Numerical Modelling of 2004 Indonesian Tsunami along the coast of Phuket

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Abstract: *In coastline, the boundary-fitted curvilinear grids are appropriate to make the grid model and build up the simple boundary conditions, which are more mathematic. The method of lines (MOL) is a general procedure for the solution of time dependent partial differential equations (PDEs. The methods of lines (MOL) are more feasible and comparable to the regular finite difference method in terms of computational time, accuracy and numerical stability. Thus, we have tried to present the new numerical solution technique to simulate the 2004 Indonesian tsunami. At first, this numerical solution technique set up on a curvilinear grid model where the vertically integrated shallow water equations are solved by using MOL. The boundary fitted curvilinear grids are presented along the coastal and island boundaries. Thereafter, we have applied the method of lines (MOL) technique to transform the shallow water equations and boundary conditions. These equations transfigure into ordinary differential equations with the initial value problem. At last, we have used Runge-Kutta 4ᵗʰ order method to solve these ordinary differential equations. To simulate the 2004 Indonesian tsunami along the Puket coast, we have compared our results with the authentic research papers and USGS, where this simulation shows excellent agreement with observing data.*
Keywords: *Boundary fitted curvilinear grid system, Method of lines (MOL), Runge-Kutta method, Nonlinear Shallow water equations, Indonesian tsunami and Indian Ocean tsunami 2004.*

I. Introduction

The 26ᵗʰ December 2004, was the terrible day beside the Indian Ocean countries. Because, on this day, the Indian Ocean earthquake occurred off the west coast of northern Sumatra in Indonesia, it was triggered a devastating tsunami along the Indian Ocean surrounding countries including the of Peninsular Malaysia. The advancement of tsunami waves propagates across the Indian Ocean. As a result, Tsunami was the creator of causing damage to the shore areas and which affected more than twelve countries. According to [16], the prevention and mitigation of tsunami hazards depend on the accurate assessment of the generation, propagation and run-up of tsunamis. So, attempts should be made to construct numerical models. In particular regional tsunami numerical models should be developed to develop the early warning system. Naturally, The west coasts of southern Thailand are curvilinear in nature and the bending is high along the coasts of this country (Figure 1). In addition, there are some offshore islands, such as Phuket in Thailand, Penang in Peninsular Malaysia and so on. According to [4], if the coast lines are curvilinear, then it is better to use the boundary-fitted grids to represent the model boundaries accurately.

Boundary–fitted curvilinear grid system is a technique which gives us an approach and its combines the best aspects of finite-difference discretization with grid flexibility. It is very useful to make the simple equations and boundary condition and also a good technique to better present the complex geometry with less number of grid points. On the other hand, this system is able to improve the finite difference schemes. However, the grid lines in a boundary-fitted curvilinear technique are curvilinear and non-orthogonal. For this reason regular finite difference scheme cannot be applied. For the regular finite difference scheme, the grid system must be rectangular. Hence, by using appropriate transformations, the curvilinear boundaries are transformed into a rectangular domain where the regular finite difference techniques can be used.

On the other hand, the method of lines (MOL) is a general technique for solving a system of partial differential equations (PDEs) by converting it to a system of ordinary differential equations (ODEs). According to [1], MOL is the very powerful approach to the numerical solution of time dependent partial differential equations. It is regarded as a special finite difference method which is more effective than the regular finite difference method in terms of accuracy and computational time [12]. It has the advantages of less computation time, no relative convergence problem and numerical stability etc. [13]. In the MOL approach, the system of PDEs is converted into a system of ODEs initial value problem by discretizing the spatial derivatives together with the boundary conditions and then the resulting ODEs with time as the independent variable is solved by using an ODE solver. The model [7] developed a shallow water model using a boundary-fitted curvilinear grid system to simulate the 2004 Indonesian tsunami. In a stair step model the coastal boundaries are approximated along the nearest finite difference grid lines of the numerical scheme and so the accuracy of a stair step model

depends on the grid size. Since very fine resolution was not considered on that stair step model, the representation of the coastal boundaries was not very accurate.

