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Infinite Series Obtained By Backward Alpha Difference Operator With Real Variable

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Abstract: *In this paper, we derive the formula for Infinite Multi-Series generated by generalized backward alpha difference equation by equating the closed and infinite summation form solutions of higher order generalized $\alpha_i -$ backward difference equation for positive and negative variable. Suitable examples are inserted to illustrate the main results.*
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I. Introduction

The modern theory of differential or integral calculus began in the 17th century with the works of Newton and Leibnitz. In 1989, K.S.Miller and Ross [11] introduced the discrete analogue of the Riemann-Liouville fractional derivative and proved some properties of the fractional derivative operator. In 2011, M.Maria Susai Manuel, et.al, [8, 11] extended the definition of Δ_α to $\Delta_{\alpha(\ell)}$ defined on $u(k)$ as $\Delta_{\alpha(\ell)}v(k) = v(k+\ell) - \alpha v(k)$, where $\alpha \neq 0$, $\ell > 0$ are fixed and $k \in [0,\infty)$ is variable. The results derived in [11] are coincide with the results in [7] when $\alpha = 1$. An equation involving both Δ and Δ_α is called mixed difference equation. Oscillatory behaviour of solutions certain types of mixed difference equations have been dicussed in [3, 4, 6, 12]. In 2014, G.Britto Antony Xavier, et al. [2], [3] proved several interesting results of geometric progression using generalized difference operator and q-difference operator.

In this paper, we obtain infinite summation form and closed form solution of higher order backward $\alpha -$difference equation for getting formula of infinite multi-series of polynomials.

II. Preliminaries

In this section, we define the generalized backward alpha difference operator and we presents certain results on its inverse alpha difference operator with polynomial and polynomial factorials for positive and negative variable k.

Definition 2.1 *If $v(k)$ is a real valued function on $[0,\infty)$, then the generalized difference operator for negative ℓ denoted by $\Delta_{\alpha(-\ell)}$ is defined as*

$$\Delta_{\alpha(-\ell)}v(k) = v(k-\ell) - \alpha v(k), \ell \in (0,\infty) \tag{1}$$

If $\Delta_{\alpha(-\ell)}v(k) = u(k)$ then the inverse generalized $\alpha -$difference equation is defined as

$$v(k) = \Delta^{-1}_{\alpha(-\ell)}u(k)$$

Definition 2.2 *The higher order generalized $\alpha_i -$difference equation is defined as*

$$\Delta_{\alpha_1(-\ell_1)}(\Delta_{\alpha_2(-\ell_2)}\cdots) = u(k), k \in [0,\infty), \ell_i > 0 \tag{2}$$

III. Infinite α Summation On Positive Variable k

In this section we derive infinite alpha summation formula for positive variable k using the inverse of generalized backward alpha difference operator

Theorem 3.1 *(Infinite α-summation formula for $k>0$) For $\ell, \alpha > 0$*

if $\Delta^{-1}_{\alpha(-\ell)} \lim_{r\to\infty} \alpha^{[\frac{k}{\ell}]} u(k+[\frac{k}{\ell}]\ell) = 0$, then we have

$$\Delta^{-1}_{\alpha(-\ell)} u(k) = \sum_{r=1}^{\infty} \alpha^{r-1} u(k+r\ell) \tag{3}$$

Proof: By taking $\Delta^{-1}_{-\ell} u(k) = v(k)$, we have $\Delta_{-\ell} v(k) = u(k)$, which gives

$$v(k-\ell) = u(k) + \alpha v(k) \tag{4}$$

Replacing k by $k+\ell$ in (4), we get

$$v(k) = u(k+\ell) + \alpha v(k+\ell) \tag{5}$$

Substituting (5) in (4), we get

$$v(k-\ell) = u(k) + \alpha u(k+\ell) + \alpha^2 v(k+\ell) \tag{6}$$

Replacing k by $k+\ell$ in (12),we obtain

$$v(k) = u(k+\ell) + \alpha u(k+2\ell) + \alpha^2 v(k+2\ell) \tag{7}$$

Substituting (7) in (4), we get

$$v(k-\ell) = u(k) + \alpha u(k+\ell) + \alpha^2 u(k+2\ell) + \alpha^3 v(k+2\ell) \tag{8}$$

