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A Realistic Approach for Studying the Effect of an Internal Heat Source on the Onset of Convection in a Newtonian Nanofluid Layer: rigid-rigid case

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Abstract: *The aim of this paper, is to use a more realistic model which incorporates the effects of Brownian motion and thermophoresis for studying the effect of a uniform heat source on the onset of convective instability in a confined medium filled of a Newtonian nanofluid layer and heated from below, this layer is assumed to have a low concentration of nanoparticles. The linear study in the rigid - rigid case which was achieved in this investigation shows that the thermal stability of Newtonian nanofluids depends of the volumetric heat delivered by the internal source, the Brownian motion, the thermophoresis and other thermos-physical properties of nanoparticles. Our problem will be solved using a technique of converting a boundary value problem to an initial value problem, in this technique we will also approach the searched solutions with polynomials of high degree.*

Keywords: *Linear stability, Nanofluid, Brownian motion, Thermophoresis, Internal heat source, Power series, Realistic approach, Rigid-Rigid case.*

I. Introduction

The nanofluid is considered as a homogeneous fluid containing colloidal suspensions of nano-sized particles named nanoparticles in the base fluid (water, ethylene glycol, oil). The nanoparticles used in nanofluids are generally prepared of metals, oxides, carbides, or carbon nanotubes. The purpose of using nanofluids is to obtain a higher values of heat transfer coefficient compared with that of the base fluid , this remarkable properties make them potentially useful in many heat transfer applications , for example micro electromechanical systems, coolant in machining, automobile radiator cooling , solar water heating, heat exchangers, nuclear reactors and in several aerospace applications.

The nanofluid term was introduced by Choi [1] in 1995 and remains usually used to characterize this type of colloidal suspension. Buongiorno [2] was the first researcher who treated the convective transport problem in nanofluids, he was established the conservation equations of a non-homogeneous equilibrium model of nanofluids for mass, momentum and heat transport. The thermal problem of instability in nanofluids with rigid-free and free-free boundaries was studied by Tzou [3, 4] using the eigenfunction expansions method. The onset of convection in a horizontal nanofluid layer of finite depth was studied by Nield and Kuznetsov [5], they found that the critical Rayleigh number can be decreased or increased by a significant quantity depending on the relative distribution of nanoparticles between the top and bottom walls.

In this paper, we will study the rigid-rigid case and examine the effect of an internal heat source which produces a constant volumetric heat on the onset of convection in a Newtonian nanofluid layer heated uniformly from below in the case where the nanoparticle flux is assumed to be zero on the impermeable boundaries instead of consider that the volumetric fraction of nanoparticles is constant at the horizontal walls. Currently, this new boundary conditions of nanoparticles is used by several authors for studying the convective problem in nanofluids [6-11] , among these authors we find D.A. Nield and A.V. Kuznetsov [6] who studied analytically the linear thermal stability in a porous medium for a Newtonian nanofluid , Shilpi Agarwal [7] treated analytically the linear thermal stability of a rotating porous layer for a Newtonian nanofluid , I.S. Shivakumara et al. [8] made a numerically investigation on the linear thermal stability of a porous layer for an Oldroyd-B nanofluid , Shilpi Agarwal and Puneet Rana [9] analyzed analytically and numerically the linear and nonlinear thermal stability of a rotating porous layer for an Oldroyd-B nanofluid. D.A. Nield and A.V. Kuznetsov [6] are considered as the first ones who were used the new boundary conditions for the nanoparticles, which are physically more realistic than the previous model which imposes a temperature and nanoparticle volume fractions at the boundaries of the layer.

To show the accuracy of our method in this study, we will check some results treated by Chandrasekhar [12] and Dhananjay Yadav et al. [13] concerning the study of the convective instability of the regular fluids in presence or in absent of an internal heat source which produces a constant volumetric heat in the rigid-rigid case.

The verification of our results will be done by using polynomials of high degree for approaching the searched solutions and increasing the convergence of our method. The contributions of Brownian motion and thermophoresis are strongly appear in the equation expressing the conservation of nanoparticles. The temperature and particle density are coupled in a particular way in which the instability is almost purely a phenomenon due to buoyancy coupled with the conservation of nanoparticles motion.

II. Mathematical Formulation

We consider an infinite horizontal layer of an incompressible Newtonian nanofluid characterized by a low concentration of nanoparticles, heated uniformly from below and confined between two identical horizontal surfaces where the temperature is constant and the nanoparticle flux is zero on the boundaries, this layer will be subjected to an internal heat source which will provide a constant volumetric heat Q_s and also to the gravity field $\vec{g}$ (see **Figure 1**) .The thermo-physical properties of nanofluid (viscosity, thermal conductivity, specific heat) are assumed constant in the analytical formulation except for the density variation in the momentum equation which is based on the Boussinesq approximations .The asterisks are used to distinguish the dimensional variables from the nondimensional variables (without asterisks) .