In this numerical model, the MOL method is suitable to solve the shallow water equation in boundary fitted curvilinear grid system and it has been developed to simulate the 2004 Indonesian tsunami along the coast of Thailand. For this reason, this numerical model depicts the two functions of southern Thailand and western open sea boundary respectively. On the other hand, the north and south open boundaries were represented as straight lines. We have used transformations in non-orthogonal curvilinear grids, thus the physical domain becomes rectangular. The deep averaged shallow water equations are transformed to the new space domain. Then we applied the method of lines (MOL) technique to transform the shallow water equations and boundary conditions. For this reason, these equations transfigure into ordinary differential equations with the initial value problem. At last, we have used 4th order Runge-Kutta method to solve these ordinary differential equations.

II.   Governing Equations And Boundary Condition For Tsunami Simulations

The nonlinear shallow water equations are used here to model the 2004 Indonesian tsunami. Consider a rectangular Cartesian co-ordinate system in which the origin O is in the undisturbed sea level (MSL), x- axis is directed towards west and y-axis is directed north on the MSL, whereas z-axis is directed vertically upwards. Let, the displaced position of the sea-surface is given by $z = \zeta(x, y, t)$ and the position of the sea-floor is given by $z = -h(x, y)$. So that depth of the fluid layer is $(\zeta + h)$.
Following [3], the vertically integrated shallow water equations are,

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}[(\zeta + h)u] + \frac{\partial}{\partial y}[(\zeta + h)v] = 0 \tag{01}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv = -g\frac{\partial \zeta}{\partial x} - \frac{c_f\, u\, (u^2 + v^2)^{1/2}}{\zeta + h} \tag{02}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu = -g\frac{\partial \zeta}{\partial y} - \frac{c_f\, v\, (u^2 + v^2)^{1/2}}{\zeta + h} \tag{03}$$

Where, u and v are the velocity components of flow particle in x and y directions, f is the Coriolis parameter, g is the acceleration due to gravity, c_f is the friction coefficient.

For numerical treatment it is convenient to express the equations (02) & (03) in the flux form by using the equation (01). The shallow water equations in flux forms are

$$\frac{\partial \zeta}{\partial t} + \frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0 \tag{04}$$

$$\frac{\partial \tilde{u}}{\partial t} + \frac{\partial(u\,\tilde{u})}{\partial x} + \frac{\partial(v\,\tilde{u})}{\partial y} - f\,\tilde{v} = -g(\zeta + h)\frac{\partial \zeta}{\partial x} - \frac{c_f\, \tilde{u}\, (u^2 + v^2)^{\frac{1}{2}}}{\zeta + h} \tag{05}$$

$$\frac{\partial \tilde{v}}{\partial t} + \frac{\partial(u\,\tilde{v}˜)}{\partial x} + \frac{\partial(v\,\tilde{v})}{\partial y} + f\,\tilde{u} = -g(\zeta + h)\frac{\partial \zeta}{\partial y} - \frac{c_f\, \tilde{v}\, (u^2 + v^2)^{\frac{1}{2}}}{\zeta + h} \tag{06}$$

Where, $(\tilde{u}, \tilde{v}) = (\zeta + h)(u, v)$

Here u and v in the bottom stress terms of (02) and (03) have been replaced by $\tilde{u}$ and $\tilde{v}$ in (05) and (06) in order to solve the equations in a semi-implicit manner and are the depth-averaged volume fluxes in the x and y directions, respectively.

The appropriate boundary condition along the coastal boundary is that the normal component of the vertically integrated velocity vanishes at the coast and following [3] this may be expressed as:

$u \cos \propto + v \sin \propto = 0$ for all $t \geq 0$ (07)

Where $\propto$ is the inclination of the outward directed normal to the x-axis. It then follows that $u = 0$ along y-directed boundaries and $v = 0$ along the x-directed boundaries.

At the open-sea boundaries the waves and disturbance, generated within the model domain, are allowed to leave the domain without affecting the interior solution. Thus the normal component of velocity cannot vanish and so a radiation type of boundary is generally used. the following radiation type of condition may be used:

$u \cos \propto + v \sin \propto = -(g/h)^{1/2}\, \zeta$ for all $t \geq 0$ (08)

Note that the velocity structure in a shallow water wave is described by $= \frac{g\zeta}{\sqrt{gh}}$, where w is the horizontal particle velocity.

