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Homotopy Analysis to Soret and Dufour Effects on Heat and Mass Transfer of a Chemically Reacting Fluid past a Moving Vertical Plate with Viscous Dissipation

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Abstract: *The objective of this paper is to study the Soret and Dufour effects on the free convection boundary layer flow of an incompressible, viscous and chemically reacting fluid over a vertical plate in the presence of viscous dissipation. The governing partial differential equations are converted to a set of ordinary differential equations using suitable similarity transformations. The resulting equations are solved analytically using homotopy analysis method (HAM). The convergence of obtained analytical solutions is explicitly discussed. The effects of various parameters on dimensionless velocity, temperature and concentration profiles are discussed with the help of graphs. The numerical values of skin friction, Nusselt number and Sherwood number for different parameters are presented in tabular form. Our results are compared with the previously published results and are found to be in good agreement.*

Keywords: *Soret and Dufour effects, viscous dissipation, HAM.*

I. Introduction

Free convection flow in porous medium has gained increasing research interest in last several years due to its wide applications in chemical and mechanical engineering. For example, in food processing, drying process, heat exchangers, metallurgy, nuclear reactors, etc. Detailed study on this subject has been made by many researchers. Helmy [1] studied MHD unsteady free convection flow past a vertical plate embedded in a porous medium. Elabashbeshy [2] analyzed the magnetic field effect on heat and mass transfer analysis along a vertical plate. Prasad et al. [3] studied natural convection in porous medium. Raptis e al. [4] investigated free convection flow through porous medium bounded by an infinite vertical porous plate with constant heat flux. Makinde [5] investigated free convection flow with thermal radiation and mass transfer past a moving vertical porous plate. Hymavathi and Shankar [6] have discussed the flow and heat transfer characteristics of an incompressible, electrically conducting visco-elastic fluid over a stretching sheet using quasilinearization method. Hymavathi [7] has studied the effect of radiation on MHD boundary layer flow of a viscous fluid over a nonisothermal stretching sheet.

In a moving fluid when heat and mass transfer occur simultaneously affecting each other causes cross diffusion effect (Soret and Dufour effects). The Soret and Dufour effects has several applications in the fields like hydrology, petrology, geosciences, etc. Kaufoussian and Williams [8] analyzed the thermal diffusion and diffusion thermal effects on mixed free force convective and mass transfer of steady laminar boundary layer flow with temperature dependent viscosity. Postelnicu [9] analyzed magnetic field effect on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Jha and Ajibade [10] discussed the heat and mass transfer aspect of the flow of a viscous incompressible fluid in a vertical channel considering the Dufour effect. Anghel et al. [11] presented the Soret and Dufour effects on free convection boundary layer flow over a vertical surface embedded in porous medium. Dursunakya and Worek [12] studied diffusion thermo and thermal diffusion effects in transient and steady natural convection from a vertical surface.

In the above mentioned work, the viscous dissipation effect has been neglected which is very important for the fluids with high velocity or high Prandtl number. This effect is generally characterized by the Eckert number Ec. Grebhart and Molllendorf [13] analysed viscous dissipation in external natural convection flows. Israel-Cookey et al. [14] investigated the influence of viscous dissipation and radiation on the problem of unsteady magnetohydrodynamic free-convection flow past an infinite vertical heated plate in an optically thin environment with time-dependent suction Vajravelu and Hadjinicalaou [15] analyzed the heat transfer characteristics over a stretching surface with viscous dissipation in the presence of internal heat generation or absorption. Pantokratoras [16] studied the effect of viscous dissipation in natural convection along a heated vertical plate. Kabir et al. [17] discussed the effects of viscous dissipation on MHD natural convection flow along a vertical wavy surface with heat generation. Recently, Prsanna Lakshmi [18] studied Soret and Dufour effects on MHD c boundary layer flow of a chemically reacting fluid past a moving vertical plate with viscous dissiapton.

The aim of this paper is to study the Soret and Dufour effects on free convective boundary layer flow of a chemically reacting fluid past a moving vertical plate with viscous dissipation. HAM [19, 20] which was proposed by Liao [21, 22] is used to solve the dimensionless equations. The effects of various parameters such as buoyancy parameters Gr and Gc, chemical reaction parameter K_r, Prandtl number Pr, Eckert number Ec, Soret number Sr, Dufour number Du and Schmidt number Sc on dimensionless velocity, temperature and concentration profiles are discussed with the help of graphs. The numerical values of skin friction, Nusselt number and Sherwood number for different parameters are presented in tabular form. Our results are compared with the previously published results and are found to be in good agreement.