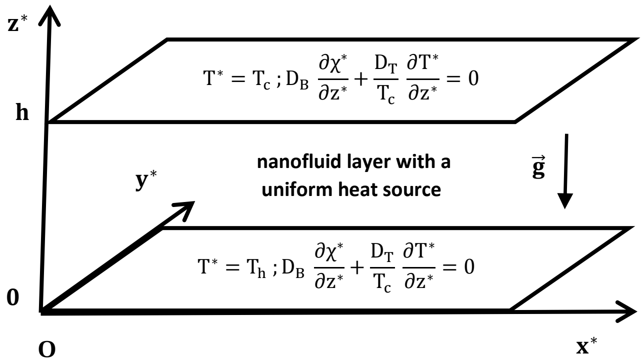

Figure 1 : Physical configuration.

Within the framework of the assumptions which were made by Buongiorno [2] and Tzou [3, 4] in their publications for the Newtonian nanofluids, we can write the basic equations of conservation which govern our problem in dimensionless form as follows:

$$\vec{\nabla}^*.\vec{V}^* = 0 \tag{1}$$

$$\rho_f\left[\frac{\partial \vec{V}^*}{\partial t^*} + (\vec{V}^*.\vec{\nabla}^*)\vec{V}^*\right] = -\vec{\nabla}^* P^* + \{\rho_0[1-\beta(T^*-T_c)](1-\chi^*) + \rho_p\chi^*\}\vec{g} + \eta{\vec{\nabla}^*}^2\vec{V}^* \tag{2}$$

$$(\rho c)_f\left[\frac{\partial T^*}{\partial t^*} + (\vec{V}^*.\vec{\nabla}^*)T^*\right] = \kappa{\vec{\nabla}^*}^2 T^* + (\rho c)_p\left[D_B\vec{\nabla}^*\chi^*.\vec{\nabla}^*T^* + \left(\frac{D_T}{T_c}\right)\vec{\nabla}^*T^*.\vec{\nabla}^*T^*\right] + Q_s \tag{3}$$

$$\frac{\partial \chi^*}{\partial t^*} + (\vec{V}^*.\vec{\nabla}^*)\chi^* = D_B{\vec{\nabla}^*}^2\chi^* + \left(\frac{D_T}{T_c}\right){\vec{\nabla}^*}^2 T^* \tag{4}$$

where ρ_f is the density of the base fluid , ρ_0 is the fluid density at reference temperature T_c , ρ_p is the nanoparticle density , β is the thermal expansion coefficient of the base fluid ,$\vec{V}^*$ is the velocity vector , $t^*$is the time , P^* is the pressure , T^* is the temperature , χ^* is the volume fraction of nanoparticles , η is the viscosity of nanofluid , κ is the thermal conductivity of nanofluid , D_B is the Brownian diffusion coefficient , D_T is the thermophoretic diffusion coefficient , $(\rho c)_f$ is the heat capacity of the base fluid , $(\rho c)_p$ is the heat capacity of the nanoparticle , (x^*, y^*, z^*) are the cartesian coordinates , $\vec{\nabla}^*$ is the vector differential operator .

If we consider the following dimensionless variables:

$$(x^*; y^*; z^*) = h(x; y; z) \; ; \; t^* = \frac{h^2}{\alpha}t \; ; \; \vec{V}^* = \frac{\alpha}{h}\vec{V} \; ; \; P^* = \frac{\eta\alpha}{h^2}P \; ; \; T^* - T_c = (T_h - T_c)T \; ; \; \chi^* - \chi_0^* = \chi_0^*\chi$$

Then, we can get from equations (1)-(4) the following adimensional forms:

$$\vec{\nabla}.\vec{V} = 0 \tag{5}$$

$$P_r^{-1}\left[\frac{\partial \vec{V}}{\partial t} + (\vec{V}.\vec{\nabla})\vec{V}\right] = -\vec{\nabla}(P + R_M z) + \vec{\nabla}^2\vec{V} + [(1-\chi_0^*)R_a T - R_N\chi - \chi_0^* R_a T\chi]\vec{e}_z \tag{6}$$

$$\frac{\partial T}{\partial t} + (\vec{V}.\vec{\nabla})T = \vec{\nabla}^2 T + N_B L_e^{-1}\vec{\nabla}\chi.\vec{\nabla}T + N_A N_B L_e^{-1}\vec{\nabla}T.\vec{\nabla}T + H_s \tag{7}$$

$$\frac{\partial \chi}{\partial t} + (\vec{V}.\vec{\nabla})\chi = L_e^{-1}\vec{\nabla}^2\chi + N_A L_e^{-1}\vec{\nabla}^2 T \tag{8}$$