Replacing k by $k+\ell$ in (8)

$$v(k) = u(k+\ell) + \alpha u(k+2\ell) + \alpha^2 u(k+3\ell) + \alpha^3 v(k+3\ell) \tag{9}$$

Proceeding like this we get

$$v(k) = u(k+\ell) + \alpha u(k+2\ell) + \alpha^2 u(k+3\ell) + \alpha^3 u(k+3\ell) + \ldots \tag{10}$$

which gives (3).

Corollary 3.2 *Let $k \in [0,\infty)$ and if $\lim_{r\to\infty} \Delta^{-1}_{-\ell} u(k) = 0$, then we have*

$$\Delta^{-1}_{-\ell} u(k)\Big|_k^{\infty} = \sum_{r=1}^{\infty} u(k+r\ell) \tag{11}$$

Proof: The proof follows by taking $\alpha = 1$ *in* (3).

Theorem 3.3 *If $\alpha_1, \alpha_2 \neq 1, k \in (0,\infty)$, then we have*

$$\Delta^{-1}_{\alpha_2(-\ell_2)} \Delta^{-1}_{\alpha_1(-\ell_1)} u(k) = \sum_{r_1=1}^{\infty}\sum_{r_2=1}^{\infty} \alpha_1^{r_1-1} \alpha_2^{r_2-1} u(k + r_1\ell_1 + r_2\ell_2) \tag{12}$$

Proof: Replacing ℓ, α by ℓ_2, α_2 in (3), we get

$$\Delta^{-1}_{\alpha_{\alpha_2(-\ell_2)}} u(k) = u(k+\ell_2) + \alpha_2 u(k+2\ell_2) + \ldots$$

Replacing k by $k+\ell_1$

$$\Delta^{-1}_{\alpha_{2(-\ell_2)}} u(k+\ell_1) = u(k+\ell_1+\ell_2) + \alpha_2 u(k+\ell_1+2\ell_2) + \alpha_2^2 u(k+\ell_1+3\ell_2) + \ldots$$

Replacing k by $k+2\ell_1$ and multiplying α_1 on both sides, we get

$$\Delta^{-1}_{\alpha_2(-\ell_2)} u(k+2\ell_1) = u(k+2\ell_1+\ell_2) + \alpha_2 u(k+2\ell_1+2\ell_2) + \ldots$$

Replacing k by $k+3\ell_1$ and multiplying α_1^2 on both sides, we get

$$\Delta^{-1}_{\alpha_{2(-\ell_2)}} u(k+3\ell_1) = u(k+3\ell_1+\ell_2) + \alpha_2 u(k+3\ell_1+2\ell_2) + \ldots$$

Replacing k by $k+4\ell_1$ and multiplying α_1^3 on both sides, we get

$$\Delta^{-1}_{\alpha_{2(-\ell_2)}}u(k+4\ell_1)=u(k+4\ell_1+\ell_2)+\alpha_2 u(k+4\ell_1+2\ell_2)+...$$

Replacing k by $k+r_1\ell_1$ and multiplying $\alpha_1^{r_1-1}$ on both sides, we get

$$\Delta^{-1}_{\alpha_2(-\ell_2)}u\Delta(k+r_1\ell_1)=u(k+r_1\ell_1+\ell_2)+\alpha_2 u(k+r_1\ell_1+2\ell_2)+...$$

Proceeding like this we get (12).