II. Mathematical formulation

In this paper, steady, two dimensional, viscous, incompressible, chemically reacting free convection flow along a semi infinite vertically moving flat plate under the influence of viscous dissipation and Soret and Dufour effects is considered. The positive x-axis is assumed along the direction of the flow. The y-axis is taken perpendicular to it. The fluid properties are assumed to be constant except the density in the buoyancy terms which is approximated according to the Boussinesq's approximation. Under the above assumptions the governing equations describing the flow are

$$\frac{\partial u}{\partial x}+\frac{\partial \mathrm{v}}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+\mathrm{v}\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\,\beta_T\left(T-T_\infty\right)+g\,\beta_c\left(C-C_\infty\right), \tag{2}$$

$$u\frac{\partial T}{\partial x}+\mathrm{v}\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{D_m k_T}{c_s C_p}\frac{\partial^2 C}{\partial y^2}+\frac{\nu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2, \tag{3}$$

species equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2}-\gamma\left(C-C_\infty\right). \tag{4}$$

The boundary conditions are

$$\begin{aligned} &u=Bx,\ \mathrm{v}=0,\ T=T_w=T_\infty+a\,x,\ C=C_w=C_\infty+b\,x \quad at\ y=0,\\ &u\to 0,\ \ T\to T_\infty,\ \ C\to C_\infty \qquad\qquad as\ y\to\infty, \end{aligned} \tag{5}$$

where B is a constant, u and v are the velocity components in x and y directions, T and C are the fluid temperature and concentration, T_w and C_w are the wall surface temperature and concentration, T_∞ and C_∞ are the fluid temperature and concentration at a distinct location from the surface, β_T and β_C are the thermal and concentration expansion coefficients, k_T is the thermal diffusion ratio, ν is the kinematic viscosity, g is the acceleration due to gravity, ρ is the density, c_p is the specific heat at constant pressure, c_s is the concentration susceptibility, T_m is the mean fluid temperature, γ is the reaction rate, the α is the thermal diffusivity, D_m is the mass diffusivity, a and b are constants.

Generally, when $T_w(x)>T_\infty$ and $C_w(x)>C_\infty$ the convective motion of the fluid is in the upward direction along the plate. In this study, without changing the boundary layer profiles we consider the case in which fluid flow in downward direction i.e., $T_w(x)<T_\infty$ and $C_w(x)<C_\infty$.

We introduce the following similarity transformations to convert the partial differential equations into ordinary differential equations:

$$\begin{aligned} &\eta=y\sqrt{\frac{\beta}{\nu}},\quad \psi=x\sqrt{\nu\beta}\ f(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\quad \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty},\\ &u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}, \end{aligned} \tag{6}$$

substituting (6) in (1) to (5), we get

$$f'''+f\,f''-(f')^2+Gr\,\theta+Gc\,\phi=0, \tag{7}$$

$$\theta''+Pr\,f\,\theta'-Pr\,f'\,\theta+Pr\,Ec\left(f''\right)^2+Pr\,Du\,\phi''=0, \tag{8}$$

$$\phi''+Sc\,\phi'\,f-Sc\,\phi\,f'-Sc\,\beta\,\phi+Sc\,Sr\,\theta''=0. \tag{9}$$

The corresponding boundary conditions are

$$\begin{aligned} &f(0)=0,\quad f'(0)=1\quad \theta(0)=1,\quad \phi(0)=1,\\ &\qquad f'(\infty)=0,\quad \theta(\infty)=0,\quad \phi(\infty)=0. \end{aligned} \tag{10}$$

Where $Gr=\dfrac{g\,\beta_T(T_w-T_\infty)}{x\,B^2}$ is the local temperature Grashof number, $Gc=\dfrac{g\,\beta_c(C_w-C_\infty)}{x\,B^2}$ is the local concentration Grashof number, $Pr=\dfrac{\nu}{\alpha}$ is the Prandtl number, $Ec=\dfrac{B^2x^2}{c_p(T_w-T_\infty)}$ is the Eckert number, $Du=\dfrac{D_m k_T(C_w-C_\infty)}{c_s\,c_p\,(T_w-T_\infty)}$ is the Dufour number, $Sc=\dfrac{\nu}{D_m}$ is the Schmidt number, $Sr=\dfrac{D_m k_T\,(T_w-T_\infty)}{\nu\,T_m\,(C_w-C_\infty)}$ is the Soret number, $\beta=\dfrac{\gamma}{B}$ is the chemical reaction parameter

Here, we also concentrate on skin friction coefficient $C_f=2(\mathrm{Re})^{-1/2}\,f''(0),$ Nusselt number $Nu=-(\mathrm{Re})^{1/2}\,\theta'(0)$ and Sherwood number $Sh=-(\mathrm{Re})^{1/2}\,\phi'(0),$ where $\mathrm{Re}=\dfrac{Ut}{\nu}$ is the Reynolds's number.