Where $P_r = \frac{\eta}{\rho_f \alpha}$ is the Prandtl number , $L_e = \frac{\alpha}{D_B}$ is the Lewis number , $H_s = \frac{Q_s h^2}{\kappa(T_h - T_c)}$ is the dimensionless constant heat source strength , $R_a = \frac{\rho_0 g \beta h^3 (T_h - T_c)}{\eta \alpha}$ is the thermal Rayleigh number , $R_M = \frac{[\rho_0(1-\chi_0^*)+\rho_p \chi_0^*] g h^3}{\eta \alpha}$ is the basic density Rayleigh number , $R_N = \frac{(\rho_p - \rho_0)\chi_0^* g h^3}{\eta \alpha}$ is the concentration Rayleigh number , $N_A = \frac{D_T}{D_B T_c}\left(\frac{T_h - T_c}{\chi_0^*}\right)$ is the modified diffusivity ratio , $N_B = \frac{(\rho c)_p}{(\rho c)_f}\chi_0^*$ is the modified particle-density increment , $\alpha = \frac{\kappa}{(\rho c)_f}$ is the thermal diffusivity of nanofluid , χ_0^* is the reference value for nanoparticle volume fraction .

2.1 Basic Solution

The basic solution of our problem is a quiescent thermal equilibrium state, it's assumed to be independent of time where the equilibrium variables are varying only in the z-direction, therefore:

$$\vec{V}_b = \vec{0} \tag{9}$$

$$T_b = 1 \quad ; \frac{d\chi_b}{dz} + N_A \frac{dT_b}{dz} = 0 \quad \text{at} \quad z = 0 \tag{10}$$

$$T_b = 0 \quad ; \frac{d\chi_b}{dz} + N_A \frac{dT_b}{dz} = 0 \quad \text{at} \quad z = 1 \tag{11}$$

If we introduce the precedent results into equations (6)-(8), we obtain:

$$\vec{\nabla}(P_b + R_M z) = [(1-\chi_0^*)R_a T - R_N \chi - \chi_0^* R_a T \chi]\vec{e}_z \tag{12}$$

$$\frac{d^2 T_b}{dz^2} + N_B L_e^{-1}\left(\frac{d\chi_b}{dz}\frac{dT_b}{dz}\right) + N_A N_B L_e^{-1}\left(\frac{dT_b}{dz}\right)^2 = -H_s \tag{13}$$

$$\frac{d^2\chi_b}{dz^2} + N_A \frac{d^2 T_b}{dz^2} = 0 \tag{14}$$

After using the boundary conditions (10) and (11), we can integrate the equation (14) between 0 and z for obtaining:

$$\chi_b = N_A(1 - T_b) + \chi_0 \tag{15}$$

Where $\chi_0 = \frac{\chi^* - \chi_0^*}{\chi_0^*}$ is the relative nanoparticle volume fraction at $z = 0$.

If we take into account the expression (15), we can get after simplification of the equation (13):

$$\frac{d^2 T_b}{dz^2} = -H_s \tag{16}$$

Finally, we obtain after an integrating of the equation (16) between 0 and 1:

$$T_b = -\frac{1}{2}H_s z^2 + \left(\frac{1}{2}H_s - 1\right)z + 1 \tag{17}$$

$$\chi_b = \frac{1}{2}N_A H_s z^2 - N_A\left(\frac{1}{2}H_s - 1\right)z + \chi_0 \tag{18}$$

2.2 Stability Analysis

For analyzing the stability of the system, we superimpose infinitesimal perturbations on the basic solutions as follows:

$$T = T_b + T' \quad ; \quad \vec{V} = \vec{V}_b + \vec{V}' \quad ; \quad P = P_b + P' \quad ; \quad \chi = \chi_b + \chi' \tag{19}$$

In the framework of the Oberbeck-Boussinesq approximations, we can neglect the terms coming from the product of the temperature and the volumetric fraction of nanoparticles in equation (6), if we suppose also that we are in the case of small temperature gradients in a dilute suspension of nanoparticles, we can obtain after introducing the expressions (19) into equations (5)-(8) the following linearized equations:

$$\vec{\nabla}.\vec{V'} = 0 \tag{20}$$

$$P_r^{-1}\frac{\partial \vec{V}'}{\partial t} = -\vec{\nabla}P' + (R_a T' - R_N \chi')\vec{e}_z + \vec{\nabla}^2\vec{V}' \tag{21}$$

$$\frac{\partial T'}{\partial t} + f_1 w' = \vec{\nabla}^2 T' + f_2\frac{\partial T'}{\partial z} + f_3\frac{\partial \chi'}{\partial z} \tag{22}$$

$$\frac{\partial \chi'}{\partial t} + f_4 w' = N_A L_e^{-1} \vec{\nabla}^2 T' + L_e^{-1} \vec{\nabla}^2 \chi' \tag{23}$$

Where $f_1 = DT_b$, $f_2 = N_B L_e^{-1} D(\chi_b + 2N_A T_b)$, $f_3 = N_B L_e^{-1} DT_b$, $f_4 = D\chi_b$ and $D = d/dz$.

After application of the curl operator twice to the equation (21) and using the equation (20), we obtain the following z -component of the momentum equation:

$$P_r^{-1} \frac{\partial}{\partial t} \vec{\nabla}^2 w' = \vec{\nabla}^4 w' + R_a \vec{\nabla}_2^2 T' - R_N \vec{\nabla}_2^2 \chi' \tag{24}$$

Where $\vec{\nabla}_2^2 = \left(\frac{\partial^2}{\partial x^2}\right) + \left(\frac{\partial^2}{\partial y^2}\right)$ is the two-dimensional Laplacian operator on the horizontal plane.