Corollary 3.4 *If* $\alpha>0, k\in(0,\infty)$*, then we have*

$$\Delta^{-1}_{\alpha(-\ell_2)}\Delta^{-1}_{\alpha(-\ell_1)}u(k)=\sum_{r_1=1}^{\infty}\sum_{r_2=1}^{\infty}\alpha^{r_1+r_2-2}u(k+r_1\ell_1+r_2\ell_2) \tag{13}$$

Proof: The proof follows by taking $\alpha_1=\alpha_2=\alpha$ in Theorem 3.3.

Corollary 3.5 *If* $\alpha=1>0, k\in(0,\infty)$*, then we have*

$$\Delta^{-1}_{(-\ell_2)}\Delta^{-1}_{(-\ell_1)}u(k)=\sum_{r_1=1}^{\infty}\sum_{r_2=1}^{\infty}u(k+r_1\ell_1+r_2\ell_2) \tag{14}$$

Proof: The proof follows by taking $\alpha=1$ in corollary 3.4.

Theorem 3.6 *If* $\alpha>0, k\in(0,\infty)$*, then we have*

$$\Delta^{-1}_{\alpha(-\ell_1)}\Delta^{-1}_{\alpha(-\ell_2)}\Delta^{-1}_{\alpha_3(-\ell_3)}u(k)=\sum_{r_1=1}^{\infty}\sum_{r_2=1}^{\infty}\sum_{r_3=1}^{\infty}\alpha_1^{r_1-1}\alpha_2^{r_1-1}\alpha_3^{r_3-1}u(k+r_1\ell_1+r_2\ell_2+r_3\ell_3) \tag{15}$$

Proof: Replacing r_1,r_2 by r_2,r_3 and ℓ_1,ℓ_2 by ℓ_2,ℓ_3 and α_1,α_2 by α_2,α_3 in(12),we get

$$\Delta^{-1}_{\alpha_3(-\ell_3)}\Delta^{-1}_{\alpha_2(-\ell_2)}u(k)=\sum_{r_2=1}^{\infty}\sum_{r_3=1}^{\infty}\alpha_2^{r_2-1}\alpha_3^{r_3-1}u(k+r_2\ell_2+r_3\ell_3)$$

Replacing k by $k+\ell_1$ and multiplying α_1 on both sides, we get

$$\Delta^{-1}_{\alpha_3(-\ell_3)}\Delta^{-1}_{\alpha_2(-\ell_2)}u(k+\ell_1)=\sum_{r_2=1}^{[\frac{k+\ell_1}{\ell_2}]}\sum_{r_3=1}^{[\frac{k+\ell_1+r_2\ell_2}{\ell_2}]}\alpha_2^{r_2-1}\alpha_3^{r_3-1}u(k+\ell_1+r_2\ell_2+r_3\ell_3)$$

Replacing k by $k+2\ell_1$ and multiplying α_1^2 on both sides, we get

$$\Delta^{-1}_{\alpha_3(-\ell_3)}\Delta^{-1}_{\alpha_2(-\ell_2)}u(k+2\ell_1)=\sum_{r_2=1}^{[\frac{k+2\ell_1}{\ell_2}]}\sum_{r_3=1}^{[\frac{k+2\ell_1+r_2\ell_2}{\ell_2}]}\alpha_2^{r_2-1}\alpha_3^{r_3-1}u(k+2\ell_1+r_2\ell_2+r_3\ell_3)$$

Proceeding like this and replacing k by $k+r_1\ell_1$ and multiplying α_1^{r-1} we get (15).

Corollary 3.7 *If* $\alpha>0, k\in(0,\infty)$*, then we have*

$$\Delta^{-1}_{\alpha(-\ell_1)}\Delta^{-1}_{\alpha(-\ell_2)}\Delta^{-1}_{\alpha(-\ell_3)}u(k)=\sum_{r_1=1}^{\infty}\sum_{r_2=1}^{\infty}\sum_{r_3=1}^{\infty}\alpha^{r_1+r_2+r_3-3}u(k+r_1\ell_1+r_2\ell_2+r_3\ell_3)$$

Proof: The proof follows by taking $\alpha_1=\alpha_2=\alpha_3=\alpha$ in Theorem 3.6.