The eastern moving coastal boundary is situated at $x = b_1(y,t)$ with the initial position at $x = b_1(y,0)$ and the western open-sea boundary is at $x = b_2(y)$. The southern and the northern open sea boundaries are at $y = 0$ and $y = L$ respectively. This configuration is shown in Fig. 1. The western, southern and northern open sea boundaries are considered as fixed.
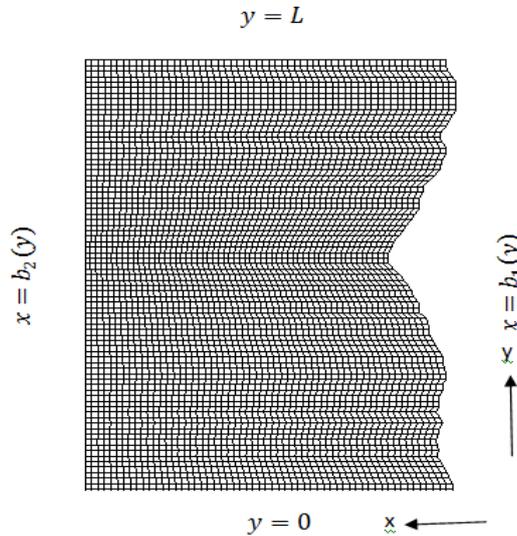

Figure 1: Boundary fitted grids in physical domain [10].

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Following Roy [3], the system of gridlines along $x = b_1(y,t)$ and $x = b_2(y)$ are given by the generalized function,

$$x = \{(k - l)\, b_1(y,t) + l\, b_2(y)\}/k \tag{09}$$

Where, $k = M$ is the number of gridlines in *x*-direction and *l* is an integer such that $0 \leq l \leq k$

The system of gridlines along $y = 0$ and $y = L$ are given by the generalized function

$$y = \{(q - p)\, 0 + p\, L\}/q \tag{10}$$

Where, $q = N$ is the number of gridlines in *y*-direction and *p* is an integer such that $0 \leq p \leq q$

We note that the equation (09/5.4) reduces to $x = b_1(y,t)$ and $x = b_2(y)$ for $l = 0$ and $l = k$ respectively. Similarly equation (10) reduces to $y = 0$ and $y = L$ for $p = 0$ and $p = q$ respectively. A suitable choice of $\{l, k\}$ and $\{p, q\}$ then appropriately generates the boundary-fitted curvilinear grid lines.

2.1 The Boundary Conditions

The coastal boundary may be of two types: the coastline consists of a vertical side wall or the shoreline moves with the same velocity as that of the approaching water. Following [14], in case of fixed coastal boundary the condition is:

$$u - v\frac{\partial b_1}{\partial y} = 0 \quad \text{along } x = b_1(y, 0) \tag{11}$$

And, in case of moving coastal boundary the condition is:

$$u - \frac{\partial b_1}{\partial t} - v\frac{\partial b_1}{\partial y} = 0 \quad \text{along } x = b_1(y, t) \tag{12}$$

Following [7], boundary conditions along the open boundaries are:

$$v + \left(\frac{g}{h}\right)^{1/2}\zeta = 0 \quad \text{along } y = 0 \tag{13}$$

$$\tag{14}$$

$$v - \left(\frac{g}{h}\right)^{1/2}\zeta = 0 \quad \text{along } y = L$$

$$u - v\frac{\partial b_2}{\partial y} - \left(\frac{g}{h}\right)^{1/2}\zeta = 0 \quad \text{along } x = b_2(y) \tag{15}$$

2.2 Coordinate Transformation

To facilitate the numerical treatment of the bending coastal boundary, according to [14], the following coordinate transformation is introduced:

$$\eta = \frac{x - b_1(y,t)}{b(y,t)}, \lambda = \frac{y}{L}, \text{ where } \quad b(y,t) = b_2(y) - b_1(y,t) \tag{16}$$