Analyzing the disturbances into normal modes, we can simplify the equations (22) - (24) by assuming that the perturbation quantities are of the form:

$$(w', T', \chi') = (\mathcal{w}(z), \mathcal{T}(z), \mathcal{X}(z)) \exp[i(k_x x + k_y y) + \sigma t] \tag{25}$$

After introducing the expressions (25) into equations (22) - (24), we obtain:

$$P_r^{-1} \sigma (D^2 - k^2) \mathcal{w} = (D^2 - k^2)^2 \mathcal{w} - k^2 R_a \mathcal{T} + k^2 R_N \mathcal{X} \tag{26}$$

$$\sigma \mathcal{T} + f_1 \mathcal{w} = (D^2 - k^2) \mathcal{T} + f_2 D\mathcal{T} + f_3 D\mathcal{X} \tag{27}$$

$$\sigma \mathcal{X} + f_4 \mathcal{w} = N_A L_e^{-1} (D^2 - k^2) \mathcal{T} + L_e^{-1} (D^2 - k^2) \mathcal{X} \tag{28}$$

Where σ is the dimensionless growth rate, k_x and k_y are respectively the dimensionless waves numbers along the x and y directions and $k = \sqrt{k_x^2 + k_y^2}$ is the resultant dimensionless wave number.

In the rigid-rigid case, the equations (26) - (28) will be solved subject to the following boundary conditions:

$$\mathcal{w} = D\,\mathcal{w} = \mathcal{T} = D(\mathcal{X} + N_A \mathcal{T}) = 0 \quad \text{at} \quad z = 0; 1 \tag{29}$$

2.3 Method of Solution

In this study we assume that the principle of exchange of stability is valid, as we are interested in a stationary stability study characterized by $\sigma = 0$, then the equations (26)-(28) become:

$$(D^2 - k^2)^2 \mathcal{w} - k^2 R_a \mathcal{T} + k^2 R_N \mathcal{X} = 0 \tag{30}$$

$$f_1 \mathcal{w} - (D^2 - k^2) \mathcal{T} - f_2 D\mathcal{T} - f_3 D\mathcal{X} = 0 \tag{31}$$

$$f_4 \mathcal{w} - N_A L_e^{-1} (D^2 - k^2) \mathcal{T} - L_e^{-1} (D^2 - k^2) \mathcal{X} = 0 \tag{32}$$

We can solve the equations (30) - (32) which are subjected to the conditions (29), by using a suitable change of variables that makes the number of variables equal to the number of boundary conditions, to obtain a set of eight first order ordinary differential equations which we can write it in the following form:

$$\frac{d}{dz} u_i(z) = a_{ij} u_j(z) ; \ 1 \le i, j \le 8 \tag{33}$$

With:

$$a_{ij} = a_{ij}(z, k, R_a, H_s, N_B, L_e, R_N, N_A)$$

The solution of the system (33) in matrix notation can be written as follows:

$$U = BC \tag{34}$$

Where $B = \left((b_{ij}(z))_{\substack{1 \le i \le 8 \\ 1 \le j \le 8}} \right)$ is a square matrix of order 8×8 , $U = \left((u_i(z))_{1 \le i \le 8} \right)^T$ is the unknown vector column of our problem, $C = \left((c_j)_{1 \le j \le 8} \right)^T$ is a constant vector column.

If we assume that the matrix B is written in the following form:

$$B = \left((u_i^j(z))_{\substack{1 \le i \le 8 \\ 1 \le j \le 8}} \right) \tag{35}$$

Therefore, the use of four boundary conditions at $z = 0$, allows us to write each variable $u_i(z)$ as a linear combination only for four functions $u_i^j(z)$, such that:

$$b_{ij}(0) = u_i^j(0) = \delta_{ij} \tag{36}$$

Where δ_{ij} is the Kronecker delta symbol .

After introducing the new expressions of the variables $u_i(z)$ in the system (33), we will obtain the following equations:

$$\frac{d}{dz}u_i^j(z) = a_{il}u_l^j(z)\,;\ \ 1 \le i, l, j \le 8 \tag{37}$$

For each value of j, we must solve a set of eight first order ordinary differential equations which are subjected to the initial conditions (36) , by approaching these variables with power series defined in the interval $[0,1]$ and truncated at the order N, such that:

$$u_i^j(z) = \sum_{p=0}^{p=N} d_p^{i,j} z^p \tag{38}$$

A linear combination of the solutions $u_i^j(z)$ satisfying the boundary conditions (29) at $z = 1$ leads to a homogeneous algebraic system for the coefficients of the combination. A necessary condition for the existence of nontrivial solution is the vanishing of the determinant which can be formally written as:

$$f(R_a\,, k\,, H_s\,, N_B\,, L_e\,, R_N\,, N_A) = 0 \tag{39}$$

If we give to each control parameter $(H_s\,, N_B\,, L_e\,, R_N\,, N_A)$ its value, we can plot the neutral curve of the stationary convection by the numerical research of the smallest real positive value of the thermal Rayleigh number R_a which corresponds to a fixed wave number k and verifies the dispersion relation (39). After that, we will find a set of points $(k\,, R_a)$ which help us to plot our curve and find the critical value $(k_c\,, R_{ac})$ which characterizes the onset of the convective stationary instability, this critical value represents the minimum value of the obtained curve.