Corollary 3.8 *If* $\alpha>0, k\in(0,\infty)$*, then we have*

$$\Delta^{-1}_{(-\ell_1)}\Delta^{-1}_{(-\ell_2)}\Delta^{-1}_{(-\ell_3)}u(k)=\sum_{r_1=1}^{\infty}\sum_{r_2=1}^{\infty}\sum_{r_3=1}^{\infty}u(k+r_1\ell_1+r_2\ell_2+r_3\ell_3)$$

Proof: The proof follows by taking $\alpha=1$ in corollary 3.9.

Theorem 3.9 *(α_i - higher order summation formula) If* $k\in[0,\infty)$ *and* $\ell_i>0$*, then*

$$\prod_{n=1}^{t}\Delta^{-1}_{\alpha_n(-\ell_n)}u(k)=\sum_{r_{[1-n]}=1}^{\infty}\prod_{n=1}^{t}\alpha_n^{r_n-1}u(k+\sum_{n=1}^{t}r_n\ell_n) \tag{16}$$

Proof: The proof follows by Theorem 3.6, Theorem 3.3 and Theorem 3.1.

Corollary 3.10 *If* $k\in[0,\infty)$ *and* $\ell_i>0$, *then we have*

$$\prod_{n=1}^{t}\Delta^{-1}_{\alpha(-\ell_n)}u(k)=\sum_{r_{[1-n]}=1}^{\infty}\prod_{n=1}^{t}\alpha^{r_n-1}u(k+\sum_{n=1}^{t}r_n\ell_n) \tag{17}$$

Proof: The proof follows by taking $\alpha_n=\alpha$ in Corollary 3.9.

Corollary 3.11 *If* $k\in(0,\infty)$ *and* $\ell_i>0$, *then we have*

$$\prod_{n=1}^{t}\Delta^{-1}_{(-\ell_n)}u(k)=\sum_{r_{[1-n]}=1}^{\infty}\prod_{n=1}^{t}u(k+\sum_{n=1}^{t}r_n\ell_n) \tag{18}$$

Proof: The proof follows by taking $\alpha=1$ in Corollary 3.10.

Theorem 3.12 *(Infinite* $\frac{1}{\alpha}$*-summation formula for* $k<0$*) For* $\ell>0$,

if $\Delta^{-1}_{\alpha(-\ell)}\lim_{r\to\infty}\frac{1}{\alpha^{[-\frac{k}{\ell}]+1}}u(k-([-\frac{k}{\ell}]+1)\ell)=0$, then we have

$$\Delta^{-1}_{\alpha(-\ell)}u(k)=\sum_{r=0}^{\infty}\frac{-1}{\alpha^{r+1}}u(k-r\ell) \tag{19}$$

Proof: By taking $\Delta^{-1}_{-\ell}u(k)=v(k)$, we have $\Delta_{-\ell}v(k)=u(k)$, which gives

$$v(k)=\frac{-1}{\alpha}u(k)+\frac{1}{\alpha}v(k-\ell) \tag{20}$$

Replacing k by $k-\ell$ in (20), we get

$$v(k-\ell)=\frac{-1}{\alpha}u(k-\ell)+\frac{1}{\alpha}v(k-2\ell) \tag{21}$$

Substituting (21) in (20)

$$v(k)=\frac{-1}{\alpha}u(k)-\frac{1}{\alpha^2}u(k-\ell)+\frac{1}{\alpha^2}v(k-2\ell) \tag{22}$$

Replacing k by $k-\ell$ in (22), we get

$$v(k-\ell)=\frac{-1}{\alpha}u(k-\ell)-\frac{1}{\alpha^2}u(k-2\ell)+\frac{1}{\alpha^2}v(k-3\ell) \tag{23}$$

Substituting (23) in (20), we obtain

$$v(k)=\frac{-1}{\alpha}u(k)-\frac{1}{\alpha^2}u(k-\ell)-\frac{1}{\alpha^3}u(k-2\ell)+\frac{1}{\alpha^3}v(k-3\ell)$$

Proceeding like this we get

$v(k)=\frac{-1}{\alpha}u(k)-\frac{1}{\alpha^2}u(k-\ell)-\frac{1}{\alpha^3}u(k-2\ell)-\frac{1}{\alpha^4}u(k-3\ell)-\dots$ which gives (19).