This mapping transforms the analysis area enclosed by $x = b_1(y,t), x = b_2(y), y = 0$ and $y = L$ into a rectangular domain given by $0 \le \eta \le 1, \ 0 \le \lambda \le 2$, which is shown in figure:
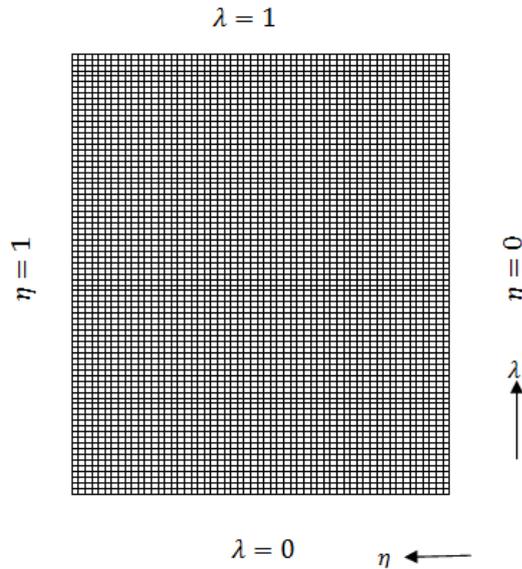
$\lambda = 1$

$\eta = 1$

$\eta = 0$

λ

$\lambda = 0$ $\quad \eta \longleftarrow$

Figure 2: Transformed domain and the rectangular grid lines

The generalized function (09) takes the form $b\eta + b_1 = \{(k - l)b_1(y,t) + lb_2(y)\}/k$, which can be written as $\eta = l(b_2 - b_1)/bk$. This gives,

$$\eta = l/k \tag{17}$$

The generalized function (10) takes the form $\lambda L = \{(q - p)0 + pL\}/q$, which implies

$$\lambda = p/q \tag{18}$$

For $l = 0$, we have the eastern coastal boundary $\eta = 0$ or $x = b_1(y,t)$ and for $l = k$ we have the western open sea boundary $\eta = 1$ or $x = b_2(y)$. Similarly, for $p = 0$, we have the southern open sea boundary $\lambda = 0$ or $y = 0$ and for $p = q$ we have the northern open-sea boundary $\lambda = 1$ or $y = L$. Thus by the proper choice of the constants k and q and the parameters l and p, rectangular grid system can be generated in the transformed domain.

2.3 Transformed Shallow Water Equations And Boundary Conditions
By using the transformations (16)

$$\frac{\partial}{\partial x} \equiv \frac{1}{b} \frac{\partial}{\partial \eta} \tag{19}$$

$$\frac{\partial}{\partial y} \equiv -\frac{1}{b} \left(\frac{db_1}{dy} + \eta \frac{db}{dy} \right) \frac{\partial}{\partial \eta} + \frac{1}{L} \frac{\partial}{\partial \lambda} \tag{20}$$

Taking η, λ, y, t as the new independent variables and using relations (19) and (20), the equations (04) - (06) transform to [5]

$$\frac{\partial(bL\zeta)}{\partial t} + \frac{\partial \tilde{U}}{\partial \eta} + \frac{\partial \tilde{V}}{\partial \lambda} = 0 \tag{21}$$

$$\frac{\partial \tilde{u}}{\partial t} + \frac{\partial(U\tilde{u})}{\partial \eta} + \frac{\partial(V\tilde{u})}{\partial \lambda} - f\tilde{v} = -gL(\zeta + h) \frac{\partial \zeta}{\partial \eta} - \frac{c_f \, \tilde{u} \, (u^2 + v^2)^{\frac{1}{2}}}{\zeta + h} \tag{22}$$

$$\frac{\partial \tilde{v}}{\partial t} + \frac{\partial (U\tilde{v})}{\partial \eta} + \frac{\partial (V\tilde{v})}{\partial \lambda} + f\tilde{u} = -g(\zeta + h)\left[b\frac{\partial \zeta}{\partial} - L\left(\frac{db_1}{dy} + \eta \frac{db}{dy}\right)\frac{\partial \zeta}{\partial \eta}\right] - \frac{c_f \, \tilde{v} \, (u^2 + v^2)^{\frac{1}{2}}}{\zeta + h} \tag{23}$$