2.4 Validation of the Method

To validate our method, we compared our results with those obtained by Chandrasekhar **[12]** and Dhananjay Yadav et al. **[13]** concerning the Rayleigh-Bénard problem for the regular fluids in the case where the internal heat source is absent or present. To make this careful comparison, we must take into consideration the restrictions $L_e^{-1} = R_N = N_A = N_B = 0$ in the governing equations of our problem. The convergence of our method is assumed when the absolute value of the difference between the critical thermal Rayleigh numbers $R_{ac}(N+1)$ and $R_{ac}(N)$ is of the order of 10^{-5} (see **Table 1-Table 2**), such that $R_{ac} = R_{ac}(N)$.

Where $R_{ac}(N)$ and $R_{ac}(N+1)$ are respectively the critical thermal Rayleigh numbers which correspond to the truncation order N and $N+1$.

Table 1 : The critical values of the Rayleigh number R_{ac} and the corresponding wave number k_c of Chandrasekhar **[12]** , Dhananjay Yadav et al. **[13]** and our results for the regular fluids $(H_s = 0)$.

Present study				Chandrasekhar		D. Yadav et all	
N	k_c	R_{ac}	$\lvert R_{ac}(N+1) - R_{ac}(N)\rvert$	k_c	R_{ac}	k_c	R_{ac}
15	3.20999	1669.44346	55.03134				
16	3.05441	1724.47480	22.88711				
17	3.14712	1701.58769	7.83433				
18	3.10666	1709.42202	2.03222				
19	3.11906	1707.38980	0.42833				
20	3.11574	1707.81813	0.05755				
21	3.11640	1707.76058	0.00185				
22	3.11632	1707.75873	0.00444				
23	3.11631	1707.76317	0.00182	**3.117**	**1707.762**	**3.116**	**1707.75923**
24	3.11632	1707.76135	0.00052				
25	3.11632	1707.76187	0.00012				
26	**3.11632**	**1707.76175**	**0.00003**				
27	3.11632	1707.76178	0.00001				
28	3.11632	1707.76177	0				
29	3.11632	1707.76177	0				
30	3.11632	1707.76177	-----------				

Table 2 : The critical values of the Rayleigh number R_{ac} and the corresponding wave number k_c of D. Yadav et al. [13] for the regular fluids and our results for the regular fluids and a nanofluid characterized by N_B=0.01, L_e = 100 , R_N = 1 and N_A = 0.1 for various values of the heat source strength H_s.

H_s	Regular fluids					Nanofluid		
	D.Yadav et all		Present study			Present study		
	k_c	R_{ac}	k_c	R_{ac}	N	k_c	R_{ac}	N
0	3.116	1707.75923	3.11632	1707.76175	26	3.10758	1692.02837	26
1	3.119	1704.52398	3.11891	1704.52648	27	3.11014	1688.78226	26
2	3.127	1694.94792	3.12656	1694.95019	31	3.11775	1679.19805	29
10	3.304	1462.86825	3.30367	1462.86090	35	3.29363	1447.11659	34
20	3.529	1118.45908	3.52913	1118.43009	39	3.51608	1102.72068	36
30	3.659	878.34427	3.65933	878.30338	39	3.64263	862.61262	37
40	3.736	717.24455	3.73587	717.19979	39	3.71530	701.51821	37
60	3.819	521.44662	3.81895	521.40318	39	3.79041	505.72725	38

According to above results, we notice that there is a very good agreement between our results and the previous works, hence the accuracy of the used method. Briefly, the convergence of the results depends greatly on the truncation order N of the power series and also of the heat source strength H_s . Finally, to ensure the accuracy of our obtained critical values for the studied nanofluid, we will take as truncation order:

$$N = 38$$

III. Tables and Figures

3.1 Tables

Table 3 : The stationary instability threshold of a nanofluid according to the values of parameters N_B and H_s for $L_e = 100$, $R_N = 1$ and $N_A = 0.1$.

