# Foreign exchange transaction exposure within a newsvendor framework under hybrid demand distribution 

Sanjay Patel ${ }^{1}$, Ravi Gor ${ }^{2}$<br>${ }^{1}$ (Mathematics Department, St. Xavier's College, Ahmedabad, India)<br>${ }^{2}$ ( Dr. Babasaheb Ambedkar Open University, Ahmedabad, India)


#### Abstract

We consider a global supply chain consisting of one retailer and one manufacturer, both from different countries. As there is a time duration between the placement of the order and the order is acquired, there is a possibility in the exchange rate fluctuation between the two countries. This affects the optimal pricing and order quantity decisions of the retailer. In this paper we elaborate the derivations of analytic expressions involving the transaction exposure when the retailer or manufacturer undertakes to share the exchange rate risk under the generalized hybrid demand in the news vendor framework.


Keywords: transaction exposure, exchange rate, global supply chain, newsvendor problem, optimal pricing and quantity, hybrid demand

## I. Introduction

Consider a manufacturer and a retailer from two different countries having different currencies are into a business. When the exchange rate between the two currencies gets an exposure to unexpected changes, there exists a financial risk and this risk is known as foreign exchange transaction exposure risk (or exchange rate risk). A transaction exposure arises only when there exists a time lag between the time of the financial obligation has been incurred and the time its due to be settled. This is because of the purchase price to buyer/ retailer on the settlement day may differ from that when it was incurred, if the debt is denominated in the manufacturer/ supplier currency. Arcelus, Gor and Srinivasan [1] have developed a mathematical model in news vendor framework to find optimum ordering and pricing policies for retailer/manufacturer, when the foreign exchange rate between the two countries doing the business, faces transaction exposure. Our main contribution in this paper is to derive analytic expressions involving the transaction exposure under the general form of hybrid demand for such a global supply chain within the newsvendor framework.

## II. Literature Review and transaction exposure model

Cases of transaction exposure when a firm has an accounts receivable or payable denominated in a foreign currency has been reported in Goel [2]. The nature of global trade is that either the buyer or the seller has to bear what is commonly known in international finance as transaction exposure, Eitemann et al.[3], Shubita et al.[4]. The very important newsvendor framework introduced by Petruzzi and Dada [5] and the price dependent demand forms in the additive and multiplicative error structures by Mills[6] and Karlin and Carr[7] have been used.

Suppose a retailer (say in India) wants to order q units from a foreign manufacturer (say in U.S) of a certain product. The retailer does not know the demand (D) of the product, which is uncertain. But it partly depends upon the price $(p)$ and partly random. The fluctuation or error in the demand can be of various types. In this paper we take the price dependent demand in the hybrid form which can be described as $D(p, \in)=g_{1}(p)+g_{2}(p) \in$, Where $g_{1}(p), g_{2}(p)$ are arbitrary deterministic demand functions and $\in$ is the demand error over the single period and it is a random variable. The $\in$ follows some distribution $f(\in)$ in $[A, B]$ with mean $\mu$.

Note that $g_{2}(p) \neq 0$ otherwise D becomes deterministic demand. So we may assume that $g_{2}(p) \neq 0$. Further observe that, if $g_{1}(p)=a-b p$ and $g_{2}(p)=1$ then demand D becomes linear with additive error and if $g_{1}(p)=0$ and $g_{2}(p)=a p^{-b}$ then demand D becomes isoelastic with multiplicative error as in ArcelusRavi Gor paper [1]. Here we develop more general foreign exchange transaction exposure model in news vendor framework under the hybrid demand as defined above.

Let the exchange rate be ' $r$ ' in the retailer currency when the order is placed by him [e.g. $1 \$=r$ Rs.]. Let $w$ be the cost of one unit of the product in the manufacturer currency. If the buyer pays immediately then he has to pay wr (Rs) per unit of the product.

But suppose there is a time lag (some fixed period) between the order is placed and the amount is paid for the product when it is acquired by the retailer. Thus there exists transaction exposure exchange rate risk, since the exchange rate (r) may get fluctuate. So the buyer has to pay more or less according to the existing rate at the time of the arrival of the product. Generally the fluctuation in the exchange rate $r$ is very small and uncertain. Also the fluctuation in $r$ is always some percentage of $r$, hence we can take the future exchange rate as, $\mathrm{r}+\mathrm{r} \epsilon_{\mathrm{r}}=\mathrm{r}\left(1+\epsilon_{\mathrm{r}}\right)$.[e.g. $1 \$=\mathrm{r}\left(1+\epsilon_{\mathrm{r}}\right)$ Rs.]. Note that $\epsilon_{\mathrm{r}}$ is also a random variable together with the random variable D . The fluctuation $\epsilon_{\mathrm{r}}$ is unknown but its distribution is known (say $\psi\left(\epsilon_{\mathrm{r}}\right)$ ).