III. Homotopy analysis solutions

In this section, we employ HAM to solve the equations (7) to (9) subject to the boundary conditions (10). We choose the initial guesses f_0, θ_0 and ϕ_0 of f,θ and ϕ respectively, in the following form

$$f_0(\eta)=1-e^{-\eta}, \tag{11}$$

$$\theta_0(\eta)=e^{-\eta}, \tag{12}$$

$$\phi_0(\eta)=e^{-\eta}. \tag{13}$$

The linear operators are selected as

$$L_1(f)=f'''-f', \tag{14}$$

$$L_2(\theta)=\theta''-\theta, \tag{15}$$

$$L_3(\phi)=\phi''-\phi. \tag{16}$$

which have the following properties

$$L_1\left(C_1+C_2e^{\eta}+C_3e^{-\eta}\right)=0, \tag{17}$$

$$L_3\left(C_4e^{\eta}+C_5e^{-\eta}\right)=0, \tag{18}$$

$$L_2\left(C_6e^{\eta}+C_7e^{-\eta}\right)=0, \tag{19}$$

where C_i $(i=1\ to\ 7)$ are the arbitrary constants.

If $p\in[0,1]$ is the embedding parameter, $\hbar_1,\hbar_2$ and $\hbar_3$ are the non-zero auxiliary parameters and $H_1(\eta),H_2(\eta)$ and $H_3(\eta)$ are auxiliary functions, then we can construct the following zeroth-order deformation equations:

$$(1-p)L_1\left(\hat f(\eta;p)-f_0(\eta)\right)=p\hbar_1\,H_1(\eta)\,N_1\left[\hat f(\eta;p),\hat\theta(\eta;p),\hat\phi(\eta;p)\right], \tag{20}$$

$$(1-p)\,L_2(\hat\theta(\eta;p)-\theta_0(\eta))=p\hbar_2H_2(\eta)\,N_2[\hat f(\eta;p),\hat\theta(\eta;p),\hat\phi(\eta;p)], \tag{21}$$

$$(1-p)\,L_3(\hat\phi(\eta;p)-\phi_0(\eta))=p\hbar_2H_3(\eta)\,N_3[\hat f(\eta;p),\hat\theta(\eta;p),\hat\phi(\eta;p)], \tag{22}$$

subject to the boundary conditions

$$\begin{aligned} &\hat{f}(0;p)=0, \qquad \hat{f}'(0;p)=1, \qquad \hat{f}'(\infty;p)=0,\\ &\hat{\theta}(0;p)=1, \qquad \hat{\theta}(\infty;p)=0,\\ &\hat{\phi}(0;p)=1, \qquad \hat{\phi}(\infty;p)=0. \end{aligned} \tag{23}$$

Based on equations (20) to (21), we define nonlinear operators as

$$\begin{aligned} N_1\left[\hat{f}(\eta;p),\hat{\theta}(\eta;p),\hat{\phi}(\eta;p)\right] &= \frac{\partial^3 \hat{f}(\eta;p)}{\partial\eta^3} + \hat{f}(\eta;p)\frac{\partial^2 \hat{f}(\eta;p)}{\partial\eta^2}\\ &-\left(\frac{\partial \hat{f}(\eta;p)}{\partial\eta}\right)^2 + Gr\,\hat{\theta}(\eta;p) + Gc\,\hat{\phi}(\eta;p), \end{aligned} \tag{24}$$

$$\begin{aligned} N_\theta\left[f(\eta;p),\theta(\eta;p)\right] &= \frac{\partial^2\theta(\eta,p)}{\partial\eta^2} + \Pr f(\eta,p)\frac{\partial\theta(\eta,p)}{\partial\eta}\\ &-\Pr\frac{\partial f(\eta,p)}{\partial\eta}\theta(\eta,p) + \Pr Ec\left(\frac{\partial^2 f(\eta,p)}{\partial\eta^2}\right)^2 + \Pr Du\frac{\partial^2\phi(\eta,p)}{\partial\eta^2}, \end{aligned} \tag{25}$$

$$\begin{aligned} N_\phi\left[\phi(\eta,p)\right] &= \frac{\partial^2\phi(\eta,p)}{\partial\eta^2} + Sc\, f(\eta,p)\frac{\partial\phi(\eta,p)}{\partial\eta} - Sc\frac{\partial f(\eta,p)}{\partial\eta}\phi(\eta,p)\\ &-Sc\,\beta\,\phi(\eta,p) + Sc\,Sr\frac{\partial^2\theta(\eta,p)}{\partial\eta^2}. \end{aligned} \tag{26}$$

For $p=0$ and $p=1$ we have

$$\hat{f}(\eta;0)=f_0(\eta), \qquad \hat{\theta}(\eta;0)=\theta_0(\eta), \qquad \hat{\phi}(\eta;0)=\phi_0(\eta), \tag{27}$$

$$\hat{f}(\eta;1)=f(\eta), \qquad \hat{\theta}(\eta;1)=\theta(\eta), \qquad \hat{\phi}(\eta;1)=\phi(\eta). \tag{28}$$

Thus, as p increases from 0 to 1 then $\hat{f}(\eta;p)$, $\hat{\theta}(\eta;p)$ and $\hat{\phi}(\eta;p)$ vary from initial approximations to the exact solutions of the original nonlinear differential equations.