H_s	$N_B = 0.001$		$N_B = 0.01$		$N_B = 0.05$		$N_B = 0.1$	
	k_c	R_{ac}	k_c	R_{ac}	k_c	R_{ac}	k_c	R_{ac}
5	3.16654	1617.17374	3.16648	1617.12214	3.16621	1616.89286	3.16586	1616.60635
10	3.29376	1447.20290	3.29363	1447.11652	3.29307	1446.73267	3.29236	1446.25307
20	3.51634	1102.83891	3.51608	1102.72062	3.51495	1102.19514	3.51354	1101.53864
30	3.64300	862.74391	3.64262	862.61267	3.64095	862.02984	3.63887	861.30179
40	3.71579	701.65629	3.71530	701.51829	3.71310	700.90555	3.71036	700.14032
50	3.76109	588.73776	3.76048	588.59567	3.75776	587.96475	3.75437	587.17742
60	3.79114	505.87202	3.79041	505.72725	3.78717	505.08454	3.78314	504.28275

Table 4 : The stationary instability threshold of a nanofluid according to the values of parameters L_e and H_s for $N_B = 0.01$, $R_N = 1$ and $N_A = 0.1$.

H_s	$L_e = 100$		$L_e = 300$		$L_e = 500$		$L_e = 700$	
	k_c	R_{ac}	k_c	R_{ac}	k_c	R_{ac}	k_c	R_{ac}
5	3.16648	1617.12214	3.14808	1585.80216	3.12922	1554.32212	3.10987	1522.67493
10	3.29363	1447.11652	3.27348	1415.90849	3.25283	1384.54569	3.23164	1353.02130
20	3.51608	1102.72062	3.49006	1071.64358	3.46336	1040.39668	3.43593	1008.97189
30	3.64262	862.61267	3.60935	831.57900	3.57508	800.34152	3.53976	768.88903
40	3.71530	701.51829	3.67426	670.49059	3.63180	639.21801	3.58784	607.68471
50	3.76048	588.59567	3.71142	557.55591	3.66042	526.22650	3.60733	494.58560
60	3.79041	505.72725	3.73316	474.66547	3.67332	443.26673	3.61069	411.50173

Table 5 : The stationary instability threshold of a nanofluid according to the values of parameters R_N and H_s for $N_B = 0.01$, $L_e = 100$ and $N_A = 0.1$.

H_s	$R_N = 1$		$R_N = 3$		$R_N = 5$		$R_N = 7$	
	k_c	R_{ac}	k_c	R_{ac}	k_c	R_{ac}	k_c	R_{ac}
5	3.16648	1617.12214	3.14808	1585.60216	3.12922	1553.92212	3.10987	1522.07493
10	3.29363	1447.11652	3.27348	1415.70849	3.25283	1384.14569	3.23164	1352.42130
20	3.51608	1102.72062	3.49006	1071.44358	3.46336	1039.99668	3.43593	1008.37189
30	3.64262	862.61267	3.60935	831.37900	3.57508	799.94152	3.53976	768.28903
40	3.71530	701.51829	3.67426	670.29059	3.63180	638.81801	3.58784	607.08471
50	3.76048	588.59567	3.71142	557.35591	3.66042	525.82650	3.60733	493.98560
60	3.79041	505.72725	3.73316	474.46547	3.66957	442.86673	3.61069	410.90173

Table 6 : The stationary instability threshold of a nanofluid according to the values of parameters N_A and H_s for $N_B = 0.01$, $L_e = 100$ and $R_N = 1$.

H_s	$N_A = 0.1$		$N_A = 0.3$		$N_A = 0.5$		$N_A = 0.7$	
	k_c	R_{ac}	k_c	R_{ac}	k_c	R_{ac}	k_c	R_{ac}
5	3.16648	1617.12214	3.14795	1585.48847	3.12895	1553.69671	3.10946	1521.73984
10	3.29363	1447.11652	3.27320	1415.51840	3.25227	1383.76935	3.23081	1351.86270
20	3.51608	1102.72062	3.48950	1071.18429	3.46225	1039.48547	3.43429	1007.61653
30	3.64262	862.61267	3.60852	831.09260	3.57346	799.37961	3.53737	767.46326
40	3.71530	701.51829	3.67318	669.99082	3.62969	638.23298	3.58475	606.23021
50	3.76048	588.59567	3.71009	557.04876	3.65783	525.23047	3.60356	493.12090
60	3.79041	505.72725	3.73158	474.15409	3.67028	442.26614	3.60630	410.03694

