IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 4 Ver. IV (Jul - Aug. 2015), PP 42-47
www.iosrjournals.org

Effects Of Hall Current On Transient Convective Mhd Flow Through Porous Medium Past An Infinite Vertical Oscillating Plate With Temperature Gradient Dependent Heat Source

Smita Sharma[1],Jitendra Singh[2],B.B.Singh[3]
ITM Universe Shitholi Gwalior M.P.[1]
Govt. K.R.G. Autonomous P.G. College Gwalior M.P.[2]

Abstract: *In this Paper, we study of an implicit finite-difference technique is applied to study on effects of hall current on the magneto hydrodynamic flow through porous medium past an infinite vertical plate oscillating in its plane in the presence of a temperature gradient dependent heat source. The results are obtained for velocity and temperature profiles. The numerical results are presented graphically for different values of the parameters entering into the problem.*
Key words :*Finite difference scheme, porous medium, MHD, heat source, heat transfer, hall current*

I. Introduction

Flow past an infinite horizontal plate oscillating in its own plane was studied by Stokes (1851) who presented an exact solution to this problem and is discussed in all the text books on viscous flow theory. Flow past a vertical plate oscillating in its own plane was first studied by Soundalgekar (1979). Exact solution by the Laplace transform technique to coupled linear system of equations was presented. However, if the vertical plate oscillates in a fluid-medium containing heat sources, how the flow is affected? This has not been studied in the literature. We study in this paper the effect of temperature gradient dependent heat source on the flow past a vertical plate oscillating in its own plane. In Section 2, the mathematical analysis is presented and in Section 3, the conclusion is set out. Jaiswal and Soundalgekar (2000) have studied on Transient convective flow past an infinite vertical Oscillating plate with temperature gradient dependent heat source. Many investigations have been developed from the past four decades for the free convection flow from various geometries (plane or axi-symmetrical shape bodies) in thermally stratified medium. Cheesewright (1967) discussed a theoretical investigation of free convection from a vertical flat plate in non-isothermal surroundings. He obtained similarity solutions of the governing equations dealing with various types of non-uniform ambient temperature distributions. Yang et al. (1972) studied natural convection heat transfer from a non-isothermal vertical flat plate immersed in a thermal stratified medium and similarity solutions have been carried out for a wide range of wall and ambient temperature distributions for a vide values of the Prandtl number between 0.1 and 20. Further, Venkatachala et al. (1981) solved the complete set of governing partial differential equations for the problem of an isothermal wall in linearly stratified atmosphere using the finite difference method. They also used series expansion and local non-similarity methods. Kulkarni et al. (1987) investigated the problem of natural convection from an iso-thermal flat plate suspended in a stable linearly stratified fluid medium using the Von-Karaman-Pohlhausen integral method. Unsteady natural convection flow over a vertical flat plate embedded in a stratified medium has been studied by Tripathi and Nath). Transient numerical study of double-diffusive free convection from a vertical surface in a thermally stratified medium is highlighted by Srinivasan and Angirasa (1998), and Angirasa and Srinivasan (1989) and boundary layer equations were solved using an explicit finite-difference method. Later, Srinivasan and Angirasa (1990) studied the unsteady laminar axisymmetric plumes that emanate from a source of combined buoyancy due to simultaneous heat and mass diffusion in thermally stratified medium and solved the boundary layer equations using an explicit finite-difference method. Also, authors (1998-1990) discussed the problem of two buoyancy-driving forces which are aid and oppose each other. Takhar et al. (2001) presented a study on the steady natural convection boundary layer flow over a continuously moving isothermal vertical surface immersed in thermally stratified medium. The non-linear coupled partial differential equations governing the non-similar flow have been solved numerically using an implicit finite difference scheme. For small values of the streamwise distance, the governing equations have been solved by using a perturbation expansion technique along with the Shanks transformation. Later, Chamkha (2002) has numerically studied the problem of steady laminar natural convection flow along a vertical permeable surface immersed in a thermally stratified environment in the presence of magnetic field and heat absorption effects. The effect of rotation of the unsteady hydro magnetic flow past a uniformly accelerated infinite vertical plate in the presence of variable temperature and mass diffusion was studied by

Muthucumaraswamy *et al*. (2013). The effect of magnetic fields on heat transfers from an accelerated plate were given by Sahin *et. al (2013*).