If the fluctuation $\epsilon_{\mathrm{r}}$ is positive buyer has to pay more and if it is negative seller will get less. So the question arises here is that who will bare the exchange rate risk? Buyer/retailer OR seller/manufacturer? In this paper we discuss the two scenarios viz. retailer an manufacturer bears the exchange rate risk under the hybrid demand distribution. In each case the retailer's optimal policy is to determine the optimum order $(q)$ and selling price(p) of the product so that his expected profit is maximum. At the same time we obtain the manufacturer's optimal policies as well.

## III. Assumptions and Notations

The following assumptions are made in the foreign exchange transaction exposure model:
(i) The standard newsvendor problem assumptions apply.
(ii) The global supply chain consists of single retailer- single manufacturer.
(iii) The error in demand is multiplicative.
(iv) Only one of the two-retailer or manufacturer- bears the exchange rate risk.

The following notations are used in the paper:
$\mathrm{q}=$ order quantity
$\mathrm{p}=$ selling price per unit
$\mathrm{D}=$ demand of the product= no. of units required
$\epsilon=$ demand error= randomness in the demand.
$v=$ salvage value per unit
$s=$ penalty cost per unit for shortage
$\mathrm{c}=$ cost of manufacturing per unit for manufacturer
$\mathrm{w}_{\mathrm{r}}=$ purchase cost for retailer
$\epsilon_{\mathrm{r}}=$ the exchange rate fluctuation= exchange rate error= randomness in exchange rate
$\Pi=$ profit function.

## Case-1: Retailer bears the exchange Rate Risk

In this case we assume that the retailer bears the exchange rate risk and manufacturer does not. Thus the manufacturer will get $w$ per unit at any point of time and the buyer will have to pay according to the existing exchange rate. So the buyer will be paying $\operatorname{wr}\left(1+\epsilon_{\mathrm{r}}\right)$ per unit, on the settlement day or when the product is acquired by him. This amount in terms of manufacturer currency is $\operatorname{wr}\left(1+\epsilon_{\mathrm{r}}\right) / \mathrm{r}=\mathrm{w}\left(1+\epsilon_{\mathrm{r}}\right)=\mathrm{w}_{\mathrm{r}}$ (say). Thus $\mathrm{w}_{\mathrm{r}}$ is the purchase cost to buyer in seller's currency.

Now the retailer/ buyer will choose the selling price $p$ and the order quantity $q$ so as to maximize his expected profit. The profit function for the retailer is given by,
$\Pi(p, q)=[$ revenue from $q$ items $]-$ [expenses for the $q$ items]
$\Pi(p, q)=\left\{\begin{array}{l}{[p D+v(q-D)]-\left[q w_{r}\right] \text { if } D \leq q \text { (overstocking) }} \\ {[p q]-\left[s(D-q)+q w_{r}\right] \text { if } D>q \quad \text { (shortage) }}\end{array}\right.$
Note that all the parameters $p, v, s, w_{r}$ are taken in manufacturer's currency. The salvage value $v$ is taken as an income from the disposal of each of the $q$-D leftover and the penalty cost is taken as the cost for each of the D-q shortages.

