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Model for Discrete Voting Tendency

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Abstract: *Prime Minister voting tendencies are of interest every 5 years. By considering three party system generate a probabilistic transition matrix, or by the system of difference equation to solve the variables.*
Keywords: *Probabilistic Transition Matrix, System of Difference Equation, Langrange's Interpolation, Markov Chain Process, Graphical Analysis.*

I. Introduction

Prime Minister voting tendencies are of interest every 5 years. Let's consider a three-party system with BarathiyaJanata Party (BJP), Indian National Congress (INC) and JathyathithaJanatha Dal Secular (JDS) or other leading III-Party.

II. History of Indian General Election Results

Table given below explains the past General Election Results of India in the year 1996, 2009 and 2014 elections respectively;

Indian General Election-1996:			
	I Party	**II Party**	**III Party**
Party	UF	BJP	INC
Alliance	-	BJP+	INC+
Leader	HD DeveGowda	AtalBihariVajapayee	PV NarasimhaRao
Popularvote	97,113,252	67,945,790	96,443,506

Indian General Election-2009:			
	I Party	**II Party**	**III Party**
Party	INC	BJP	CPI(M)
Alliance	UPA	NDA	TF
Leader	Manmohan Singh	Lal Krishna Advani	Prakash Karat
Popularvote	153,482,356	102,689,312	88,174,229

Indian General Election-2014:			
	I Party	**II Party**	**III Party**
Party	BJP	INC	AIADMK
Alliance	NDA	UPA	-
Leader	NarendraModi	Rahul Gandhi	J.Jayalalitha
Popular vote	171,459,286	106,760,001	18,115,825

III. Problem Identification

Can we find the long-term behavior of voters in General (Lok-sabha) election of India?

By considering the results of the above elections to form probabilistic transition matrix. From the transition matrix we can construct the system of difference equation. By solving the system from known numerical technique we get the solution after some iterations. This iteration represents the long-term behavior of voters.

3.1. Assumption:

1. Not considering the general election held in 1999 and 2003 as the seats won by third parties is not reckonable.
2. Considering the general election held in 1996, 2009 and 2014 as the seats won by both the mainalliance (NDA AND UPA) along with third parties (AIADMK, AITMC and BJD) is reckonable.

3.2. Consideration

During the past decade, the trends have been to vote less strictly along party lines. We provide hypothetical historical data for voting trends in the past 3 results of state-wide voting.

	BJP	INC	III Party
BJP	57.86	36.03	06.11
INC	29.82	44.57	25.61
III Party	25.98	36.88	37.14

IV. Formulation of the Problem

Three-state Markov Chain for the voting tendencies is represented as;

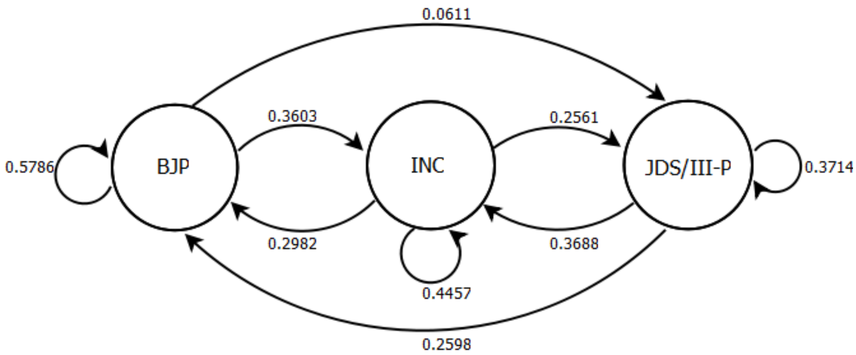


Fig (1): Representation of Markov Chain Using Schematic diagram.