Therefore, the aim of this chapter is to study on effects of hall current on transient convective MHD flow of viscous incompressible fluid through a porous medium confined in an infinite vertical with temperature gradient dependent heat source. The velocity and temperature profiles have been analyzed for variations in the different parameters involved in the problem.

II. Mathematical Analysis

Consider a vertical infinite plate held in an infinite mass of viscous incompressible fluid. Initially both the plate and the fluid are assume to be at the same temperature T'_∞. At time $t' > 0$, the plate starts oscillating in its own plane and the temperature of the plate is raised to, T'_w such that $T'_w > T'_\infty$ and hence $T'_w - T'_\infty$ is the temperature-difference which causes the flow of free-convection currents near the plate. The $x'-$axis is taken along the plate in the vertically upward direction and the $y'-$axis is taken normal to the plate. A uniform magnetic field of intensity B_0^2 is applied in the $y'-$direction. As the plate is infinite in extent, the flow variables are functions of y' and t' only. Then the flow can be shown to be governed by the following non-dimensional equations under usual Boussinesq's approximation:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \theta - \left(\frac{M}{1+m^2} + \frac{1}{K}\right)u \qquad ...(1)$$

$$\Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + \Pr S\frac{\partial \theta}{\partial y} \qquad ...(2)$$

With initial and boundary conditions:

$$\left.\begin{array}{ll} t \le 0: & u = 0, \quad \theta = 0 \quad for\ all \quad y \\ t > 0: & u = \cos\omega t, \quad \theta = 1 \quad at \quad y = 0 \\ & u = 0, \quad \theta = 0 \quad as \quad y \to \infty \end{array}\right\} \qquad ...(3)$$

Here Pr is the Prandtl number, *S* the heat source parameter, *M* is the applied magnetic field, *m* is the hall current parameter, *K* is the permeability of porous medium and t_R, L_R and U_0 are the reference time, length and velocity respectively. The equations (1) and (2) are to be solved subject to conditions (3). If the Laplace-transform technique is applied, it leads to a very complicated inverse transform. Hence we employ implicit finite difference technique to solve these coupled equations, as this is always a stable and convergent scheme (1969).

The non-dimensional quantities are defined as follows:

$$\left.\begin{array}{l} u = \dfrac{u'}{U_0}, \quad Y = \dfrac{y'}{L_R}, \quad t = t' t_R, \quad M = \dfrac{\sigma B_0^2 \mu^2 t_R}{\rho}, \ K = \dfrac{K' t_R}{\upsilon} \\ S = \dfrac{S'}{\rho C_p U_0}, \quad \Pr = \dfrac{\mu C_p}{k}, \quad \theta = \dfrac{T' - T'_\infty}{T'_w - T'_\infty} \end{array}\right\} \qquad ...(4)$$

and u_R, L_R, t_R are respectively reference velocity, length and time defined as follows:

$$u_R = (\upsilon g\beta\Delta T)^{1/3}, \qquad L_R = \upsilon^{2/3}(g\beta\Delta T)^{-1/3}$$

$$t_R = \upsilon^{1/3}(g\beta\Delta T)^{-2/3}$$

III. Method of Solution

The governing Equations (1) and (2) are to be solved under the initial and boundary conditions of equation (3). The finite difference method is applied to solve these equations.

The equivalent finite difference scheme of equations (1) and (2) are given by

$$\left[\frac{u_{i,j+1}-u_{i,j}}{\Delta t}\right]=\left[\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{(\Delta y)^2}\right]+\left(\theta_{i,j}\right)-\left[\frac{M}{1+m^2}+\frac{1}{K}\right]u_{i,j} \quad \ldots(5)$$

$$\left[\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t}\right]=\frac{1}{\Pr}\left[\frac{\theta_{i+1,j}-2\theta_{i,j}+\theta_{i-1,j}}{(\Delta y)^2}\right]+S\left[\frac{\theta_{i+1,j}-\theta_{i,j}}{\Delta y}\right] \quad \ldots(6)$$

Here, index *i* refers to *y* and *j* to time. The mesh system is divided by taking, $\Delta y = 0.1$

From the initial conditions in Equation (3), we have the following equivalent.