Corollary 3.13 *(Infinite summation formula for* $k<0$ *) For* $\ell>0$,

if $\Delta^{-1}_{(-\ell)}\lim_{r\to\infty}u(k-([-\frac{k}{\ell}]+1)\ell)=0$, then we have

$$\Delta^{-1}_{(-\ell)}u(k)=-\sum_{r=0}^{\infty}u(k-r\ell) \tag{24}$$

Proof: The proof follows by $\alpha=1$ in Theorem 3.12.

Theorem 3.14 *If* $\alpha_1,\alpha_2\neq 0, k\in(0,\infty)$, *then we have*

$$\Delta^{-1}_{\alpha_2(-\ell_2)}\Delta^{-1}_{\alpha_1(-\ell_1)}u(k)=\sum_{r_1=0}^{\infty}\sum_{r_2=0}^{\infty}\frac{-1}{\alpha_1^{r_1+1}}\frac{-1}{\alpha_2^{r_2+1}}u(k-r_1\ell_1-r_2\ell_2) \tag{25}$$

Proof: Replacing $\ell=\ell_1$ and $\alpha=\alpha_1$ in (24)

$$\Delta^{-1}_{\alpha_1(-\ell_1)}u(k)=\frac{-1}{\alpha_1}u(k)-\frac{-1}{\alpha_1^2}u(k-\ell_1)-\frac{1}{\alpha_1^3}u(k-2\ell_1)...$$

Replacing ℓ_1 by ℓ_2 and α_1 by α_2 and multiplying $\frac{-1}{\alpha_1}$ on both sides, we get

$$\Delta^{-1}_{\alpha_2(-\ell_2)}u(k)=\frac{-1}{\alpha_2}u(k)-\frac{-1}{\alpha_2^2}u(k-\ell_2)-\frac{1}{\alpha_2^3}u(k-2\ell_2)...$$

Replacing k by $k-\ell_1$ and multiplying $\frac{-1}{\alpha_1^2}$ on both sides, we get

$$\Delta^{-1}_{\alpha_2(-\ell_2)}u(k-\ell_1)=\frac{-1}{\alpha_2}u(k-\ell_1)-\frac{1}{\alpha_2^2}u(k-\ell_1-\ell_2)\ \ -\frac{1}{\alpha_2^3}u(k-\ell_1-2\ell_2)...$$

Replacing k by $k-2\ell_1$ and multiplying $\frac{-1}{\alpha_1^3}$ on both sides, we get

$$\Delta^{-1}_{\alpha_2(-\ell_2)}u(k-2\ell_1)=\frac{-1}{\alpha_2}u(k-2\ell_1)-\frac{1}{\alpha_2^2}u(k-2\ell_1-\ell_2)-\frac{1}{\alpha_2^3}u(k-2\ell_1-2\ell_2)...$$

Replacing k by $k-3\ell_1$ and multiplying $\frac{-1}{\alpha_1^4}$ on both sides, we get

$$\Delta^{-1}_{\alpha_2(-\ell_2)}u(k-3\ell_1)=\frac{-1}{\alpha_2}u(k-3\ell_1)-\frac{1}{\alpha_2^2}u(k-3\ell_1-\ell_2)-\frac{1}{\alpha_2^3}u(k-3\ell_1-2\ell_2)...$$

Replacing k by $k-r\ell_1$ and multiplying $\frac{-1}{\alpha_1^{r_1+1}}$ on both sides, we get

$$\Delta^{-1}_{\alpha_2(-\ell_2)}u(k-r_1\ell_1)=\frac{-1}{\alpha_2}u(k-r_1\ell_1)-\frac{1}{\alpha_2^2}u(k-r_1\ell_1-\ell_2)-\frac{1}{\alpha_2^3}u(k-r_1\ell_1-2\ell_2)...$$

Proceeding like this we get (25).