Where,

$U = \frac{1}{b}\left[u - \left(\frac{db_1}{dy} + \eta \frac{db}{dy}\right)v\right], V = \frac{v}{L}, \ (\tilde{u}, \tilde{v}, \tilde{U}, \tilde{V}) = bL(\zeta + h)(u, v, U, V)$

At $x = b_1(y,t)$ i.e. $\eta = 0$, $U = \frac{1}{b}\left[u - \left(\frac{db_1}{dy} + \eta \frac{db}{dy}\right)v\right] = \frac{1}{b}\left[u - \frac{db_1}{dy}v\right] = 0$

And at $x = b_2(y)$ i,e. $\eta = 1$, $U = \frac{1}{b}\left[u - \left(\frac{db_2}{dy} + 1.\frac{db}{dy}\right)v\right] = \frac{1}{b}\left[u - \frac{db_2}{dy}v\right] = 0$, which gives,

$$bU = u - \frac{db_2}{dy}v$$

Therefore the boundary conditions reduces to,

$U = 0$ at $\eta = 0$ (24)

$bU - \left(g/h\right)^{1/2}\zeta = 0$ at $\eta = 1$ (25)

$VL + \left(g/h\right)^{1/2}\zeta = 0$ at $\lambda = 0$ (26)

$VL - \left(g/h\right)^{1/2}\zeta = 0$ at $\lambda = 1$ (27)

At each boundary of an island, the normal component of the velocity vanishes. Thus, the boundary conditions of an island are given by

$U = 0$ at $\eta = l_1/k$ and $\eta = l_2/k$ (28)
$V = 0$ at $\lambda = p_1/q$ and $\lambda = p_2/q$ (29)

2.4 Numerical Discretisation In Transformed Domain
In the physical domain, the curvilinear grid system is generated through Equations (09) and (10); and in the transformed domain, the corresponding rectangular grid system is generated through Equations. (16) and (17) with an appropriate choice of l, k, p and q. Discrete coordinate points in the transformed domain at the respective grid widths $\Delta \chi$, $\Delta \lambda$ and we define the grid points (η_i, λ_j) in the domain by

$\chi_i = (i-1)\Delta \chi$ $\qquad i = 1, 2, 3, \ldots, m$ (30)

$\lambda_j = (j-1)\Delta \lambda$ $\qquad j = 1, 2, 3, \ldots, n$ (31)

The sequence of discrete times using the time step Δt is
$t_k = k\Delta t$ $\qquad k = 1, 2, 3, \ldots\ldots$ (32)
A staggered grid system, similar to Arakawa C system is used in the transformed domain where there are three distinct types of computational points. These three types of points are defined as follows: For every discrete grid point (χ_i, λ_j), if i is even and j is odd, the point is a ζ-point at which ζ is computed. If i is odd and j is odd, the point is a u-point at which u is computed. If i is even and j is even, the point is a v-point at which v is computed. In a staggered grid system, since every dependent variable is computed at any one of these three types of points rather than at every point, the CPU time is reduced. Moreover, a staggered grid system is favorable in filtering out sub-grid scale oscillations.
The transformed shallow water equations (21) - (23) together with the boundary conditions (25) - (26) are discretized by finite difference (Forward Time Centered Space) and are solved by a conditionally stable semi-implicit method.
The curvilinear and the corresponding rectangular boundaries and grids are shown in Fig. 3. The χ-axis is directed towards west at an angle $15°$ (anticlockwise) with the latitude line and the λ-axis is directed towards north inclined at an angle $15°$ (anticlockwise) with the longitude line.
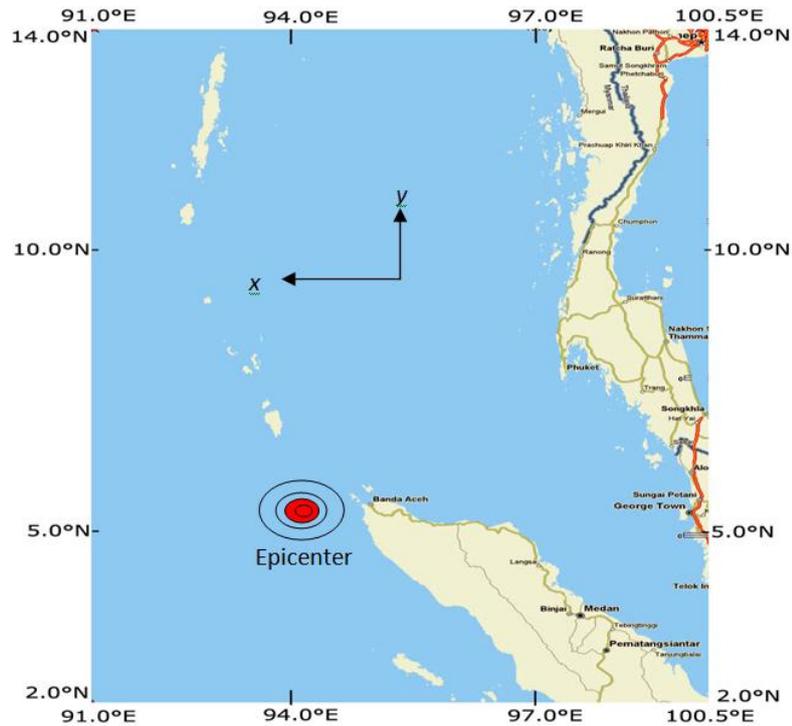
Figure 3: Model Domain including the coastal geometry and the epicenter of the 2004 earthquake
(Retrieved from [7]).