3.2 Figures

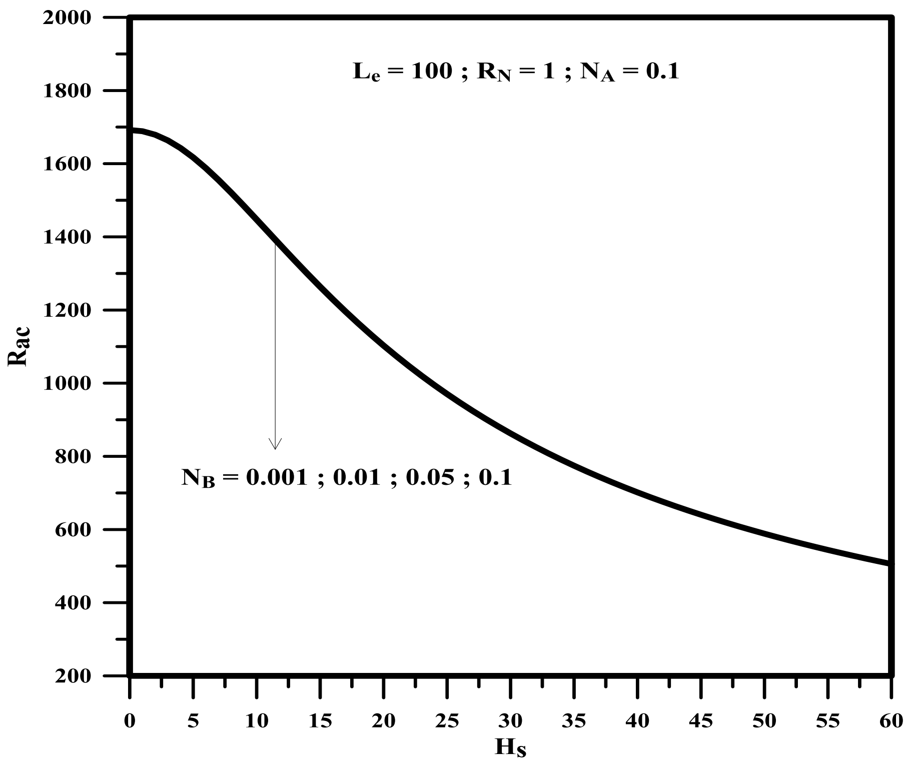

Figure 2 : Plot of R_{ac} as a function of H_s for different values of N_B .

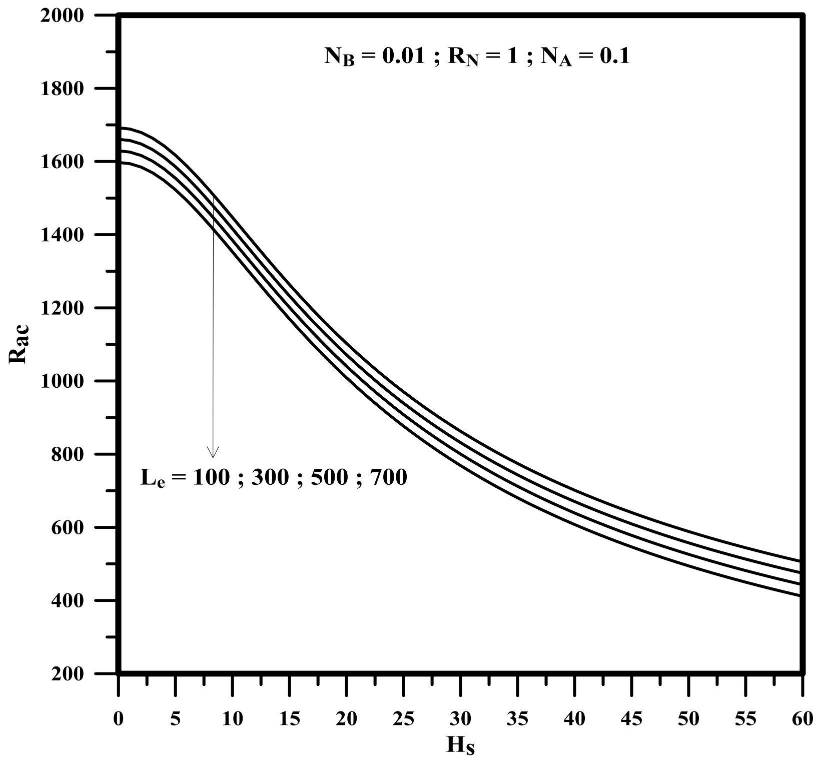

Figure 3 : Plot of R_{ac} as a function of H_s for different values of L_e .