Corollary 3.15 *If* $\alpha>0, k\in(0,\infty)$*, then we have*

$$\Delta^{-1}_{\alpha(-\ell_2)}\Delta^{-1}_{\alpha(-\ell_1)}u(k)=\sum_{r_1=0}^{\infty}\sum_{r_2=0}^{\infty}\frac{-1}{\alpha^{r_1+1}}\frac{-1}{\alpha^{r_2+1}}u(k-r_1\ell_1-r_2\ell_2) \tag{26}$$

Proof: The proof follows by Theorem 3.14, when $\alpha_1=\alpha_2=\alpha$.

Corollary 3.16 *If* $\ell>0, k\in(0,\infty)$*, then we have*

$$\Delta^{-1}_{(-\ell_2)}\Delta^{-1}_{(-\ell_1)}u(k)=\sum_{r_1=0}^{\infty}\sum_{r_2=0}^{\infty}u(k-r_1\ell_1-r_2\ell_2) \tag{27}$$

Proof: The proof follows by corollary3.15, when $\alpha=1$.

Theorem 3.17 *If* $\frac{1}{\alpha}>0, k\in(0,\infty)$*, then we have*

$$\Delta^{-1}_{\alpha_1(-\ell_1)}\Delta^{-1}_{\alpha_2(-\ell_2)}\Delta^{-1}_{\alpha_3(-\ell_3)}u(k)=\sum_{r_1=0}^{\infty}\sum_{r_2=0}^{\infty}\sum_{r_3=0}^{\infty}\frac{-1}{\alpha_1^{r_1+1}}\frac{-1}{\alpha_2^{r_2+1}}\frac{-1}{\alpha_3^{r_3+1}}u(k-r_1\ell_1-r_2\ell_2-r_3\ell_3) \tag{28}$$

Proof: Replacing r_1,r_2 by r_2,r_3 and ℓ_1,ℓ_2 by ℓ_2,ℓ_3 and α_1,α_2 by α_2,α_3 in (25), we get

$$\Delta^{-1}_{\alpha_3(-\ell_3)}\Delta^{-1}_{\alpha_2(-\ell_2)}u(k)=\sum_{r_2=0}^{\infty}\sum_{r_3=0}^{\infty}\frac{-1}{\alpha_2^{r_2+1}}\frac{-1}{\alpha_3^{r_3+1}}u(k-r_2\ell_2-r_3\ell_3)$$

Replacing k by $k-\ell_1$ and multiplying $\frac{1}{\alpha_1}$ on both sides

$$\Delta^{-1}_{\alpha_3(-\ell_3)}\Delta^{-1}_{\alpha_2(-\ell_2)}u(k-\ell_1)=\sum_{r_2=0}^{\infty}\sum_{r_3=0}^{\infty}\frac{-1}{\alpha_2^{r_2+1}}\frac{-1}{\alpha_3^{r_3+1}}u(k-\ell_1-r_2\ell_2-r_3\ell_3)$$

Replace k by $k-2\ell_1$ and multiplying $(\frac{1}{\alpha_1^2})$ on both sides

$$\Delta^{-1}_{\alpha_3(-\ell_3)}\Delta^{-1}_{\alpha_2(-\ell_2)}u(k-2\ell_1)=\sum_{r_2=0}^{\infty}\sum_{r_3=0}^{\infty}\frac{-1}{\alpha_2^{r_2+1}}\frac{-1}{\alpha_3^{r_3+1}}u(k-2\ell_1-r_2\ell_2-r_3\ell_3)$$

Procuding like this and replacing k by $k-r_1\ell_1$ and multiplying $\frac{1}{\alpha_1^{r-1}}$ we get (25).