Since the demand, $D(p, \in)=g_{1}(p)+g_{2}(p) \in$ the retailer's profit function (1) for ordering $q$ units and keeping selling price $p$ is given by,
$\Pi(p, q)= \begin{cases}{[p D+v(q-D)]-\left[q w_{r}\right]} & \text { if } D \leq q \\ {[p q]-\left[s(D-q)+q w_{r}\right]} & \text { if } D>q\end{cases}$
$\Rightarrow \Pi(p, q)=\left\{\begin{array}{c}{\left[p\left(g_{1}(p)+g_{2}(p) \in\right)+v\left\{q-\left(g_{1}(p)+g_{2}(p) \in\right)\right\}\right]-\left[q w_{r}\right] \text { if } D \leq q} \\ {[p q]-\left[s\left(\left\{g_{1}(p)+g_{2}(p) \in\right\}-q\right)+q w_{r}\right] \text { if } D>q}\end{array}\right.$
$\Rightarrow \Pi(p, q)=\left\{\begin{array}{c}p\left(g_{1}+g_{2} \in\right)+v q-v\left(g_{1}+g_{2} \in\right)-q w_{r} \text { if } D \leq q \\ p q-s\left(g_{1}+g_{2} \in\right)+s q-q w_{r} \text { if } D>q\end{array}\right.$
Let us change the parameter q to z by defining $z=\frac{q-g_{1}}{g_{2}}$ that is $q=g_{1}+g_{2} z$.
Now $D \leq q \Leftrightarrow g_{1}+g_{2} \in \leq q \Leftrightarrow g_{2} \in \leq q-g_{1} \Leftrightarrow \in \leq \frac{q-g_{1}}{g_{2}} \Leftrightarrow \in \leq z$. Similarly $D>q \Leftrightarrow \in>z$
$\Rightarrow \Pi(\mathrm{z}, p)=\left\{\begin{array}{c}p\left(g_{1}+g_{2} \in\right)+v\left(g_{1}+g_{2} z\right)-v\left(g_{1}+g_{2} \in\right)-w_{r}\left(g_{1}+g_{2} z\right) \text { if } z \leq \epsilon \\ p\left(g_{1}+g_{2} z\right)-s\left(g_{1}+g_{2} \in\right)+\mathrm{s}\left(g_{1}+g_{2} z\right)-w_{r}\left(g_{1}+g_{2} z\right) \text { if } \in>z\end{array}\right.$
$\Rightarrow \Pi(\mathrm{z}, p)= \begin{cases}p\left(g_{1}+g_{2} \in\right)+v g_{2}(\mathrm{z}-\epsilon)-w_{r}\left(g_{1}+g_{2} z\right) & \text { if } z \leq \epsilon \\ p\left(g_{1}+g_{2} z\right)-s g_{2}(\in-z)-w_{r}\left(g_{1}+g_{2} z\right) & \text { if } \in>z\end{cases}$
The equation (2) describes the profit function for the retailer in the manufacturer currency. Note that the parameter $q$ is replaced by $z$. Now the retailer wants to find the optimal order quantity q say q* and optimal price $\mathrm{p}=\mathrm{p}^{*}$ to maximize his expected profit. In order to do this he must find optimal values of the price p and the parameter z , say $\mathrm{p}^{*}$ and $\mathrm{z}^{*}$ respectively which maximizes his expected profit so that he can determine the optimal order $q^{*}=g_{1}\left(p^{*}\right)+g_{2}\left(p^{*}\right) z^{*}$. So when the retailer is risk taker his expected profit for the demand error $\in$ with support $[A, B]$ is given by,
$E[\Pi(z, p)]=\int_{A}^{B} \Pi(z, p) f(\in) d \in$
$\Rightarrow E[\Pi(z, p)]=\int_{A}^{Z} \Pi(z, p) f(\epsilon) d \in+\int_{z}^{B} \Pi(z, p) f(\epsilon) d \in$, as $A \leq z \leq B$
Let us replace the variable $\in$ by u for simplicity in the above equation and also in (2).

$$
\begin{align*}
& \Rightarrow E[\Pi(z, p)]=\int_{A}^{Z} \Pi(z, p) f(u) d u+\int_{z}^{B} \Pi(z, p) f(u) d u \\
& \Rightarrow E[\Pi(z, p)]=\int_{A}^{Z}\left[p\left(g_{1}+g_{2} u\right)+v g_{2}(z-u)-w_{r}\left(g_{1}+g_{2} z\right)\right] f(u) d u+\int_{Z}^{B}\left[p\left(g_{1}+g_{2} z\right)-s g_{2}(u-z)-w_{r}\left(g_{1}+g_{2} z\right)\right] f(u) d u \\
& \Rightarrow E[\Pi(z, p)]=\int_{A}^{Z}\left[p\left(g_{1}+g_{2} u\right)+v g_{2}(z-u)\right] f(u) d u+\int_{z}^{B}\left[p\left(g_{1}+g_{2} z\right)-s g_{2}(u-z)\right] f(u) d u \\
& -w_{r}\left(g_{1}+g_{2} z\right)\left[\int_{A}^{Z} f(u) d u+\int_{z}^{B} f(u) d u\right] \\
& \Rightarrow E[\Pi(z, p)]=\int_{A}^{Z}\left[p\left(g_{1}+g_{2} u\right)+v g_{2}(z-u)\right] f(u) d u+\int_{z}^{B}\left[p\left(g_{1}+g_{2} z\right)-s g_{2}(u-z)\right] f(u) d u-w_{r}\left(g_{1}+g_{2} z\right) \tag{3}
\end{align*}
$$