Let's define variables B_n, C_n and J_n as follows:

B_n =The percentage of voters to vote BJP in period n
C_n =The percentage of voters to vote INC in period n
J_n =The percentage of voters to vote III-Party in period n

Using the previous data and the ideas on discrete dynamical system, can formulate the following system of equations giving the percentage of voters who vote BJP, INC or III-Party at each time period:

$$\begin{aligned} B_{n+1} &= 0.5786\, B_n + 0.2982\, C_n + 0.2598\, J_n \\ C_{n+1} &= 0.3603\, B_n + 0.4457\, C_n + 0.3688\, J_n \\ J_{n+1} &= 0.0611\, B_n + 0.2561\, C_n + 0.3714\, J_n \end{aligned} \quad \cdots (1)$$

V. Analysis and Solution of the Model

5.1. Equilibrium Values and Stability

To find the Equilibrium values (B, C, J) by taking $B_{n+1} = B_n = B, C_{n+1} = C_n = C$ and $J_{n+1} = J_n = J$; simultaneously. Substituting into the dynamical system (1) yields,

$$\begin{aligned} 0.4214\, B - 0.2982\, C - 0.2598\, J &= 0 \\ -0.3603\, B + 0.5543\, C - 0.3688\, J &= 0 \\ -0.0611\, B - 0.2561\, C + 0.6286\, J &= 0 \end{aligned} \quad \cdots (2)$$

There are an infinite number of solutions to this system of equations (2). Letting $J = 1$, the system is satisfied if $B = 2.0138$ and $C = 1.9746$.

Suppose the system has 498, 840, 000 voters. Then $B = 201,380,000$; $C = 197,460,000$ and $J = 100,000,000$ voters should approximate the equilibrium values.

The total voters in the system are 498, 840,000, with the initial voting as follows:

	BJP	INC	III Party
Case 1	201380000	197460000	100000000
Case 2	206380000	202460000	90000000
Case 3	100000000	100000000	298840000
Case 4	0	0	498840000

The numerical solutions for the starting values are graphed below;

Case (1):

n	**BJP**($\times$ **10⁴**)	**INC**($\times$ **10⁴**)	**III-Party**($\times$ **10⁴**)
0	20138	19746	10000
1	20138	19745	10001
2	20138	19744	10002
3	20138	19744	10002

The tabular values converge within 3 iterations.

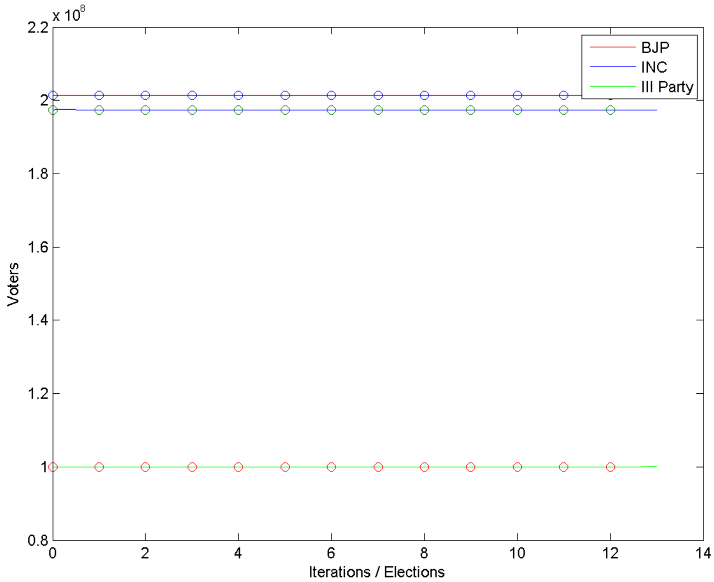


Fig (2): Iterations / Elections vs. Number of voters when the initial voters are$B_n = 201,380,000$; $C_n = 197,460,000$ and $J_n = 100,000,000$.

Case (2):

n	**BJP** ($\times$ **10⁴**)	**INC** ($\times$ **10⁴**)	**III-Party**($\times$ **10⁴**)
0	20638	20246	9000
1	20317	19779	9789
2	20196	19746	9942
3	20157	19744	9983
4	20144	19744	9996
5	20140	19744	10000
6	20139	19744	10001
7	20138	19744	10002
8	20138	19744	10002

The tabular values converge within 8 iterations.

