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Solving Fuzzy Transportation problem with Generalized Hexagonal Fuzzy Numbers

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Abstract: *In this paper we introduce a fuzzy transportation problem (FTP) in which the values of transportation costs are represented by generalized hexagonal fuzzy numbers. Here the FTP is converted to crisp one by ranking function of fuzzy numbers. The initial basic feasible solution and optimal solutions are derived without solving the original FTP. Hence it reduces the computational complexity of deriving the solutions.*
Keywords: *Fuzzy Transportation Problem, Generalized Hexagonal Fuzzy Number, Ranking Index, Optimal Solution*

I. Introduction

Transportation problem was originally introduced and developed by Hitchcock in 1941, in which the parameters like transportation cost, demand and supply are crisp values. But in the present world the transportation parameters may be uncertain due to many uncontrolled factors. So to deal the problems with imprecise information Zadeh [1] introduced the concept of fuzziness. Many authors discussed the solution of FTP with various fuzzy numbers. Pandian and Natarajan [2] proposed a new algorithm namely fuzzy zero point method to find optimal solution of a FTP with trapezoidal fuzzy numbers. Chen [3] introduced the concept of generalized fuzzy numbers to deal problems with unusual membership function. Many researchers applied generalized fuzzy numbers to solve the real life problems. Kaur and Kumar[4,5] solved FTP with generalized trapezoidal fuzzy numbers. In the present paper a FTP with generalized hexagonal fuzzy numbers is introduced with suitable solution algorithms. The paper is organized as follows: In section 2, we recall the basic concepts. In section 3, we introduce generalized hexagonal fuzzy numbers and its properties. In section 4, we introduce FTP in terms of generalized hexagonal fuzzy costs and we propose Initial Basic Feasible Solution (IBFS) and the optimal solution algorithms. In section 5, an application is given to check the solutions and then they are physically interpreted. Finally, the paper is concluded with future work in section 6.

II. Preliminaries

2.1. Fuzzy Number

A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set on the real line R such that:

- There exist atleast one $x \in$ R with $\mu_{\tilde{A}}(x) = 1$
- $\mu_{\tilde{A}}(x)$ is piecewise continuous

2.2. Triangular Fuzzy Number

A fuzzy number $\tilde{A}$ is a TFN [6] denoted by $\tilde{A} = (a_1, a_2, a_3)$ where a_1, a_2 and a_3 are real numbers and its membership function is given below,

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & for\ a_1 \le x \le a_2 \\ 1 & for\ x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & for\ a_2 \le x \le a_3 \\ 0, & otherwise \end{cases}$$

2.3. Trapezoidal Fuzzy Number

A fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a TrFN [7] where a_1, a_2, a_3 and a_4 are real numbers and its membership function is given below,

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & for\ a_1 \le x \le a_2 \\ 1, & for\ a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3}, & for\ a_3 \le x \le a_4 \\ 0, & otherwise \end{cases}$$

2.4. Hexagonal Fuzzy Number

A fuzzy number $\tilde{A}_H$ is a HFN [8] denoted by $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ where a_1, a_2, a_3, a_4, a_5 and a_6 are real numbers and its membership function is given below,

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & for\ a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right), & for\ a_2 \le x \le a_3 \\ 1, & for\ a_3 \le x \le a_4 \\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), & for\ a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right), & for\ a_5 \le x \le a_6 \\ 0, & otherwise \end{cases}$$

2.5. Arithmetic operations on Hexagonal Fuzzy Number

If $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{B}_H = (b_1, b_2, b_3, b_4, b_5, b_6)$ are two HFN's, then the following three operations can be performed as follows:

- Addition: $\tilde{A}_H + \tilde{B}_H = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5, a_6+b_6)$
- Subtraction: $\tilde{A}_H - \tilde{B}_H = (a_1-b_6, a_2-b_5, a_3-b_4, a_4-b_3, a_5-b_2, a_6-b_1)$
- Multiplication: $\tilde{A}_H * \tilde{B}_H = (a_1*b_1, a_2*b_2, a_3*b_3, a_4*b_4, a_5*b_5, a_6*b_6)$