$$\left.\begin{array}{l} u(0,0)=\cos\omega t, \quad \theta(0,0)=1 \\ u(i,0)=0, \quad \theta(i,0)=0 \quad for\ all\ i\ except\ i=0 \end{array}\right\} \quad \ldots(7)$$

The boundary conditions from equation (7) are expressed in finite difference form are as follows:

$$\left.\begin{array}{l} u(0,j)=\cos\omega t, \quad \theta(0,j)=1 \quad for\ all\ j \\ u(1,j)=\cos\omega t, \quad \theta(1,j)=1 \quad for\ all\ j \end{array}\right\} \quad \ldots(8)$$

Here, infinity is taken as *y* = 4.1. First, the velocity at the end of time step namely $u(i, j+1),\ i=1$ to 10 is computed from equation (5) and temperature $\theta(i, j+1),\ i=1$ to 10 from equation (6). The procedure is repeated until *t* = 1 (i.e., *j* = 1000). During computation, Δt was chosen to be 0.0005. These computations are carried out for different values of parameters Pr, *S, M, m, K, t* and *ωt*. To judge the accuracy of the convergence of the finite difference scheme, the same program was run with smaller values of Δt, i.e., Δt = 0.0009, 0.001 and no significant change was observed. Hence, we conclude that the finite difference scheme is stable and convergent.

IV. Results and Discussion

Numerical calculations have been carried out for velocity and temperature profiles for different values of parameters and are displayed in Figures-(1) to (10).

Figures-(1) to (7) represent the velocity profiles for different parameters. Figure-(1) shows the variation of velocity *u* with magnetic parameter *M*. It is observed that the velocity decreases as *M* increases. The velocity profile for time variable *t* is shown in Figure-(2). It is clear that an increase in *t* leads to an increase in *u*. Figure-(3) shows that an increase in permeability parameter *K* causes an increase in velocity. Figure-(4) shows that an increase in hall current parameter *m* causes an increase in velocity profile *u*. From Figure-(5) shows the variation of velocity *u* with Prandtl number Pr. It is observed that the velocity decreases as Pr increases. In figure-(6), the velocity profile decreases due to increasing strength of heat source parameter *S*. From Figure-(7), it is observed that the velocity decreases as the *ωt* increase. From Figure-(8), it is observed that increase in Prandtl number Pr causes decrease in temperature. Figure-(9) shows that an increase in strength of heat source parameter *S* causes a decrease in temperature profile. Temperature profiles are shown. Temperature is also found to fail with increasing the strength of the heat source parameter *S*, and the Prandtl number Pr. From Figure-(10), it is noticed that an increase in time *t* leads to increase in temperature profile.

V. Conclusion

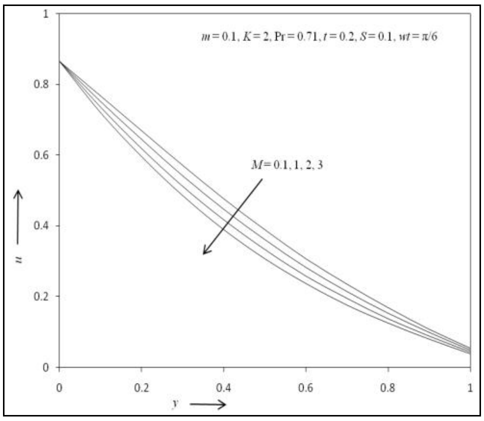
The theoretical solution for an effects of hall current on the magneto hydrodynamic flow through porous medium past an infinite vertical plate oscillating in its plane in the presence of a temperature gradient dependent heat source. The solutions are in terms of an implicit finite-difference technique. The study concludes the following results.

(i) The velocity decreases as M increases.

(ii) An increase in t leads to an increase in u.

(iii) An increase in permeability parameter K causes an increase in velocity.
(iv) An increase in hall current parameter m causes an increase in velocity profile u.
(v) The velocity decreases as Pr increases.
(vi) The velocity profile decreases due to increasing strength of heat source parameter S
(vii) The velocity decreases as the ωt increase..
(viii) An increase in Prandtl number Pr causes decrease in temperature.
(ix) An increase in strength of heat source parameter S causes a decrease in temperature profile.
(x) An increase in time t leads to increase in temperature profile.