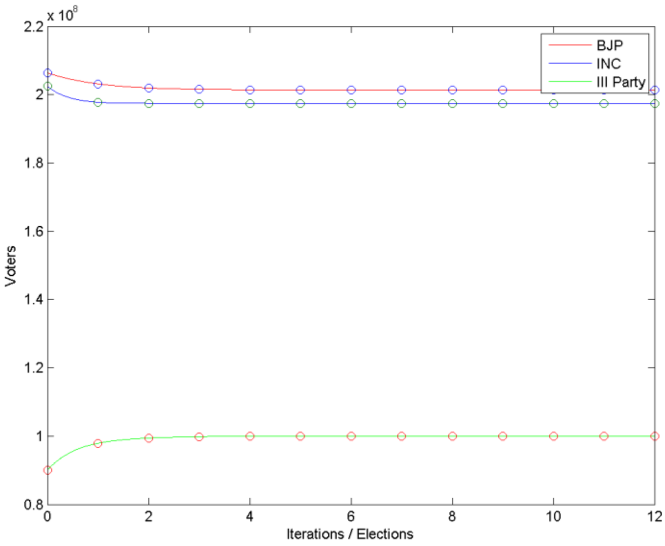


Fig (3): Iterations / Elections vs. Number of voters when the initial voters are $B_n = 206,380,000$; $C_n = 202,460,000$ and $J_n = 90,000,000$.

Case (3):

n	BJP ($\times 10^4$)	INC ($\times 10^4$)	III-Party($\times 10^4$)
0	10000	10000	29884
1	16532	19081	14271
2	18963	19724	11197
3	19763	19753	10369
4	20019	19748	10117
5	20100	19746	10038
6	20126	19745	10013
7	20134	19745	10005
8	20137	19744	10003
9	20138	19744	10002
10	20138	19744	10002

The tabular values converge within 10 iterations.

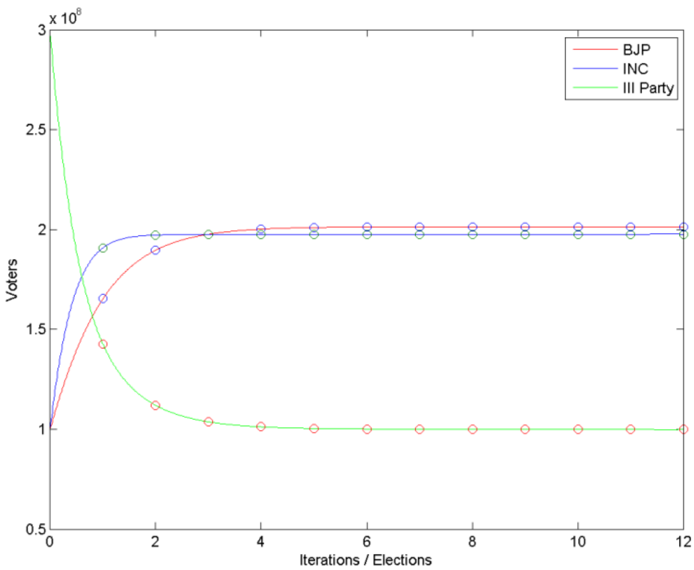


Fig (4): Iterations / Elections vs. Number of voters when the initial voters are $B_n = 100,000,000$; $C_n = 100,000,000$ and $J_n = 298,840,000$.

Case (4):

n	BJP ($\times 10^4$)	INC ($\times 10^4$)	III-Party($\times 10^4$)
0	0	0	49884
1	12960	18397	18527
2	17798	19702	12384
3	19390	19761	10733
4	19900	19752	10232
5	20063	19747	10074
6	20114	19745	10025
7	20130	19745	10009
8	20136	19744	10004
9	20137	19744	10002
10	20138	19744	10002
11	20138	19744	10002

The tabular values converge within 11 iterations.