III. Generalized Hexagonal fuzzy number

3.1. Generalized Hexagonal Fuzzy Number

If a generalized hexagonal fuzzy number denoted by $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6; w)$ where a_1, a_2, a_3, a_4, a_5 and a_6 are real numbers and w is its maximum membership degree, its membership function is given below

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{w}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & for\ a_1 \le x \le a_2 \\ \frac{w}{2}+\frac{w}{2}\left(\frac{x-a_2}{a_3-a_2}\right), & for\ a_2 \le x \le a_3 \\ w, & for\ a_3 \le x \le a_4 \\ w-\frac{w}{2}\left(\frac{x-a_4}{a_5-a_4}\right), & for\ a_4 \le x \le a_5 \\ \frac{w}{2}\left(\frac{a_6-x}{a_6-a_5}\right), & for\ a_5 \le x \le a_6 \\ 0, & otherwise \end{cases}$$

3.2. Ranking Generalized Hexagonal Fuzzy Numbers based on centroid

In a hexagonal fuzzy number, the hexagon is divided into two triangles AQB and RFE and a hexagon BQREDCB as shown in Figure 1. The centroid of the hexagonal fuzzy number is the centre point (balancing point) of the hexagon ABCDEFA.

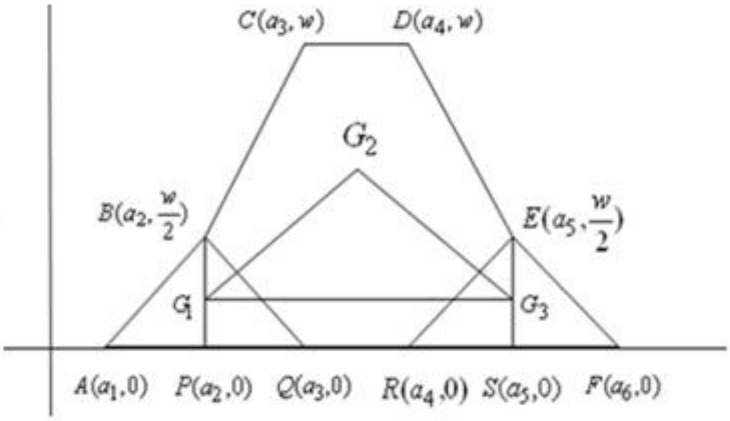

Figure 1

Using the centroid of the above three figures, the centroid $G_{\tilde{A}_H}(\bar{x}_0, \bar{y}_0)$ of the triangle with vertices G_1, G_2 and G_3 of the hexagonal fuzzy number $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ is defined as $G_{\tilde{A}_H}(\bar{x}_0, \bar{y}_0) = \left(\frac{2a_1+3a_2+4a_3+4a_4+3a_5+2a_6}{18}, \frac{5w}{18}\right)$.

The ranking index [9] of a generalized hexagonal fuzzy number $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6; w)$ which maps every fuzzy number to a of real number as follows: $R(\tilde{A}_H) = \left(\frac{2a_1+3a_2+4a_3+4a_4+3a_5+2a_6}{18}\right)\left(\frac{5w}{18}\right)$.

3.3. Arithmetic operations on Hexagonal Fuzzy Numbers

If $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6; w_1)$ and $\tilde{B}_H = (b_1, b_2, b_3, b_4, b_5, b_6; w_2)$ are two generalized hexagonal fuzzy numbers then

(i) $\tilde{A}_H + \tilde{B}_H = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6; \min(w_1, w_2))$

(ii) $\tilde{A}_H - \tilde{B}_H = (a_1 - b_6, a_2 - b_5, a_3 - b_4, a_4 - b_3, a_5 - b_2, a_6 - b_1; \min(w_1, w_2))$

(iii) $\tilde{A}_H * \tilde{B}_H = (a_1 * b_1, a_2 * b_2, a_3 * b_3, a_4 * b_4, a_5 * b_5, a_6 * b_6; \min(w_1, w_2))$

(iv) $\lambda\tilde{A}_H = \begin{cases} (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4, \lambda a_5, \lambda a_6; w_1), \text{if } \lambda > 0 \\ (\lambda a_6, \lambda a_5, \lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1; w_1), \text{if } \lambda < 0 \end{cases}$

IV. Fuzzy Transportation Problem

4.1. Introduction

In classical transportation problem, the decision maker is sure about the transportation cost, demand and supply. But in the real world it not possible to state the precise information about a problem. For example, if a new product is launched in the market, nobody is sure about the transportation cost, demand and supply of the product. Hence there exist uncertainties about the total transportation cost.