Theorem 3.18 *(α - higher order summation formula) If* $k\in(0,\infty)$ and $\ell_i>0$, then we have

$$\prod_{n=1}^{t}\Delta^{-1}_{\alpha_n(-\ell_n)}u(k)=\sum_{r_{[1-t]}}^{\infty}\prod_{n=1}^{t}\alpha_n^{r_n-1}u(k-\sum_{n=1}^{t}r_t\ell_t)\text{, where } t=1,2,3...,\infty$$

Proof: The proof follows by Lemma 3.12, Theorem 3.14 and Theorem 3.17.

Corollary 3.19 *If* $k\in[0,\infty)$ *and* $\ell_i>0$*, then we have*

$$\prod_{n=1}^{t}\Delta^{-1}_{\alpha(-\ell_n)}u(k)=\sum_{r_{[1-t]}}^{\infty}\prod_{n=1}^{t}\alpha^{r_n-1}u(k-\sum_{n=1}^{t}r_t\ell_t)\text{, where } t=1,2,3...,\infty$$

Proof: The proof follows by Theorem 3.18, when $\alpha_i=\alpha$.

Theorem 3.20 *If* $\alpha\neq1,\ell\in(0,\infty)$*,then we have*

$$\frac{1}{k^{(n)}_{-\ell}}\Big|_j^k=\Delta^{-1}_{\alpha(-\ell)}\left\{\frac{k(1-\alpha)+\ell((n-1)+\alpha)}{(k-\ell)^{(n+1)}_{-\ell}}\right\}\Big|_j^k \qquad (29)$$

Proof: From the Definition $\Delta_{-\ell}$ we have

$$\Delta_{\alpha(-\ell)}\frac{1}{k^{(1)}_{-\ell}}=\frac{1}{(k-\ell)^{(1)}_{-\ell}}-\alpha\frac{1}{k^{(1)}_{-\ell}}\ \Rightarrow\frac{1}{k^{(1)}_{-\ell}}=\Delta^{-1}_{\alpha(-\ell)}\left\{\frac{k(1-\alpha)+\alpha\ell}{(k-\ell)^{(2)}_{-\ell}}\right\}$$

Second

$$\Delta_{\alpha(-\ell)}\frac{1}{k^{(2)}_{-\ell}}=\frac{1}{(k-\ell)^{(2)}_{-\ell}}-\alpha\left\{\frac{1}{k^{(2)}_{-\ell}}\right\}\ \Rightarrow\frac{1}{k_{-\ell}}=\Delta^{\alpha(-\ell)}_{-1}\left\{\frac{k(1-\alpha)+\ell(3+\ell)}{(k-\ell)^{(}_{-\ell}n+1)}\right\}$$

Third

$$\Delta_{\alpha(-\ell)}\frac{1}{k^{(3)}_{-\ell}}=\frac{1}{(k-\ell)^{(3)}_{-\ell}}-\alpha\left\{\frac{1}{k^{(3)}_{-\ell}}\right\}\ \Rightarrow\frac{1}{k^{(3)}_{-\ell}}\Big|_j^k=\Delta^{\alpha(-\ell)}_{-1}\left\{\frac{k(1-\alpha)+\ell(2+\alpha)}{(k-\ell)^{(4)}_{-\ell}}\right\}\Big|_j^k$$

Similarly

$$\frac{1}{k^{(4)}_{-\ell}}\Big|_j^k=\Delta^{\alpha(-\ell)}_{-1}\left\{\frac{k(1-\alpha)+\ell(3+\alpha)}{(k-\ell)^{(5)}_{-\ell}}\right\}\Big|_j^k$$ which gives (29).

IV. Conclusion

We have obtained formulas for several infinite $\alpha -$ series on polynomial using inverse of generalized alpha difference operator.

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