$\rightarrow$ Define $\Lambda(z)=\int_{A}^{z}(z-u) f(u) d u \quad$ [expected leftovers] and

$$
\Phi(z)=\int_{Z}^{B}(u-z) f(u) d u \quad \text { [expected shortages] }
$$

Let $\mu=\int_{A}^{B} u f(u) d u$ be the expected value of the randomness $u$ in the demand $D$. Then from (3) we get
$E[\Pi(z, p)]=\int_{A}^{Z}\left[p\left(g_{1}+g_{2} u\right)+v g_{2}(z-u)\right] f(u) d u+\int_{z}^{B}\left[p\left(g_{1}+g_{2} z\right)-s g_{2}(u-z)\right] f(u) d u-w_{r}\left(g_{1}+g_{2} z\right)$

$$
\begin{aligned}
& \Rightarrow E[\Pi(z, p)]=\int_{A}^{Z}\left[p\left(g_{1}+g_{2} u\right)+v g_{2}(z-u)\right] f(u) d u+\int_{z}^{B}\left[p\left(g_{1}+g_{2} z\right)-s g_{2}(u-z)\right] f(u) d u-w_{r}\left(g_{1}+g_{2} z\right) \int_{A}^{B} f(u) d u \\
& =\int_{A}^{Z}\left[p\left(g_{1}+g_{2} u\right)+v g_{2}(z-u)\right] f(u) d u+\int_{Z}^{B}\left[p\left(g_{1}+g_{2} z\right)-s g_{2}(u-z)\right] f(u) d u-w_{r}\left[g_{1} \int_{A}^{B} f(u) d u+g_{2} z \int_{A}^{B} f(u) d u\right] \\
& =\int_{A}^{Z}\left[p\left(g_{1}+g_{2} u\right)+v g_{2}(z-u)\right] f(u) d u+\int_{z}^{B}\left[p\left(g_{1}+g_{2} z\right)-s g_{2}(u-z)\right] f(u) d u-w_{r}\left[g_{1} \int_{A}^{B} f(u) d u+g_{2} \int_{A}^{B} z f(u) d u\right] \\
& =\int_{A}^{z}\left[p\left(g_{1}+g_{2} u\right)+v g_{2}(z-u)\right] f(u) d u+\int_{Z}^{B}\left[p\left(g_{1}+g_{2} z\right)-s g_{2}(u-z)\right] f(u) d u \\
& -w_{r} g_{1} \int_{A}^{B} f(u) d u-w_{r} g_{2}\left[\int_{A}^{B}(z-u+u) f(u) d u\right] \\
& =\int_{A}^{Z}\left[p\left(g_{1}+g_{2} u\right)+v g_{2}(z-u)\right] f(u) d u+\int_{Z}^{B}\left[p\left(g_{1}+g_{2} z\right)-s g_{2}(u-z)\right] f(u) d u \\