Consider a FTP with m sources and n destinations with HFN's. The mathematical formulation of the FTP whose parameters are HFN's under the case that the total supply is equivalent to the total demand is given by:

$$Minimize\ \tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij} \quad \text{Subject to}$$

$$\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i \quad i = 1, 2, \ldots, m \quad , \quad \sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j \quad j = 1, 2, \ldots, n$$

$$\sum_{i=1}^{m} \tilde{a}_i \approx \sum_{j=1}^{n} \tilde{b}_j \quad i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n \text{ and } \tilde{x}_{ij} \geq \tilde{0}$$

In which the transportation costs $\tilde{c}_{ij}$, supply $\tilde{a}_i$ and demand $\tilde{b}_j$ are fuzzy quantities. Here we consider a FTP in which the transportation costs are generalized hexagonal fuzzy numbers, demand and supply are crisp values.

4.2. Generalized Fuzzy North West Corner Rule

Generalized Fuzzy North West Corner Rule is extended [4] from the classical North West Corner Rule and is used to find IBFS. The algorithm is given below:

Step 1: Construct a crisp transportation table of the given FTP by replacing all the generalized hexagonal fuzzy costs by its ranking index as in 3.2 where $w = \wedge w_i, 1 = 1,2,\dots k$ (minimum of all w's in the generalized hexagonal fuzzy costs) and k varies over the generalized hexagonal fuzzy costs in the fuzzy transportation table.

Step 2: Apply North West Corner rule of the classical transportation problem to the crisp transportation table obtained from step 1.

4.3. Generalized Fuzzy Improved Zero Suffix Method Algorithm

This algorithm is used to find the generalized fuzzy optimum solution. The algorithm is given below.

Step1: Construct a crisp transportation table of the given FTP by replacing all the generalized hexagonal fuzzy costs by its ranking index as in 3.2 where $w = \wedge w_i, 1 = 1,2,\dots k$ and k varies over the generalized hexagonal fuzzy costs in the fuzzy transportation table.

Step2: In each row, subtract the row minimum from each row entries. The same process must be done for each column of the transportation table.

Step3: In the reduced crisp cost matrix, there will be at least one zero in each row and each column. Find the index of all the zero's in i th row and j th column as follows:

$$Index(0_{ij}) = \frac{Sum\ of\ non\ zero\ costs\ in\ i\ th\ row\ and\ j\ th\ column}{Number\ of\ zeros\ in\ \ i\ th\ row\ and\ j\ th\ column}$$

Step4: Choose the maximum of S, if it has one maximum value then first supply to that demand corresponding to the cell. If it has more equal values then select {ai,bj} and supply to that demand maximum possible.

Step5: After the above steps, the exhausted demands or supplies to be trimmed. The resultant matrix must possess at least one zero is each row and column, otherwise repeat step 2.

Step6: Repeat steps 3 to 5 until the optimal solution is obtained.

V. Application

5.1. Numerical Example

Consider the following transportation problem with generalized fuzzy costs.

Table 1

Destination / Origin	D1	D2	D3	D4	Supply
O1	(3,7,11, 15,19,24;0.5)	(13,18,23, 28,33,40;0.7)	(6,13,20, 28,36,45;0.4)	(15,20,25, 31,38,45;0.8)	16
O2	(16,19,24, 29,34,39;0.2)	(3,5,7, 9,10,12;0.5)	(5,7,10, 13,17,21;0.6)	(20,23,26, 30,35,40;0.4)	36
O3	(11,14,17, 21,25,30;0.7)	(7,9,11, 14,18,22;0.6)	(2,3,4, 6,7,9;0.5)	(5,7,8, 11,14,17;0.9)	20
Demand	24	18	20	10	

5.2. Generalized Fuzzy IBFS

In Table1, the total demand and supply are equal. Therefore the given FTP is balanced one. The FTP is converted into a crisp transportation problem using the ranking index of generalized hexagonal fuzzy numbers and is as follows:

Table 2

Destination / Origin	D1	D2	D3	D4	Supply
O1	0.73	1.45	1.36	1.6	16
O2	1.48	0.43	0.67	1.6	36
O3	1.08	0.74	0.28	0.56	20
Demand	24	18	20	10	

By applying Generalized NWCR, we get the following allotment table.