Now expanding $\hat{f}(\eta;p)$, $\hat{\theta}(\eta;p)$ and $\hat{\phi}(\eta;p)$ in Taylor's series w.r.to p we have

$$\hat{f}(\eta;p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta)\,p^m, \tag{29}$$

$$\hat{\theta}(\eta;p) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta)\,p^m, \tag{30}$$

$$\hat{\phi}(\eta;p) = \phi_0(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta)\,p^m, \tag{31}$$

where

$$f_m(\eta) = \frac{1}{m!}\left.\frac{\partial^m \hat{f}(\eta;p)}{\partial p^m}\right|_{p=0}, \tag{32}$$

$$\theta_m(\eta) = \frac{1}{m!}\left.\frac{\partial^m \theta(\eta;p)}{\partial p^m}\right|_{p=0}, \tag{33}$$

$$\phi_m(\eta) = \frac{1}{m!}\left.\frac{\partial^m \phi(\eta;p)}{\partial p^m}\right|_{p=0}. \tag{34}$$

If the initial approximations, auxiliary linear operators and non-zero auxiliary parameters are chosen in such a way that the series (29) to (31) are convergent at $p=1$, then

$$f(\eta)= f_0(\eta)+\sum_{m=1}^{\infty} f_m(\eta), \tag{35}$$

$$\theta(\eta)=\theta_0(\eta)+\sum_{m=1}^{\infty}\theta_m(\eta), \tag{36}$$

$$\phi(\eta)=\phi_0(\eta)+\sum_{m=1}^{\infty}\phi_m(\eta). \tag{37}$$

Differentiating equations (20) to (22) m times w.r.to p, setting $p=0$ and finally dividing with $m!$, we get the mth-order deformation equations as follows:

$$L_1\left(f_m(\eta)-\chi_m\, f_{m-1}(\eta)\right)=\hbar_1\, H_1(\eta)\, R_m^f(\eta), \tag{38}$$

$$L_2\left(\theta_m(\eta)-\chi_m\, \theta_{m-1}(\eta)\right)=\hbar_2\, H_2(\eta)\, R_m^\theta(\eta), \tag{39}$$

$$L_3\left(\phi_m(\eta)-\chi_m\, \phi_{m-1}(\eta)\right)=\hbar_3\, H_3(\eta)\, R_m^\phi(\eta), \tag{40}$$

with the following boundary conditions

$$\begin{aligned} &f_m(0)=0, \quad f_m^{'}(0)=0, \quad f_m^{'}(\infty)=0,\\ &\theta_m(0)=0, \quad \theta_m(\infty)=0,\\ &\phi_m(0)=0, \quad \phi_m(\infty)=0, \end{aligned} \tag{41}$$

where

$$R_m^f(\eta)= f_{m-1}^{'''}+\sum_{i=0}^{m-1} f_{m-1-i}\, f_i^{''}-\sum_{i=0}^{m-1} f_{m-1-i}^{'}\, f_i^{'}+Gr\,\theta_{m-1}+Gc\,\phi_{m-1}, \tag{42}$$

$$\begin{aligned} R_m^\theta(\eta)=\theta_{m-1}^{''}+\Pr\sum_{i=0}^{m-1} f_{m-1-i}\,\theta_i^{'}-\Pr\sum_{i=0}^{m-1} f_{m-1-i}^{'}\,\theta_i \\ +\Pr Ec\sum_{i=0}^{m-1} f_{m-1-i}^{''}\, f_i^{''}+\Pr Du\,\phi_{m-1}^{''}, \end{aligned} \tag{43}$$

$$\begin{aligned} R_m^\phi(\eta)=\phi_{m-1}^{''}+Sc\sum_{i=0}^{m-1} f_{m-1-i}\,\phi_i^{'}-Sc\sum_{i=0}^{m-1} f_{m-1-i}^{'}\,\phi_i-Sc\,\beta\,\phi \\ +Sr\ Sc\ \theta_{m-1}^{''}, \end{aligned} \tag{44}$$

and

$$\chi_m=\begin{cases}0, & m\le 1,\\ 1, & m>1.\end{cases} \tag{45}$$

We choose the auxiliary functions as follows:

$$\begin{aligned} &H_1(\eta)=1,\\ &H_2(\eta)=1,\\ &H_3(\eta)=1. \end{aligned} \tag{46}$$