& -w_{r} g_{1}-w_{r} g_{2}\left[\int_{A}^{B}(z-u) f(u) d u+\int_{A}^{B} u f(u) d u\right] \\
& =\int_{A}^{Z}\left[p\left(g_{1}+g_{2} u\right)+v g_{2}(z-u)\right] f(u) d u+\int_{z}^{B}\left[p\left(g_{1}+g_{2} z\right)-s g_{2}(u-z)\right] f(u) d u \\
& -w_{r} g_{1}-w_{r} g_{2}\left[\int_{A}^{z}(z-u) f(u) d u+\int_{Z}^{B}(z-u) f(u) d u+\mu\right] \\
& =p g_{1} \int_{A}^{Z} f(u) d u+p g_{2} \int_{A}^{Z} u f(u) d u+v g_{2} \int_{A}^{Z}(z-u) f(u) d u \\
& +p g_{1} \int_{z}^{B} f(u) d u+p g_{2} \int_{z}^{B} z f(u) d u-s g_{2} \int_{z}^{B}(u-z) f(u) d u \\
& -w_{r} g_{1}-w_{r} g_{2} \int_{A}^{z}(z-u) f(u) d u-w_{r} g_{2} \int_{Z}^{B}(z-u) f(u) d u-w_{r} g_{2} \mu \\
& =p g_{1}\left[\int_{A}^{Z} f(u) d u+\int_{z}^{B} f(u) d u\right]-w_{r} g_{1}+p g_{2}\left[\int_{A}^{Z} u f(u) d u+\int_{z}^{B} z f(u) d u\right]-w_{r} g_{2} \mu \\
& +v g_{2} \int_{A}^{z}(z-u) f(u) d u-w_{r} g_{2} \int_{A}^{z}(z-u) f(u) d u \\
& -s g_{2} \int_{z}^{B}(u-z) f(u) d u-w_{r} g_{2} \int_{z}^{B}(z-u) f(u) d u \\
& =p g_{1}-w_{r} g_{1}+p g_{2}\left[\int_{A}^{z} u f(u) d u+\int_{z}^{B}(z-u+u) f(u) d u\right]-w_{r} g_{2} \mu+v g_{2} \Lambda-w_{r} g_{2} \Lambda-s g_{2} \Phi+w_{r} g_{2} \int_{Z}^{B}(u-z) f(u) d u \\
& =p g_{1}-w_{r} g_{1}+p g_{2}\left[\int_{A}^{z} u f(u) d u+\int_{z}^{B} u f(u) d u-\int_{z}^{B}(u-z) f(u) d u\right]-w_{r} g_{2} \mu+v g_{2} \Lambda-w_{r} g_{2} \Lambda-s g_{2} \Phi+w_{r} g_{2} \Phi \\
& =g_{1}\left(p-w_{r}\right)+p g_{2}[\mu-\Phi]-w_{r} g_{2} \mu+g_{2}\left(v-w_{r}\right) \Lambda-g_{2}\left(s-w_{r}\right) \Phi
\end{aligned}
$$