The number of grids in χ and λ - directions are respectively $m = 230$ and $n = 319$ and the grid size is chosen to be equal to 4 km.

The model area includes the source region of Indonesian tsunami 2004. The time step of computation is determined to satisfy the stability condition (Courant condition). It is set to 10 s in this computation. Following [6], the value of the friction coefficient C_f taken as 0.0033 throughout the model area. The depth data for the model area are collected from the Admiralty bathymetric charts. The depth at the entire rest grid points of the mesh are computed by some averaging process. The bathymetry of the model domain is shown in the figure 4.
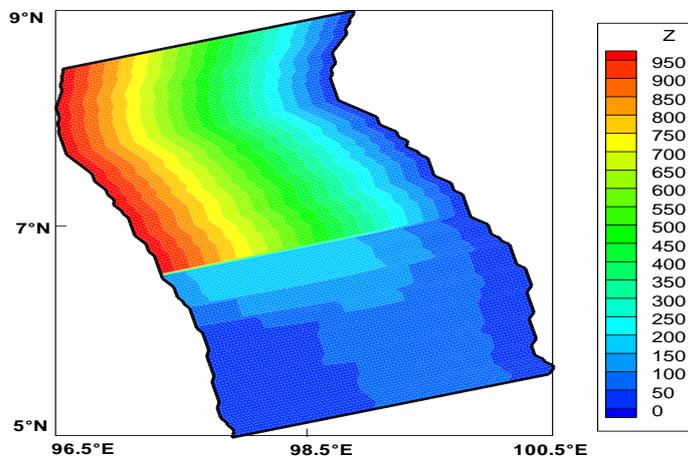

Figure 4: Bathymetry used in the numerical simulation (depth unit: m).

III. Simulation Of 2004 Tsunami Through The Method Of Lines
3.1 Propagation Of Tsunami Towards Phukets Island Arrival Time

In this section, we have showed the speared of 2004 Indian tsunami wave along the coastal belt of southern Thailand with our desired island Phuket. By using graphic design figure, we have represented travelling time and water level of tsunami in various coastal location in southern Thailand.
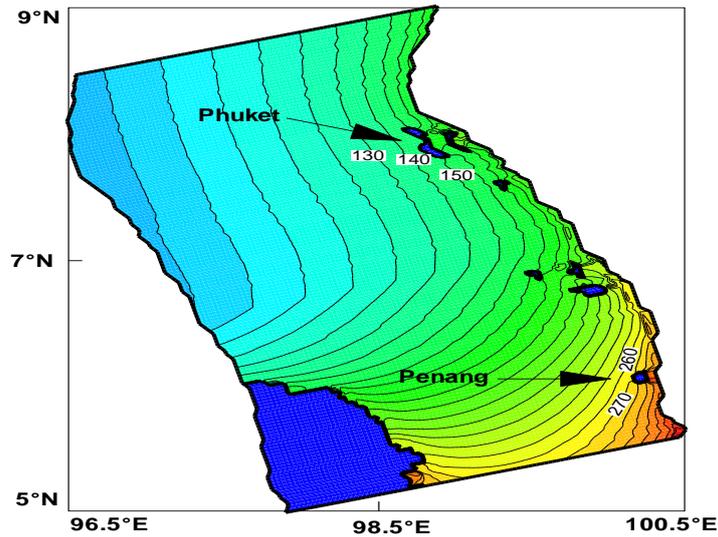
Figure 5: Tsunami propagation time in minutes towards Phuket due to the boundary condition.