If we let $f_m^*(\eta)$, $\theta_m^*(\eta)$ and $\phi_m^*(\eta)$ as the special solutions of the mth order deformation equations, then the general solutions are given by

$$f_m(\eta)= f_m^*(\eta)+C_1+C_2e^{\eta}+C_3e^{-\eta}, \tag{47}$$

$$\theta_m(\eta)=\theta_m^*(\eta)+C_4e^{\eta}+C_5e^{-\eta}, \tag{48}$$

$$\phi_m(\eta)=\phi_m^*(\eta)+C_6e^{\eta}+C_7e^{-\eta}, \tag{49}$$

where the integral constants $C_i\ (i=1\ to\ 7)$ are determined by the boundary conditions (41).

Now it is easy to solve the linear non-homogeneous equations (39) and (41) using MATHEMATICA software one after the other by considering $m=1,2,\ldots$

IV. Convergence of homotopy solutions

It is important to note that the convergence and rate of approximation of the obtained series solutions are dependent on the auxiliary parameters $\hbar_1$, $\hbar_2$ and $\hbar_3$. So to find $\hbar_1$, $\hbar_2$ and $\hbar_3$, $\hbar$-curves are drawn for the 20th order approximations in Fig. 1. From these curves, it is clear that the admissible ranges of $\hbar_1$, $\hbar_2$ and $\hbar_3$ are

$$-1.6 \le \hbar_f \le 0.0, \qquad -1.5 \le \hbar_\theta \le 0.0, \qquad -1.5 \le \hbar_\phi \le 0.0.$$

V. Results And Discussion

Numerical and graphical results are obtained for the non dimensional velocity, temperature and concentration profiles under various parameters. The parameters that arise in this study are the thermal Grashof number Gr, solutal Grashof number Gc, Chemical reaction parameter β, Dufour number Du, Soret number Sr, Prandtl number $\Pr$, Eckert number Ec and Schmidt number Sc.

Figs. 2 to 7 show the effect of thermal Grashof number Gr and solutal Grashof number Gc on velocity, temperature and concentration profiles. It is noticed from the figures that the velocity increases by increasing the Grashof number Gr and the solutal Grashof number Gc while temperature and concentration decrease.

Figs. 8 to 10 depict the effect of $\Pr$ on velocity, temperature and concentration distributions. It is noticed that the effect of $\Pr$ on velocity and concentration is very less but temperature is decreasing with the increases of $\Pr$.

Figs. 11 to 13 depict the behavior of velocity, temperature and concentration profiles for different values of Eckert number Ec. From the figures it is clear that there is a slight increase in velocity and a slight decrease in concentration profiles with an increase in Ec. It is also observed that temperature increases with Ec.

Figs. 14 to 16 illustrate the behavior of velocity, temperature and concentration fields for different values of Dufour number Du. From the figures it is clear that there is a slight increase in velocity and a slight decrease in concentration profiles with an increase in Du. It is also noticed that there is recognizable increase in temperature with Du.

Figs. 17 to 19 show the dimensionless velocity, temperature and concentration profiles for different values of Schmidt number Sc. It is clear that the velocity and concentration profiles decrease with the increase of Schmidt number Sc. Further, it is observed that the temperature monotonically increases with the increase of Schmidt number Sc.

The effect of K_r on $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ is shown in Figs. 20 to 22. From the figures it is clear that there is a slight decrease in velocity and a slight increase in temperature profiles with an increase in β. It is also noticed that there is significant decrease in concentration with an increase in β.

Figs. 23 to 25 depict the velocity, temperature and concentration profiles for different values of the Soret number Sr. It is observed that an increase in the Soret number Sr causes an increase in the velocity and concentration but a decrease in temperature.

Table 1 shows the convergence of the solutions with increasing order of approximations. To ensure the accuracy of our method, we have compared our results with the available results of Ibrahim and Makinde [23] in Table 2. This comparison proves the accuracy and validity of our results. Table 3 presents the numerical values of $-f''(0), -\theta'(0)$ and $-\phi'(0)$ for different values of various parameters.