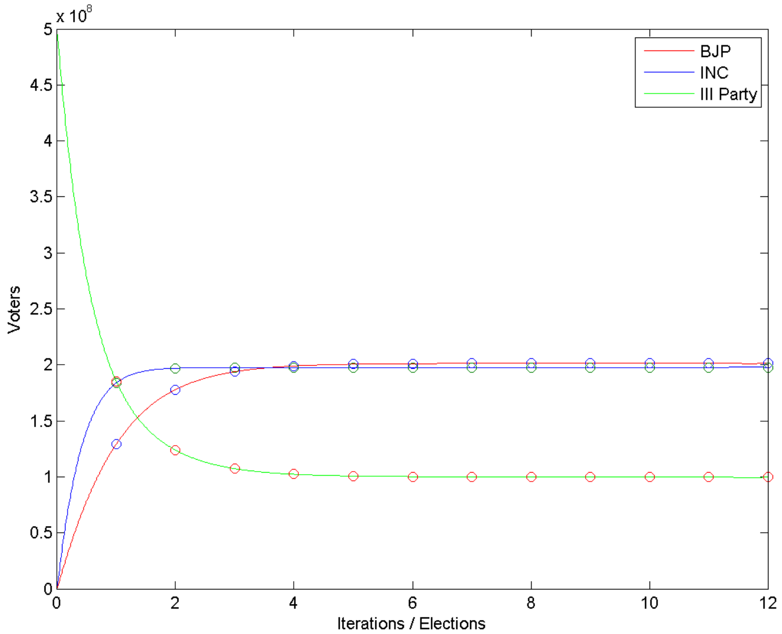


Fig (5): Iterations / Elections vs. Number of voters when the initial voters are $B_n = 0;\ C_n = 0$ and $J_n = 498,840,000$.

5.1.1. Sensitivity to Initial Conditions and Long-Term Behavior

Suppose initially there are 498,840,000 voters in the system and all remain in the system. At least for the starting values we investigated, the system approaches the same result, even if initially there are no BJP's or INC's in the system. The equilibrium investigated appears to be stable.

5.2. Comparison of the Model by using M.M on Probability

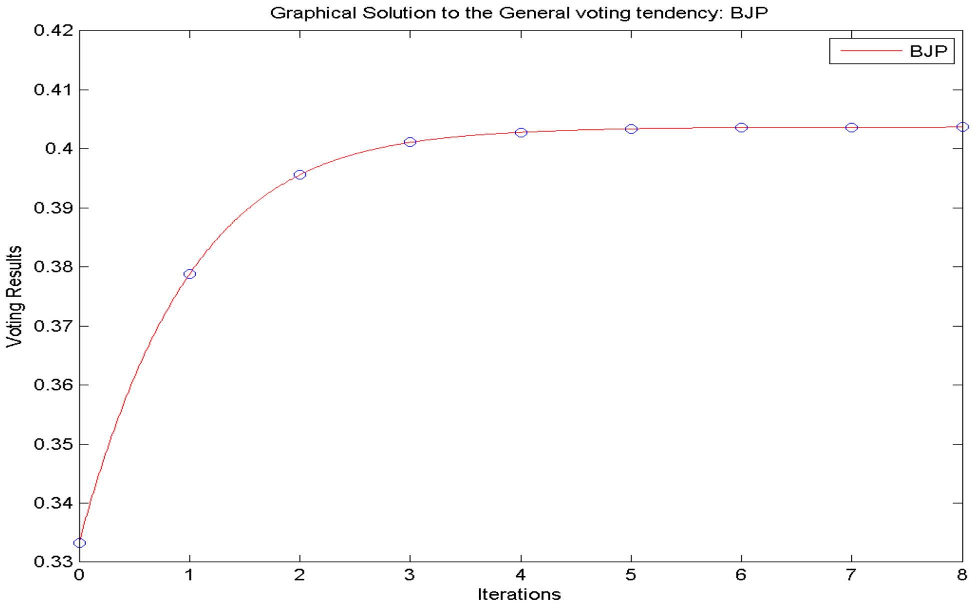
Assume that initially 1/3 of the voters are BJP, 1/3 are INC, and 1/3 are JDS (or other leading III-Party). We then obtain the numerical results shown for the percentage of voters in each group at each period n.

n	BJP	INC	III Party
0	0.3333	0.3333	0.3333
1	0.3788	0.3916	0.2295
2	0.3956	0.3957	0.2087
3	0.4011	0.3958	0.2030
4	0.4028	0.3958	0.2013
5	0.4034	0.3958	0.2007
6	0.4036	0.3958	0.2006
7	0.4036	0.3958	0.2005
8	0.4037	0.3958	0.2005
9	0.4037	0.3958	0.2005