According to our numerical modeling, the calculated time travel of tsunami in Phuket is nearly 110 minutes. On the other hand, USGS observed that, the tsunami waved reached in Phuket from tsunami source within two hours or 120 minutes. Hence, our numerical model compute the arrival time, which is not far from the USGS data.

3.2 Estimation Of Maximum Water Level

Figure (6) below describes computed maximum water level along Phuket coastal belt region. Our model shows the maximum water level is 6m to 11.5m approximately in Phuket area, which is situated in northern part of the domain. According to our model, the amplitudes are bigger in north western region. We observes that, some locations nearly 50km from Phuket coastal area, where the tsunami waves varies from 18m to 20 m. Tsuji et al. [15] showed that, the greatest tsunami wave reached in 19.6m at Ban Thung Dao area, it is 50km north from the Phuket. Finally, we can say that, our model shows similar results with the Tsuji et al. [15].
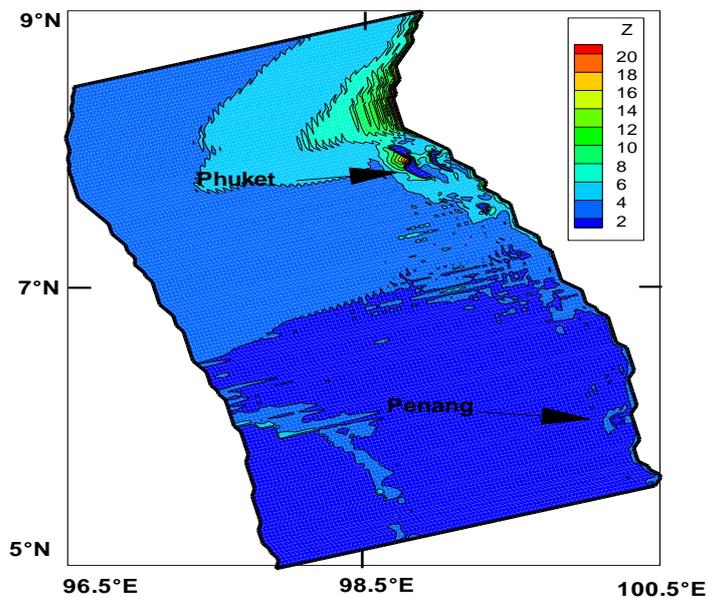