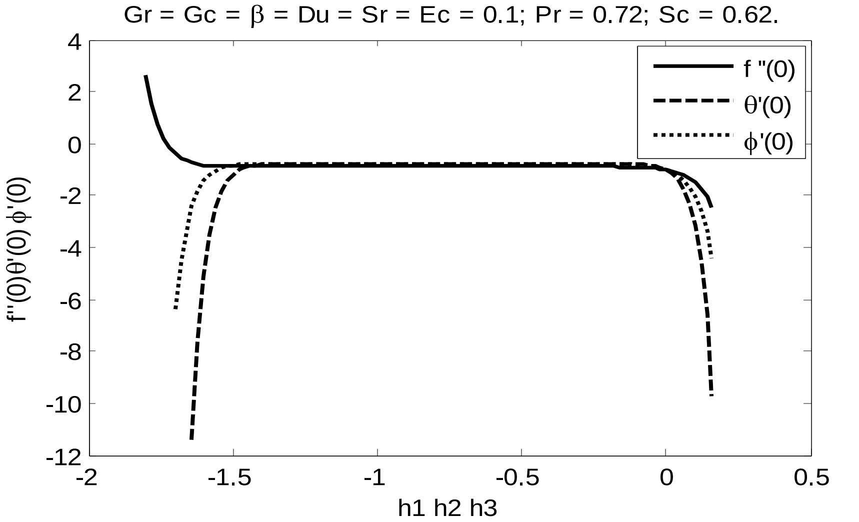

Fig. 1: $\hbar$ -curves for 20th order approximations.

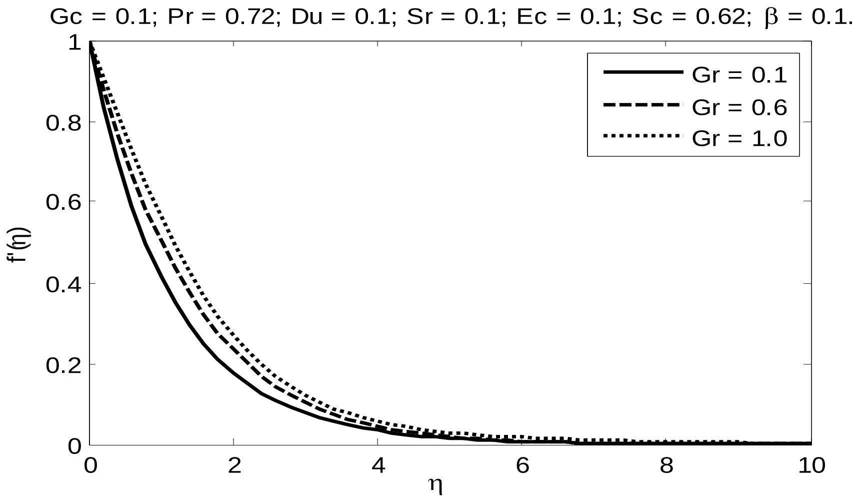

Fig. 2: Velocity profiles for different values of Gr .

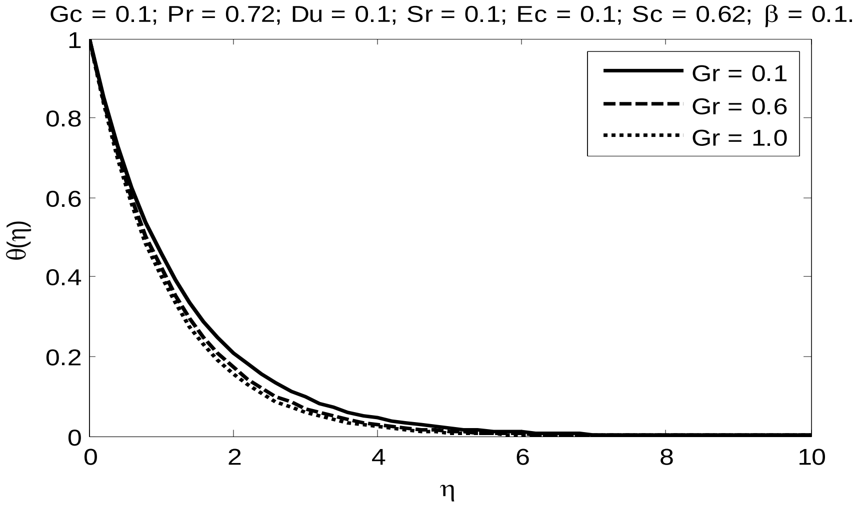

Fig. 3: Temperature profiles for different values of Gr .