Table: Iterated solutions

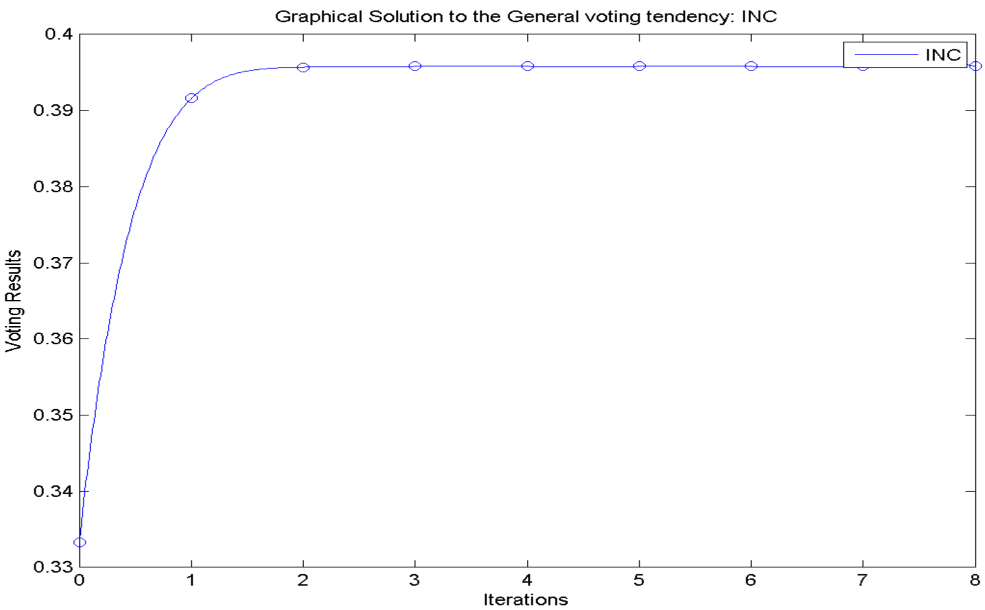
VI. Discussions by Graphs

Following Graphs explains the solution of general voting tendencies. All the Graphs are drawn by Lagrange's Interpolation function.



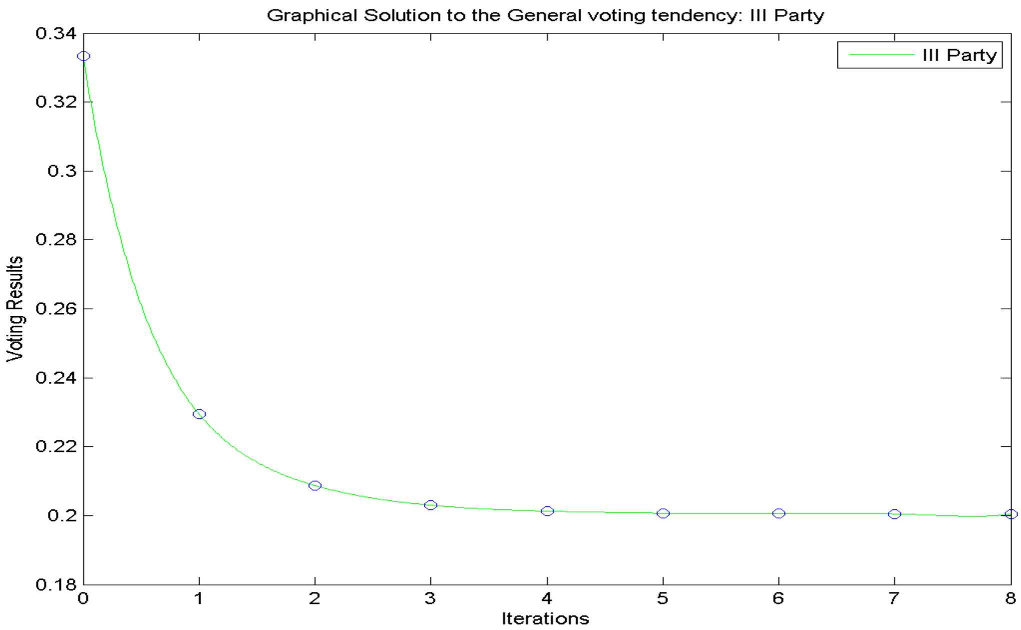
Graph (1): Iterations / Elections vs. Number of voters of BJP (B_n).