Table 3

Destination / Origin	D1	D2	D3	D4	Supply
O1	**16** 0.73	1.45	1.36	1.6	16
O2	**8** 1.48	**18** 0.43	**10** 0.67	1.6	36
O3	1.08	0.74	**10** 0.28	**10** 0.56	20
Demand	24	18	20	10	

Therefore the IBFS in terms of generalized HFN's is ,

$$x_{11} = 16, x_{21} = 8, x_{22} = 18, x_{23} = 10, x_{33} = 10, x_{34} = 10$$

with the total minimum fuzzy cost

$$Minimize\ \tilde{Z} = c_{11}x_{11} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + c_{33}x_{33} + c_{34}x_{34}$$

= 16(3,7,11,15,19,24;0.5) + 8(16,19,24,29,34,39;0.2) + 18(3,5,7,9,10,12;0.5) + 10(20,23,26,30,35,40;0.6) + 10(2,3,4,6,7,9;0.5) + 10(5,7,8,11,14,17;0.9)

= (350,524,714,934,1136,1382 ;0.2)

5.3. Generalized Fuzzy optimum solution

The optimum solution is obtained by generalized fuzzy improved zero suffix method as follows.
The crisp transportation table for the given FTP in Table 1 is given below.

Table 4

Destination / Origin	D1	D2	D3	D4	Supply
O1	0.73	1.45	1.36	1.6	16
O2	1.48	0.43	0.67	1.6	36
O3	1.08	0.74	0.28	0.56	20
Demand	24	18	20	10	

Now using the step 2 of optimum solution algorithm 4.2, we get the following table

Table 5

Destination / Origin	D1	D2	D3	D4	Supply
O1	0	0.72	0.63	0.59	16
O2	1.05	0	0.24	0.89	36
O3	0.8	0.46	0	0	20
Demand	24	18	20	10	

Calculate the index of each zero and the maximum index occurs at the cell(1,1). Now allocate the maximum possible units 16 in the cell (1,1). And write remaining in column 1. After removing the first row repeats the step 2, we get the following table

Table 6

Destination / Origin	D1	D2	D3	D4	Supply
O1	**16**	-	-	-	-
O2	0.25	0	0.24	0.89	36
O3	0	0.46	0	0	20
Demand	8	18	20	10	

In the above table the maximum zero index is at the cell(2,2). Now allocate the maximum possible units 18 in the cell (2,2). And write remaining in row 2. After removing the second column repeats the step 2 to step 6 until all fuzzy supply points are fully used and fuzzy demand points are fully received, we get the following allocation table

Table 7

Destination / Origin	D1	D2	D3	D4	Supply
O1	**16**	-	-	-	-
O2		**18**	**18**		36
O3	**8**		**2**	**10**	20
Demand	24	18	20	10	

Therefore, the fuzzy optimum solution in terms of generalized hexagonal fuzzy number is

$x_{11} = 16, x_{22} = 18, x_{23} = 18, x_{31} = 8, x_{33} = 2, x_{34} = 10$

$Minimize\ \tilde{Z} = c_{11}x_{11} + c_{22}x_{22} + c_{23}x_{23} + c_{31}x_{31} + c_{33}x_{33} + c_{34}x_{34}$

= 16 (3,7,11,15,19,24;0.5) + 18(3,5,7,9,10,12;0.5) +18(5,7,10,13,17,21;0.6) + 8(11,14,17,21,25,30;0.7) + 2(2,3,4,6,7,9;0.5) + 10(5,7,8,11,14,17;0.9)

= (334,516,706,928,1144,1406;0.2)