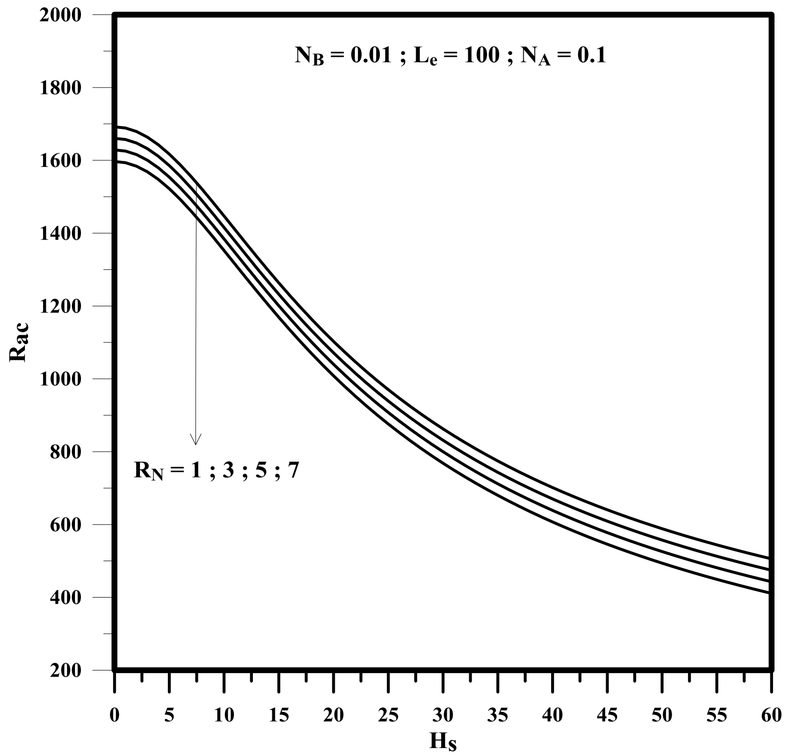

Figure 4 : Plot of R_{ac} as a function of H_s for different values of R_N .

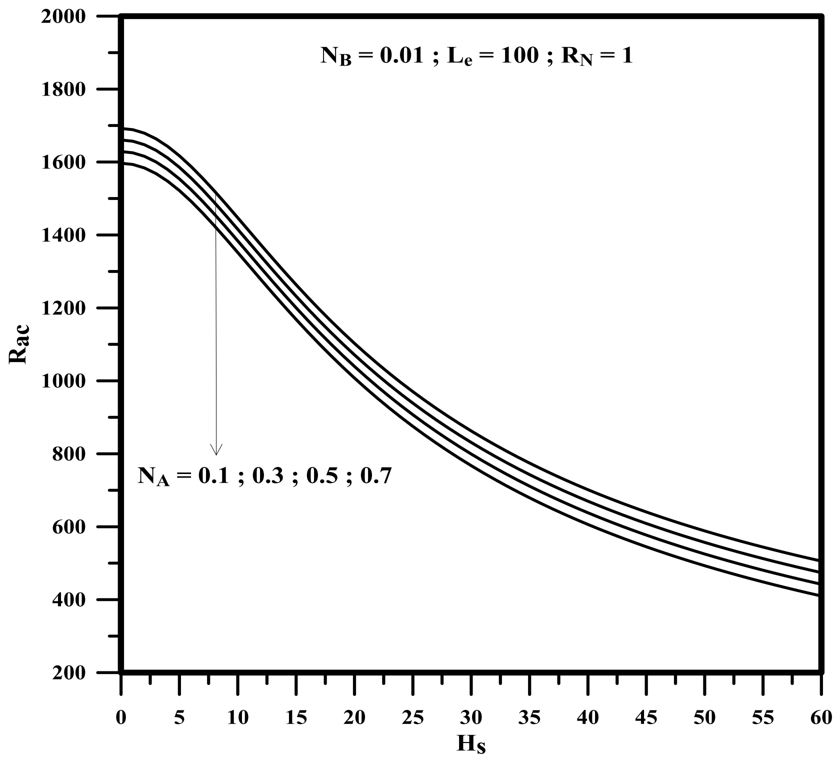

Figure 5 : Plot of R_{ac} as a function of H_s for different values of N_A .

IV. Results and Conclusion

The reported results in the **Tables (3-6)** and in their corresponding **Figures (2-5)** show that the variation in the critical thermal Rayleigh number R_{ac} with the heat source strength H_s is generally a decreasing function whatever the value taken for each parameter nanofluid (N_B, L_e, R_N, N_A) , this result can be interpreted as an increase in the heat source strength H_s amounts to increase in energy supply to the system , hence increases the driving force which accelerates the onset of the convection.

The precedent tables and figures show also that an increase either in the Lewis number L_e , in the concentration Rayleigh number R_N or in the modified diffusivity ratio N_A allows us to accelerate the onset of the convection, hence they have a destabilizing effect.

According to Buongiorno [2], Nield and Kuznetsov [5] and I.S. Shivakumara et al. [8], we have for the majority of the nanofluids $N_B \sim 10^{-3} - 10^{-1}$ and $L_e \sim 10^2 - 10^3$. The modified particle-density increment N_B appears only in the perturbed energy equation (22) as a product with the inverse of the Lewis number L_e near the temperature gradient and the volume fraction gradient of nanoparticles, so the effect of this parameter on the onset of convection in nanofluids will be very small, this result is confirmed in **Table 3** and **Figure 2**, such that an increase in the modified particle-density increment N_B allows us to destabilize somewhat the nanofluids.

In this investigation, we find that an increase in the volume fraction of nanoparticles destabilizes the nanofluids, because an increase in this parameter, increases the Brownian motion and the thermophoresis of the nanoparticles, which cause the destabilizing effect. When the temperature difference between the horizontal plates increases, the critical thermal Rayleigh number R_{ac} decreases, this result can be explained by the increase in the buoyancy forces which destabilizes the system. To ensure the stability of the nanofluids, we can use the less dense nanoparticles or the ones which are having a small heat capacity.

The used method to solve the convection problem with a new model of boundary conditions of nanoparticles in presence of a uniform heat source is more accurate, as it gives an absolute error of the order of 10^{-5} to the critical values characterizing the onset of the convection.

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