$=g_{1}\left(p-w_{r}\right)+p g_{2} \mu-p g_{2} \Phi-w_{r} g_{2} \mu+g_{2}\left(v-w_{r}\right) \Lambda-g_{2}\left(s-w_{r}\right) \Phi$
$=g_{1}\left(p-w_{r}\right)+\left(p-w_{r}\right) g_{2} \mu-g_{2}\left(w_{r}-v\right) \Lambda-g_{2}\left(p+s-w_{r}\right) \Phi$
$=\left(p-w_{r}\right)\left(g_{1}+g_{2} \mu\right)-g_{2}\left(w_{r}-v\right) \Lambda-g_{2}\left(p+s-w_{r}\right) \Phi$
$=\left(p-w_{r}\right)\left(g_{1}+g_{2} \mu\right)-g_{2}\left[\left(w_{r}-v\right) \Lambda+\left(p+s-w_{r}\right) \Phi\right]$
Thus $E[\Pi(z, p)]=\left(p-w_{r}\right)\left(g_{1}+g_{2} \mu\right)-g_{2}\left[\left(w_{r}-v\right) \Lambda+\left(p+s-w_{r}\right) \Phi\right]$
The equation (4) represents the expected profit of the retailer as a function of $z$ and $p$. We use Whitin's method [8] to maximize the expected profit function. In this method first we keep $p$ fixed in (4) and use the second order optimality conditions $\frac{\partial E}{\partial z}=0$ and $\frac{\partial^{2} E}{\partial z^{2}}<0$ to find the optimum value of $\mathrm{z}^{*}$ as a function of $p$. Then we substitute the value of $z^{*}$ in the expected profit (4) so that it becomes a function of single variable $p$ and hence the optimal $\mathrm{p}^{*}$ can also be obtained.
$\rightarrow \operatorname{Now} E[\Pi(z, p)]=\left(p-w_{r}\right)\left(g_{1}+g_{2} \mu\right)-g_{2}\left[\left(w_{r}-v\right) \Lambda+\left(p+s-w_{r}\right) \Phi\right]$
$\Rightarrow \frac{\partial E}{\partial z}=0-g_{2}(p)\left(w_{r}-v\right) \frac{\partial \Lambda}{\partial z}-g_{2}(p)\left(p+s-w_{r}\right) \frac{\partial \Phi}{\partial z}$
But $\Lambda(z)=\int_{A}^{z}(\mathrm{z}-\mathrm{u}) f(u) d u \Rightarrow \frac{\partial \Lambda}{\partial z}=\int_{A}^{z}(1) f(u) d u+0-0=\int_{A}^{z} f(u) d u$ and
$\Phi(z)=\int_{z}^{B}(\mathrm{u}-\mathrm{z}) f(u) d u \Rightarrow \frac{\partial \Phi}{\partial z}=\int_{z}^{B}(-1) f(u) d u+0-0=-\int_{z}^{B} f(u) d u$.
$\Rightarrow \frac{\partial E}{\partial z}=-g_{2}(p)\left(w_{r}-v\right) \int_{A}^{z} f(u) d u+g_{2}(p)\left(p+s-w_{r}\right) \int_{z}^{B} f(u) d u$
$\Rightarrow \frac{\partial E}{\partial z}=-g_{2}(\mathrm{p})\left(w_{r}-v\right)\left\{\int_{A}^{z} f(u) d u+\int_{z}^{B} f(u) d u-\int_{z}^{B} f(u) d u\right\}+g_{2}(\mathrm{p}) \int_{z}^{B}\left(p+s-w_{r}\right) f(u) d u$
$\Rightarrow \frac{\partial E}{\partial z}=-g_{2}(\mathrm{p})\left(w_{r}-v\right) \int_{A}^{B} f(u) d u+g_{2}(\mathrm{p})\left(w_{r}-v\right) \int_{z}^{B} f(u) d u+g_{2}(\mathrm{p}) \int_{z}^{B}\left(p+s-w_{r}\right) f(u) d u$
$\Rightarrow \frac{\partial E}{\partial z}=-g_{2}(\mathrm{p})\left(w_{r}-v\right)+g_{2}(\mathrm{p}) \int_{z}^{B}\left[w_{r}-v+p+s-w_{r}\right] f(u) d u$
$\Rightarrow \frac{\partial E}{\partial z}=-g_{2}(\mathrm{p})\left(w_{r}-v\right)+g_{2}(\mathrm{p}) \int_{z}^{B}[p+s-v] f(u) d u$
$\Rightarrow \frac{\partial E}{\partial z}=-g_{2}(p)\left(w_{r}-v\right)+g_{2}(p)(p+s-v) \int_{z}^{B} f(u) d u$
$\rightarrow$ If we use the $\operatorname{CDF} F(z)=\int_{A}^{z} f(u) d u$ then $1-F(z)=\int_{z}^{B} f(u) d u$.
Thus $\frac{\partial E}{\partial z}=-g_{2}(p)\left(w_{r}-v\right)+g_{2}(p)(p+s-v)[1-F(z)]$
$\rightarrow$ For optimal value of the expected profit we must have $\frac{\partial E}{\partial z}=0$.
$\Rightarrow-g_{2}(p)\left(w_{r}-v\right)+g_{2}(p)(p+s-v)[1-F(z)]=0 \quad($ from (5))
$\Rightarrow(p+s-v)[1-F(z)]=\left(w_{r}-v\right)$
$\Rightarrow 1-F(z)=\frac{w_{r}-v}{p+s-v}$
$\Rightarrow F(z)=1-\frac{w_{r}-v}{p+s-v}$
$\Rightarrow F(z)=\frac{p+s-w_{r}}{p+s-v}$
$\Rightarrow z=F^{-1}\left(\frac{p+s-w_{r}}{p+s-v}\right)=z^{*}$ (say)