Graph (1) represents the Number of voters of BJP (B_n) with respect to iterated elections (n).Thistendency is exponentially increases with elections and after 8^{th} election, it converges to 0.4037.



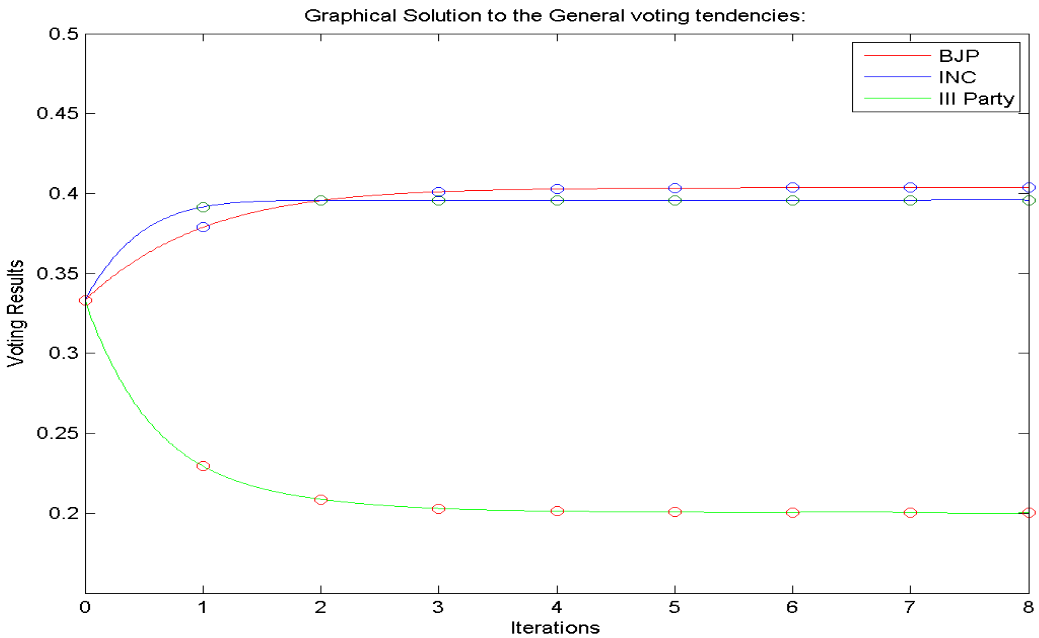
Graph (2): Iterations / Elections vs. Number of voters of INC (C_n).

Graph (2) represents the Number of voters of INC (C_n) with respect to iterated elections (n). This tendency is also exponentially increases with elections and after 3^{rd} election, it converges to 0.3958.



Graph (3): Iterations / Elections vs. Number of voters of III Party (J_n).

Graph (3) represents the Number of voters of III Party (J_n) with respect to iterated elections (n). This tendency is exponentially decreases with elections and after 7^{th} election, it converges to 0.2005.



Graph (4): Iterations / Elections vs. Number of voters of all parties.

Graph (4) represents the Number of voters of all parties with respect to iterated elections (n). After8^{th} election, it converges to 0.4037, 0.3958 and 0.2005.

VII. MATLAB Function: Lagrange's Interpolation

```
function y1 = lagrange(x1, x, y)
    n = size(x, 2);
    L = ones(n, size(x1, 2));
    if (size(x, 2) ≠ size(y, 2))
        fprintf (1, 'ERROR!: x and y must have the same number of elements');
        y = NaN;
    else
        for i = 1 : n
            for j = 1 : n
                if (i ≠ j)
                    L(i, :) = L(i, :) .* (x1 − x(j)) / (x(i) − x(j));
                end
            end
        end
        y1 = 0;
        for i = 1 : n
            y1 = y1 + y(i) * L(i, :);
        end
    end
```

VIII. Conclusion

From the available data (number of voters in different parties) our model predicts that after 8 to 9 time periods (8 to 9 elections) approximately 40.37% of the voters cast their ballots for the BJP candidate, 39.58% vote INC and 20.05% vote leading III-Party.

And also observe that the equilibrium values of the system of equations (1) are start with any initial values of B_n , C_n and J_n and reaches to the same point. Hence the equilibrium values are stable. And our model is also a stable one.

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