VI. Figures



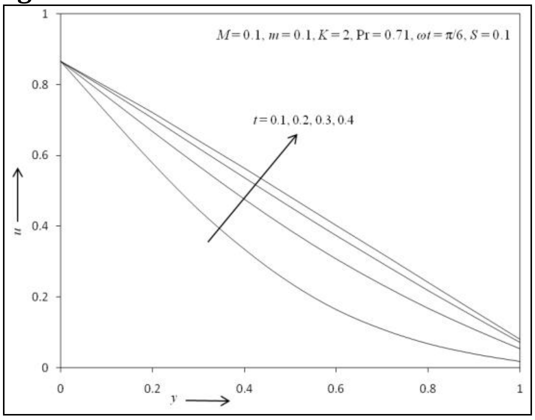
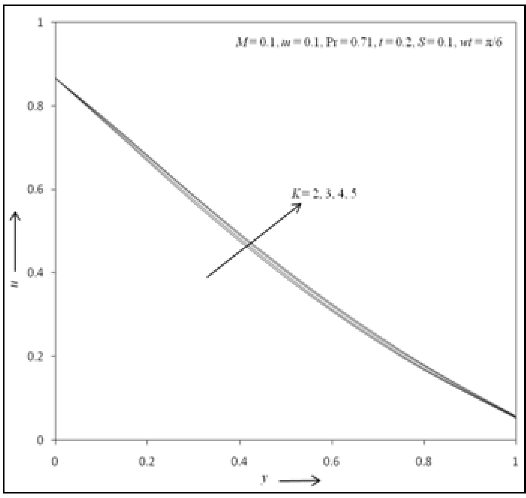


Fig (1). Velocity profile for different value of M **Fig (2).**Velocity profile for different value of *t*



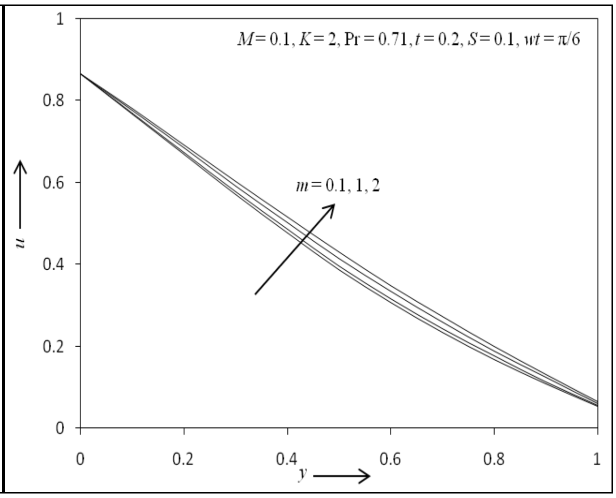
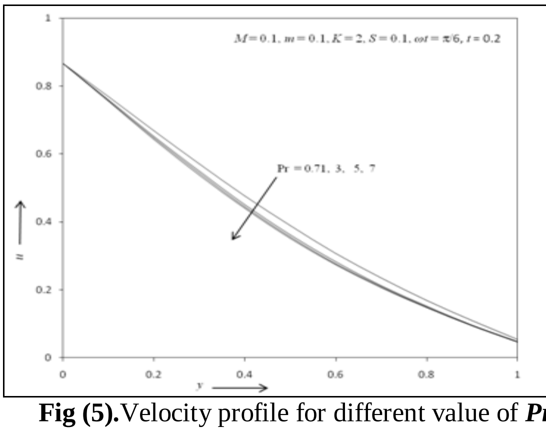


Fig (3). Velocity profile for different value of *K* **Fig (4).**Velocity profile for different value of *m*



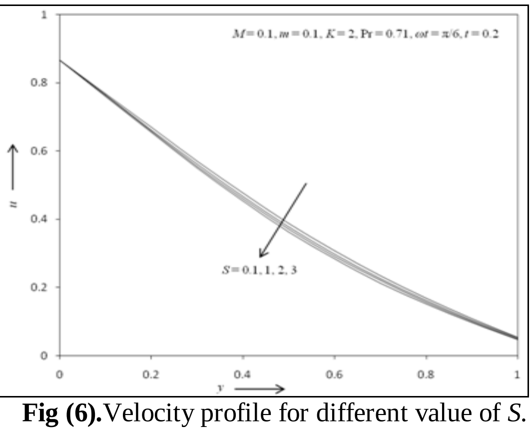


Fig (5). Velocity profile for different value of ***Pr*** **Fig (6).**Velocity profile for different value of *S.*