Again differentiating equation (5) w.r.t. z we get,
$\frac{\partial^{2} E}{\partial z^{2}}=0+g_{2}(p)(p+s-v)\left[-\frac{\partial F}{\partial z}\right]$
But $F(z)=\int_{A}^{z} f(u) d u \Rightarrow \frac{\partial F}{\partial z}=\int_{A}^{z} \frac{\partial(f(u))}{\partial z} d u+f(z) \frac{\partial(z)}{\partial z}-f(\mathrm{~A}) \cdot 0 \Rightarrow \frac{\partial F}{\partial z}=f(z)$
$\frac{\partial^{2} E}{\partial z^{2}}=-g_{2}(p)(p+s-v) f(z)$
$\Rightarrow$ Since $\frac{\partial^{2} E}{\partial z^{2}}<0$ for any value of z it follows that the $\mathrm{z}^{*}$ gives the optimum solution for maximum profit.
$\rightarrow$ Now we substitute this value of $z^{*}$ back in the profit function in (4) so that it becomes function of single variable p and hence it can be optimized using second order optimality conditions to determine optimal price $\mathrm{p}^{*}$.

Hence the retailer's optimal order $\mathrm{q}=\mathrm{q} *$ is given by,
$q^{*}=g_{1}\left(p^{*}\right)+g_{2}\left(p^{*}\right) z^{*}=g_{1}\left(p^{*}\right)+g_{2}\left(p^{*}\right) F^{-1}\left(\frac{p^{*}+s-w_{r}}{p^{*}+s-v}\right)$
Where $\mathrm{F}^{-1}$ is the inverse CFD.
$\rightarrow$ Now the manufacturer's profit when the buyer bears the risk as $\Pi_{m}=(w-c) q^{*} \quad$ [ (selling price of seller)-( cost of purchase to seller) $\} \mathrm{x}$ no. of units sold]

## Case-2: Manufacturer bears the exchange Rate Risk

In the case- 2 we assume that the manufacturer bears the exchange rate risk and retailer does not. Thus the retailer pays $w$ per unit in manufacturer's currency at any point of time and the manufacturer will get according to the existing exchange rate. So the manufacturer will be getting $w r /\left(r\left(1+\epsilon_{r}\right)\right)=w_{m}$ per unit on the settlement day in his currency. Now the retailer's profit function, his expected profit and optimal policies to get maximum expected profit can be obtained by replacing wr by w in case-1. So we get the retailer's profit as,
$\Pi(p, q)=\left\{\begin{array}{l}{[p D+v(q-D)]-[q w] \text { if } D \leq q \text { (overstocking) }} \\ {[p q]-[s(D-q)+q w] \text { if } D>q \quad \text { (shortage) }}\end{array}\right.$
And his expected profit as,
$E[\Pi(z, p)]=(p-w)\left(g_{1}+g_{2} \mu\right)-g_{2}[(w-v) \Lambda+(p+s-w) \Phi]$
Where $z=\frac{q-g_{1}(p)}{g_{2}(p)}$. The optimal policy $\mathrm{z}^{*}$ is given by $z^{*}=F^{-1}\left(\frac{p^{*}+s-w}{p^{*}+s-v}\right)$ and hence the optimum
order quantity is $q^{*}=g_{1}\left(p^{*}\right)+g_{2}\left(p^{*}\right) z^{*}=g_{1}\left(p^{*}\right)+g_{2}\left(p^{*}\right) F^{-1}\left(\frac{p^{*}+s-w}{p^{*}+s-v}\right)$.
where $\mathrm{F}^{-1}$ is the inverse CFD.
Also the manufacturer's profit when the he bears the risk is [(selling price of seller)-( purchase cost to seller $)] \times$ no. of units sold. $\Pi_{m}=\left(w_{m}-c\right) q^{*}=\left[\frac{w r}{r\left(1+\epsilon_{r}\right)}-c\right] q^{*}$
Both the cases are summarized in the following table for the hybrid demand:

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|  | Retailer bears the risk | Manufacturer bears the risk |
| :--- | :--- | :--- |
| Hybrid Demand | $D=g_{1}(p)+g_{2}(p) \in$ | $D=g_{1}(p)+g_{2}(p) \in$ |
| Selling price of <br> seller (in\$) | w | $w_{m}=\frac{w r}{r\left(1+\epsilon_{r}\right)}$ |
| Purchase cost to <br> buyer(in $\$)$ | $\mathrm{w}_{\mathrm{r}}=\mathrm{wr}\left(1+\epsilon_{\mathrm{r}}\right) / \mathrm{r}=\mathrm{w}\left(1+\epsilon_{\mathrm{r}}\right)$ | w |
| Expected profit of <br> buyer | $\left(p-w_{r}\right)\left(g_{1}(p)+g_{2}(p) \mu\right)-$ <br> $g_{2}(p)\left[\left(w_{r}-v\right) \Lambda+\left(p+s-w_{r}\right) \Phi\right]$ | $(p-w)\left(g_{1}(p)+g_{2}(p) \mu\right)-$ <br> $g_{2}(p)[(w-v) \Lambda+(p+s-w) \Phi]$ |
| Optimum Order <br> quantity $\mathbf{q}^{*}$ | $q^{*}=g_{1}\left(p^{*}\right)+g_{2}\left(p^{*}\right) F^{-1}\left(\frac{p+s-w_{r}}{p+s-v}\right)$ | $q^{*}=g_{1}\left(p^{*}\right)+g_{2}\left(p^{*}\right) F^{-1}\left(\frac{p+s-w}{p+s-v}\right)$ |
| Profit of the seller <br> $\Pi_{\mathrm{m}}$ | $\Pi_{m}=(w-c) q^{*}$ | $\Pi_{m}=\left[\frac{w r}{r\left(1+\epsilon_{r}\right)}-c\right] q^{*}$ |