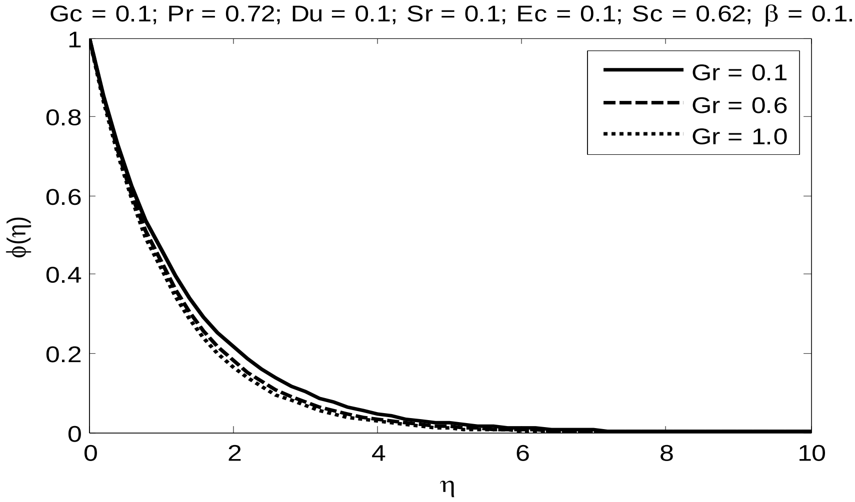

Fig. 4: Concentration profiles for different values of Gr .

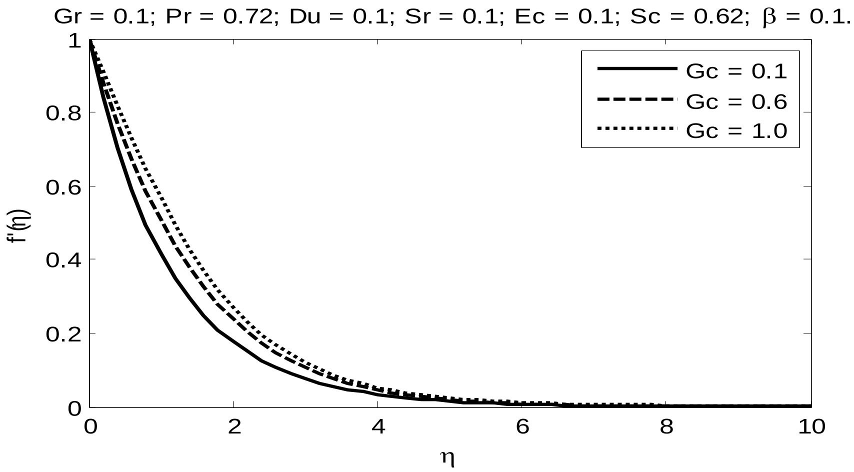

Fig. 5: Velocity profiles for different values of Gc .

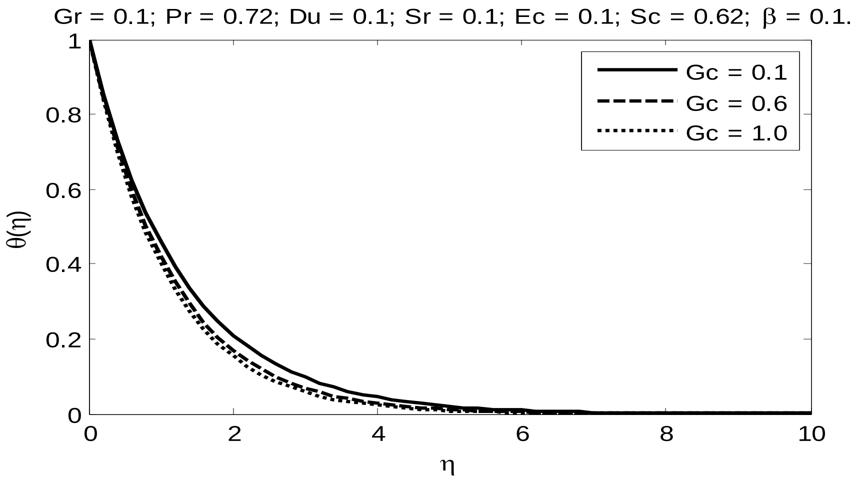

Fig. 6: Temperature profiles for different values of Gc .

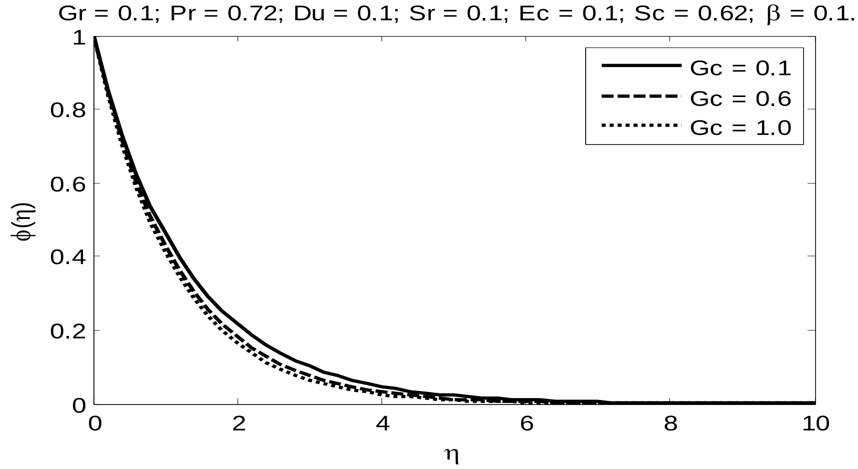

Fig. 7: Concentration profiles for different values of Gc .