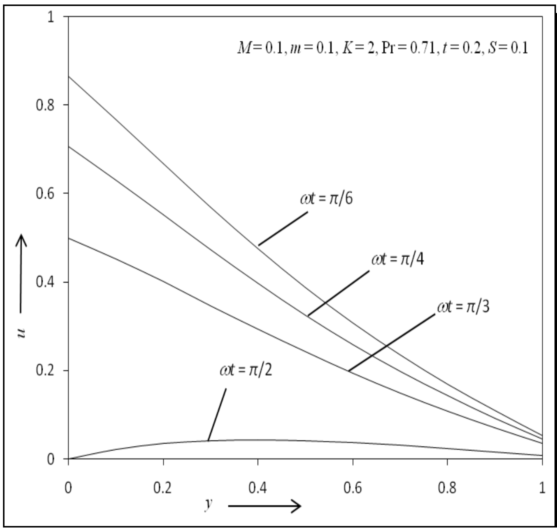


Fig (7). Velocity profile for different value of *ωt.*

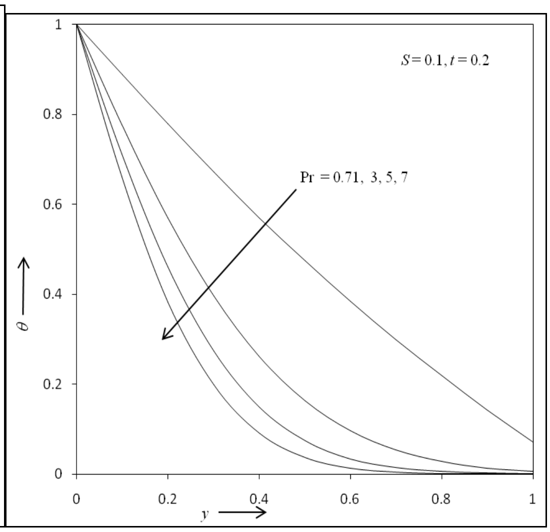


Fig (8). Temprature profile for different value of *Pr.*

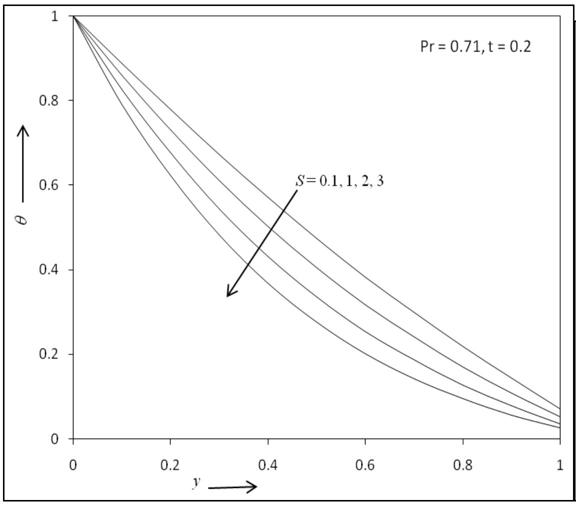


Fig (9). Temperature profile for different value of S**.**

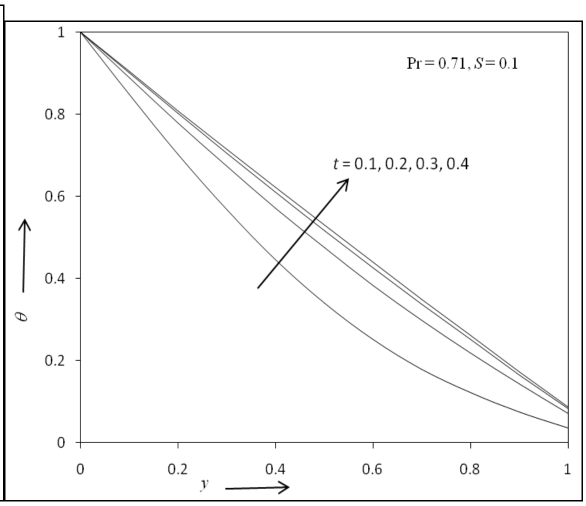


Fig (10). Temperature profile for different value of *t.*

Reference

[1]. Stokes, G.G., 1851, On the effect of the internal friction of fluids on the motion of pendulums: Cambridge Philosophical Society, Transactions, v. 9, no. 8, p.287.