5.4. Results and discussions

According to the above IBFS (350,524,714,934,1136,1382;0.2) , the total minimum transportation cost will be greater than 350 and less than 1382. For the total minimum transportation cost lies between 714 to 934, the overall level of acceptance or satisfaction is 20%.And for the remaining values of total minimum transportation cost the level of acceptance is calculated as follows: If x denotes the total cost then the overall level of acceptance is $\mu(x) \times 100$ where

$$\mu(x) = \begin{cases} \frac{0.2}{2}\left(\frac{x-350}{174}\right), & for\ 350 \leq x \leq 524 \\ \frac{0.2}{2} + \frac{0.2}{2}\left(\frac{x-524}{190}\right), & for\ 524 \leq x \leq 714 \\ \mathbf{0.2}, & \mathbf{for\ 714 \leq x \leq 934} \\ 0.2 - \frac{0.2}{2}\left(\frac{x-934}{202}\right), & for\ 934 \leq x \leq 1136 \\ \frac{0.2}{2}\left(\frac{1382-x}{246}\right), & for\ 1136 \leq x \leq 1382 \\ 0, & otherwise \end{cases}$$

Similarly in the optimum solution (334,516,**706,928**,1144,1406;0.2), the minimum cost is 334 and the maximum is 1406. But the cost interval corresponding to highest overall acceptance level 20% is 706 to 928. But it is 714 to 934 in IBFS. Hence the overall acceptance level for the optimum solution is $\mu(x) \times 100$ where

$$\mu(x) = \begin{cases} \frac{0.2}{2}\left(\frac{x-334}{182}\right), & for\ 334 \leq x \leq 516 \\ \frac{0.2}{2} + \frac{0.2}{2}\left(\frac{x-516}{190}\right), & for\ 516 \leq x \leq 706 \\ \mathbf{0.2}, & \mathbf{for\ 706 \leq x \leq 928} \\ 0.2 - \frac{0.2}{2}\left(\frac{x-928}{216}\right), & for\ 928 \leq x \leq 1144 \\ \frac{0.2}{2}\left(\frac{1406-x}{262}\right), & for\ 1144 \leq x \leq 1406 \\ 0, & otherwise \end{cases}$$

VI. Conclusion

In this paper IBFS and the optimum solution of a FTP with generalized hexagonal fuzzy costs are proposed by ranking of generalized hexagonal fuzzy numbers. It is very easy to understand and with less computational complexity like the algorithms of classical transportation problem. And in future study, the algorithms may be modified to reach the solution with a good and maximum level of satisfaction.

References

[1]. L.A.Zadeh, Fuzzy sets, *Information and Control*, *8*, (1965), 338-353.

[2]. P.Pandian and G.Natarajan, A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problem, *Applied Mathematical Sciences*, *4(2)*, 2010, 79-90.

[3]. S.H.Chen, Operations on fuzzy numbers with function principle , *Tamkang Journal of Management Sciences*, 6, 1985, 13-25.

[4]. A.Kaur and A.Kumar, A new approach for solving fuzzy transportation problems using generalized trapezoidal fuzzy numbers, *Applied soft computing*, *12(3)*, 2012, 1201-1213.

[5]. A.Kaur and A.Kumar, A method for solving fuzzy transportation problems using ranking function, *Applied Mathematical Modelling*, *35(12)*, 2011, 5652-5661.

[6]. R.Jhon Paul Antony, S.Johnson Savarimuthu and T.Pathinathan, Method for solving Transportation Problem Using Triangular Intuitionistic Fuzzy Number, *International Journal of Computing Algorithm,03*, 2014, 590-605 .

[7]. Abhinav Bansal, Trapezoidal Fuzzy numbers (a,b,c,d):Arithmetic behavior, *International Journal of Physical and Mathematical Sciences*, *ISSN- 2010-1791*, 2011,39-44.

[8]. P.Rajarajeswari, A.Sahaya Sudha and R.Karthika, A New Operation on Hexagonal Fuzzy Number, *International Journal of Fuzzy Logic Systems*, *3(3)*, 2013, 15-26.

[9]. P.Rajarajeswari and A.Sahaya Sudha, Ranking of Hexagonal Fuzzy Numbers using Centroid, *AARJMD*, *1(17)*, 265-277.