)!} y^{(j)}(t)\frac{(x-t)^j}{j!} y(x)\,dx + \int_0^t \frac{(t-x)^{n-1}f(x)}{(n-1)!}dx$$
$$+ \sum_{j=0}^{m}\frac{1}{(n-1)!}\int_0^t y^{(j)}(t)\frac{(x-t)^j}{j!}\int_x^t k(s,x)\,(s-x)^{n-1}ds\,dx + Q_n(t) \tag{4.4}$$

Or further;

$$k_{00}(t)\,y(t) + k_{01}(t)y'(t) + \cdots + k_{0m}(t)y^{(m)}(t) = f_{(n)}(t). \tag{4.5}$$

Where

$$k_{0j} = \frac{(-1)^j t^{n-\alpha+j}}{(n-\alpha+j)\Gamma(n-\alpha)j!} - \frac{1}{(n-1)!\,j!}\int_0^t (x-t)^j \int_x^t k(s,x)\,(s-x)^{n-1}ds\,dx$$
$$- \frac{(-1)^j}{(n-1)!}\int_0^t (t-x)^{n+j-1}p(x)\,dx\,, j = 0,1,\dots,m. \tag{4.6}$$

$$f_{(n)}(t) = \frac{1}{(n-1)!}\int_0^t (t-x)^{n-1}f(x) + Q_n(t) \tag{4.7}$$

Thus equation (4.1) becomes an m^{th}order, linear, ordinary differential equation with variable coefficients for $y(t)$ and it s derivative up to m. So, will be determined $y(t), \dots, y^{(m)}(t)$ by solving linear equations. Now, other m independent linear equations for $y(t), \dots, y^{(m)}(t)$ are needed. This can be achieved by integrating both

$$\int_0^t \frac{(t-x)^{n-\alpha}}{\Gamma(n+1-\alpha)} y(x)dx$$
$$= \int_0^t \frac{(t-x)^n\,p(x)}{(n)!} y(x)dx + \int_0^t (\frac{(t-x)^n f(x)}{(n)!} + Q_n(x)\,)\,dx$$
$$+ \frac{1}{(n)!}\int_0^t y(x)\int_x^t k(s,x)\,(s-x)^n ds\,dx. \tag{4.8}$$

Where replace variable s with t. Applying Taylor expansion again and substituting (4.3) for $y(x)$ into equation (4.8) gives

$$k_{10}(t)y(t) + k_{11}(t)y'(t) + \cdots + k_{1m}(t)\,y^{(m)}(t) = f_{(n+1)}(t). \tag{4.9}$$

Where

$$k_{1j} = \frac{(-1)^j t^{n-\alpha+j+1}}{(n-\alpha+1+j)\Gamma(n+1-\alpha)j!} - \frac{1}{n!\,j!}\int_0^t (x-t)^j \int_x^t k(s,x)\,(s-x)^n ds\,dx$$
$$-\frac{(-1)^j}{n!\,j!}\int_0^t (t-x)^{n+j} p(x)\,dx\,, j = 0,1,\dots,m. \tag{4.10}$$

$$f_{(n+1)}(t) = \int_0^t \frac{(t-x)^n f(x)}{n!} + Q_n(x)\ )dx \tag{4.11}$$

By repeating the above integration process for $i(i \in N^+, 1 < i \le m)$ times.

$$k_{i0}(t)y(t) + k_{i1}(t)y'(t) + \cdots + k_{im}(t)y^{(m)}(t) = f_{(n+i)}(t). \tag{4.12}$$

Where

$$k_{ij} = \frac{(-1)^j t^{n-\alpha+j+i}}{(n-\alpha+i+j)\Gamma(n+i-\alpha)j!} - \frac{1}{(n+i-1)!\,j!}\int_0^t (x-t)^j \int_x^t k(s,x)\,(s-x)^{n+i-1} ds\,dx$$
$$-\frac{(-1)^j}{(n+i-1)!\,j!}\int_0^t (t-x)^{n+j+i-1}\,p(x)\,dx\,. \tag{4.13}$$

$$f_{(r)}(t) = \int_0^t f_{(r-1)}(x)dx\,, \qquad r > n+1, r \in N^+ \tag{4.14}$$

Therefore, equations (4.6), (4.9), (4.12) form a system $(m+1)$ linear equations for $(m+1)$ unknown functions $y(t), \dots, y^{(m)}(t)$. For simplicity, the system can be written as:

$$k_{mm}(t)\,Y_m(t) = F_m(t) \tag{4.15}$$

Where $k_{mm}(t)$ is an $(m+1)\times(m+1)$ square matrix function in t, $Y_m(t), F_m(t)$ are two vectors of length $(m+1)$, and these are defined as