[2]. V.M. Soundalgekar, S.K. Gupta, N.S. Birajdar 1979, Effects of mass transfer and free convection currents on MHD Stokes' problem for a vertical plate Nuclear Engineering and design, Volume53, Issue3, Pages 339-346

[3]. Jaiswal, B.S., and Soundalgekar, V.M., 2000,Transient convective flow past an infinite vertical oscillating plate with temperature gradient dependent heat source". ul. Alld. Math.Soc., 15, 25-29.

[4]. R. Cheesewright,‖ Natural Convection from a Plane Vertical Surface in Non-Isothermal Surroundings‖, Int. J. Heat Mass Transfer, vol. **10** , Pp. 1847–1859, 1967

[5]. A K. T. Yang, J. L. Novotny and Y. S. Cheng,‖ Laminar Free Convection from a Nonisothermal Plate Immersed in a Temperature Stratified Medium‖, Int. J. Heat Mass Transfer , vol. **15,** Pp. 1097-1109, 1972 **.**

[6]. B. J. Venkatachala and G. Nath,‖ Nonsimilar Laminar Natural Convection in a Thermally Stratified Fluid‖, Int. J. Heat Mass Transfer,vol. **24,** Pp. 1848- 1850, 1981

[7]. A. K. Kulkarni, H. R. Jacobs and J. J. Hwang,‖ Similarity Solution For Natural Convection Flow Over an Isothermal Vertical Wall Immersed in Thermally Stratified Medium‖, Int. J. Heat Mass Transfer, Vol. **30,** Pp. 691-698, 1987

[8]. R.K. Tripathi and G. Nath, ―Unsteady Natural Convection Flow Over a Vertical Plate Embedded in a Stratified Medium‖, Int. J. Heat Mass Transfer Vol. **36,** Pp. 1125-1128,1993.

[9]. J. Srinivasan and D. Angirasa,‖ Numerical Study Of Double-Diffusive Free Convection from a Vertical Surface‖, Int. J. Heat Mass Transfer,Vol. 31, Pp. 2033-2038,1998.

[10]. D. Angirasa and J. Srinivasan,‖ Natural Convection Flows Due to the Combined Buoyancy Of Heat and Mass Diffusion in Thermally Stratified Medium‖, ASME Journal of Heat Transfer, Vol. 111, Pp. 657- 663,1989.

[11]. J. Srinivasan and D. Angirasa,‖ Laminar Axisymmetric Multicomponent Buoyant Plumes in a Thermally Stratified Medium‖, Int. J. Heat Mass Transfer ,Vol. 33, Pp. 1751-1757.,1990

[12]. H. S. Takhar, A. J. Chamkha and G. Nath,‖ Natural Convection Flow from a Continuously Moving Vertical Surface Immersed in a Thermally Stratified Medium‖ ,Heat and Mass Transfer , ,Vol. 38 ,Pp. 17-24 ,2001.

[13]. Ali. J. Chamkha, ―Laminar Hydromagnetic Natural Convection Flow Along a Heated Vertical Surface in a Stratified Environment with Internal Heat Absorption, Can. J. Phys. Vol. 80 ,Pp. 1145-1155, 2002.

[14]. Muthucumaraswamy R.,Tina Lal and Ranganayakulu D., MHD Flow past an Accelerated Vertical Plate with Variable Heat and Mass Diffusion in the Presence of Rotation,International Journal of Innovative Research in Science, Engineering and Technology, Vol. 2, Issue 10, 2013.

[15]. Sahin Ahmed and Abdul Batin, Convective Laminar Radiating Flow over an Accelerated Vertical Plate Embedded in a Porous Medium with an External Magnetic Field, International Journal of Engineering and Technology Volume 3 No. 1, January, 2013.