Figure 6: Contour of maximum water elevation around the west coast of Thailand

3.3 Time Histories Of Water Surface Fluctuations At Phuket Island

In this section, our numerical modelling in Phuket describes the water surface fluctuation in Indonesian tsunami 2004 at various regions of the coastal belt of Phuket. Figure 7 delineates that the time sequence of water elevation of Phuket region in south west Thailand. The maximum water height at weast coast of Phuket is 11.5m where the water surface continues to sway for long time figure 7(a). Figure 7 also shows that the time series at south coast of Phuket region started with depression of -3.5m and height of water level get to 6m figure 7(b).
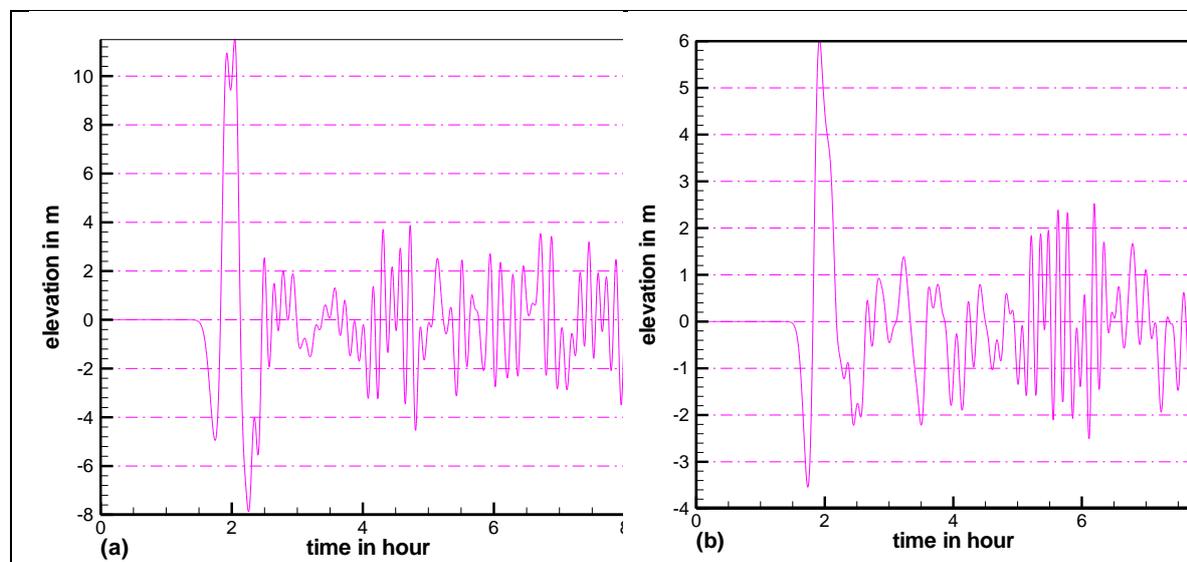

Figure 7: Time histories of water surface fluctuations at two coastal locations of Phuket: (a) West Phuket, (b) South Phuket.

3.4 Comparison Between Model Results And Observed Data

In this section we will compare our simulation result with two research model and United state geological survey (USGS). The two research models are boundary-fitted curvilinear grid model [11] and the MOL in stair step model. From the table below, all developed model have been applied in rectangular region between 20N to 140N and 910E to 100.50E. All the models showed the wave propagation and arrival time of 2004 Indian ocean tsunami along the coast belt of Penang and Phuket, while this paper shows the wave propagation and arrival time of 2004 Indonesian tsunami along the Phuket coastal area to get the more accurate results.

The arrival time of tsunami and the maximum water levels computed by the above mentioned three models are compared with the data available in USGS website.

Table 1. Computed and observed / USGS tsunami propagation time and water levels for Phuket

Model name Phuket Island	Numerical Modelling in Phuket	MOL in Stair Step	Boundary - Fitted	USGS
Propagati on time (min)	110	90	110	< 120
Max. water level (m)	6 – 11.5	5 – 9	6 – 12	7 – 11

The arrival time of 2004 indian ocean tsunami for Phuket is calculated by Numerical modelling in Phuket, MOL in stair steps model, and boundary-fitted curvilinear model, that are 110min, 90min and 240min respectively. According to USGS website, we saw that the tsunami surge arrived in Phuket Island within two hours from the tsunami source. On the other hand, maximum water level in Phuket is 6-11.5m, 5-9m and 6-12m by numerical modelling, MOL stair steps model, Boundary-fitted model respectively. So, overall deliberation, it is obvious that the arrival time and maximum water surge along the Phuket island is very similar with the referred models and USGS website.

IV.   Conclusion

In this paper, we have presented a new numerical model by using the method of lines technique in a boundary-fitted curvilinear grid model, which is used to solve the vertically integrated shallow water equations. Our numerical model has been considered to simulate the 2004 Indonesian tsunami along the Phuket coastal area, which is situated in southern Thailand. In this research, we have considered only Phuket coastal belt area because we have tried to get more accurate results than other developed models. In addition, our simulation result is nearest to the other developed models and USGS. So, the author shows in this paper that, simulation result in one area is nearer to two or more than two areas in a same domain. Moreover, the author believes that, this research will be very helpful for the novice researcher and master's thesis students. By reading this paper any neophyte researcher can develop their knowledge appropriately. Overall, it is a good method to develop the regional tsunami early warning system.

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