$\rightarrow$ We see in the next two tables the conclusions of the foreign exchange transaction paper by Arcelus, Ravi Gor and Shrinivasan as a special case of hybrid demand, viz. linear demand with additive error(LDAE) and isoelastic demand with multiplicative error(IDME).
(1) Deductions for Linear Demand and Additive Error [LDAE]:

|  | Retailer bears the risk | Manufacturer bears the risk |
| :--- | :--- | :--- |
| LDAE | $D=g_{1}(p)+g_{2}(p) \in_{\text {with }} g_{1}(p)=a-b p_{\text {and }} g_{2}(p)=1_{\text {i.e. }} D=(a-b p)+\in$ |  |
| Selling price <br> of seller (in\$) | w | $w_{m}=\frac{w r}{r\left(1+\epsilon_{r}\right)}$ |
| Purchase cost <br> to buyer(in $\$$ ) | $\mathrm{w}_{\mathrm{r}}=\mathrm{wr}\left(1+\epsilon_{\mathrm{r}}\right) / \mathrm{r}=\mathrm{w}\left(1+\epsilon_{\mathrm{r}}\right)$ | w |
| Expected <br> profit of <br> buyer | $\left(p-w_{r}\right)\left(g_{1}(p)+\mu\right)-$ <br> $\left[\left(w_{r}-v\right) \Lambda+\left(p+s-w_{r}\right) \Phi\right]$ | $\left(p-w_{r}\right)\left(g_{1}(p)+\mu\right)-$ <br> $[(w-v) \Lambda+(p+s-w) \Phi]$ |
| Optimum <br> Order <br> quantity $\mathrm{q}^{*}$ | $q^{*}=g_{1}\left(p^{*}\right)+F^{-1}\left(\frac{p+s-w_{r}}{p+s-v}\right)$ | $\Pi_{m}=\left[\frac{w r}{r\left(1+\epsilon_{r}\right)}-c\right] q^{*}$ |
| Profit of the <br> seller $\Pi_{\mathrm{m}}$ | $\Pi_{m}=(w-c) q^{*}$ |  |

(2) Deductions for Isoelastic Demand and Multiplicative Error [IDME]:

|  | Retailer bears the risk | Manufacturer bears the risk |
| :--- | :--- | :--- |
| IDME | $D=g_{1}(p)+g_{2}(p) \in_{\text {with }} g_{1}(p)=0_{\text {and }} g_{2}(p)=a p^{-b}{ }_{\text {i.e. }} D=a p^{-b} \cdot \in$ |  |
| Selling price <br> of seller <br> (in\$) | w | $w_{m}=\frac{w r}{r\left(1+\in_{r}\right)}$ |
| Purchase <br> cost to <br> buyer(in $\$$ ) | $\mathrm{w}_{\mathrm{r}}=\mathrm{wr}\left(1+\epsilon_{\mathrm{r}}\right) / \mathrm{r}=\mathrm{w}\left(1+\epsilon_{\mathrm{r}}\right)$ | w |
| Expected <br> profit of <br> buyer | $\left(p-w_{r}\right)\left(g_{2}(p) \mu\right)-$ <br> $g_{2}(p)\left[\left(w_{r}-v\right) \Lambda+\left(p+s-w_{r}\right) \Phi\right]$ | $(p-w)\left(g_{2}(p) \mu\right)-$ <br> $g_{2}(p)[(w-v) \Lambda+(p+s-w) \Phi]$ |
| Optimum <br> Order <br> quantity $\mathrm{q}^{*}$ | $q^{*}=g_{2}\left(p^{*}\right) F^{-1}\left(\frac{p+s-w_{r}}{p+s-v}\right)$ | $\Pi_{m}=\left[\frac{w r}{r\left(1+\epsilon_{r}\right)}-c\right] q^{*}$ |
| Profit of the <br> seller $\Pi_{\mathrm{m}}$ | $\Pi_{m}=(w-c) q^{*}$ |  |

## IV. Conclusion

We consider a global supply chain consisting of one retailer and one manufacturer from different countries in a newsvendor framework and derive analytic expressions when the retailer or supplier bears the transaction exposure risk. The main contribution of this research is the derivation of analytical expressions of the optimal policies under hybrid demand distribution which includes both additive and multiplicative demand errors as a special cases. This work will be useful to carry forward applications of the model to more complex situations using the optimal decisions arrived at. Attempts to formulate the model using more realistic forms of demand could be a possible future work. More sophisticated work using various forms of the error distributions and using simulations could also give useful contribution to the body of literature of exchange rate risks.

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