$$k_{mm}(t) = \begin{pmatrix} k_{00}(t) & k_{01}(t) & \cdots & k_{0m}(t) \\ k_{10}(t) & k_{11}(t) & \cdots & k_{1m}(t) \\ \vdots & \vdots & \ddots & \vdots \\ k_{m0}(t) & k_{m1}(t) & \cdots & k_{mm}(t) \end{pmatrix}, \tag{4.16}$$

$$Y_m(t) = \begin{pmatrix} y(t) \\ y'(t) \\ \cdots \\ y^{(m)}(t) \end{pmatrix}, \qquad F_m(t) = \begin{pmatrix} f_{(n)}(t) \\ f_{(n+1)}(t) \\ \cdots \\ f_{(n+m)}(t) \end{pmatrix} \tag{4.17}$$

By using Cramer's rule, obtain the approximate solution $y(t)$ as:

$$y(t) = \frac{\det\,(M_{mm}(t))}{\det\,(K_{mm}(t))}. \tag{4.18}$$

Where

$$M_{mm}(t) = \begin{pmatrix} f_n(t) & k_{01}(t) & \cdots & k_{0m}(t) \\ f_{n+1}(t) & k_{11}(t) & \cdots & k_{1m}(t) \\ \vdots & \vdots & \ddots & \vdots \\ f_{n+m}(t) & k_{m1}(t) & \cdots & k_{mm}(t) \end{pmatrix}. \qquad (4.19)$$

V. Numerical Examples

In this paper, according to He's variational iteration method, calculate the numerical result $y_{App}(t)$ by using equation (3.8) and the absolute errors can be written as $e = |y_{App}(t) - y_{Exact}(t)|$ where $y_{App}(t) = \sum_{k=0}^{n+1} y_k(t)$. While, according to Taylor method, choose $m = 3$. So, calculate the numerical result $y_{App}(t)$ by using equation (4.18) and the absolute errors can be written as $e = |y_{App}(t) - y_{Exact}(t)|$. All results are obtained by using Maple 16

Example 5.1. Consider the fractional integro-differential equation

$$D^{0.5}y(t) = \frac{1}{2} + \frac{t^{0.5}}{\Gamma(1.5)} + \frac{t^{-0.5}}{\Gamma(0.5)} - \frac{1}{2}e^t \cos t + \frac{1}{2}te^t \sin t + e^t \sin t - \frac{1}{2}te^t \cos t - e^t \sin t\, y(t) + \int_0^t e^s \cos s\, y(s)\, ds, \qquad y(0) = 1. \qquad (5.1)$$

With the exact solution $y(t) = t + 1$ (Show table 1.)

Table 1. The absolute errors of Example 5.1.

T	Abs.E (TEM)	Abs.E (VIM)
0.1	1.10^{-9}	0
0.2	1.110^{-8}	0
0.3	4.10^{-9}	0
0.4	3.10^{-9}	0
0.5	2.10^{-9}	0
0.6	2.510^{-8}	0
0.7	3.010^{-8}	0
0.8	5.510^{-8}	0
0.9	7.710^{-8}	0
1.0	2.10^{-9}	0

Example 5.2. Consider the fractional integro-differential equation

$$D^{0.5}y(t) = -\frac{1}{5} + \frac{t^{0.5}}{\Gamma(1.5)} - \frac{4}{15}\frac{t^{2.5}}{\Gamma(1.5)} + \frac{16}{945}\frac{t^{4.5}}{\Gamma(1.5)} + \frac{1}{5}e^t \cos 2t - \frac{1}{10}e^t \sin 2t + e^t \sin^2 t - e^t \sin t\, y(t) + \int_0^t e^s \cos s\, y(s)\, ds, \qquad y(0) = 0. \qquad (5.2)$$

With the exact solution $y(t) = \sin t$ (Show table 2.

Table 2. The absolute errors of Example 5.2.

T	Abs.E (TEM)	Abs.E (VIM)
0.1	0.02879569995	0
0.2	0.0608277528	0
0.3	0.0981010982	0
0.4	0.1424796175	0
0.5	0.1954737984	0
0.6	0.2579200116	0
0.7	0.3297150269	0
0.8	0.4096924611	0
0.9	0.4956751284	0
1.0	0.5846951772	0

VI. Conclusion

In this paper, we made a comparison between the problems results in VIM and TEM. It showed that the problems results in VIM are better than the results in TEM where in VIM give the exact solution.

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