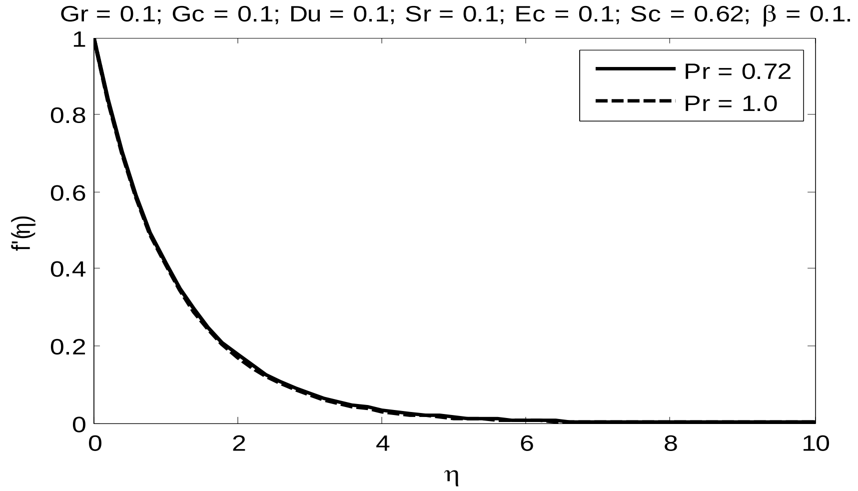

Fig. 8: Velocity profiles for different values of Pr .

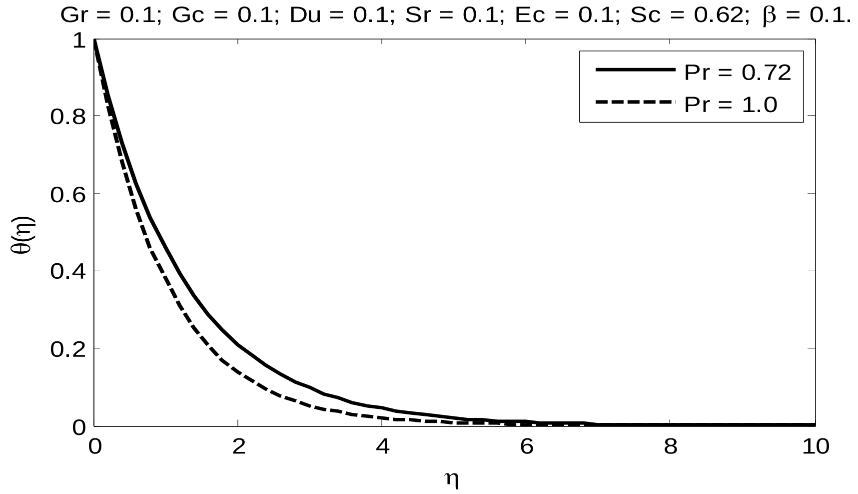

Fig. 9: Temperature profiles for different values of Pr .

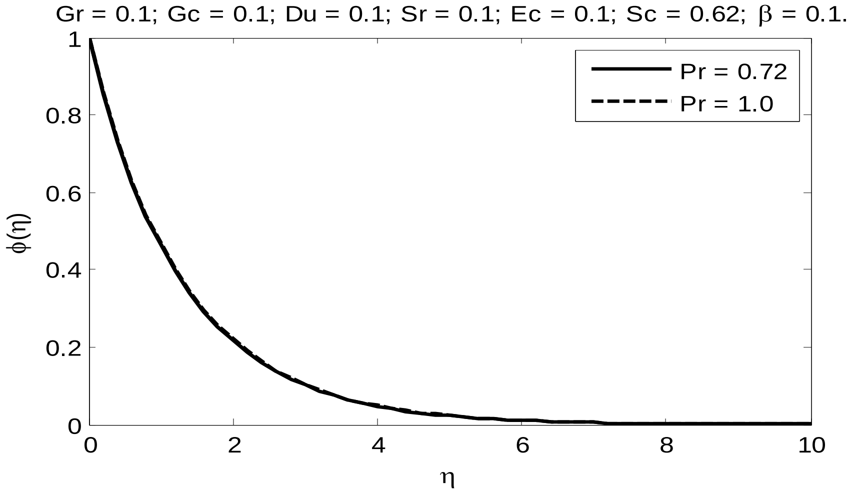

Fig. 10: Concentration profiles for different values of Pr .

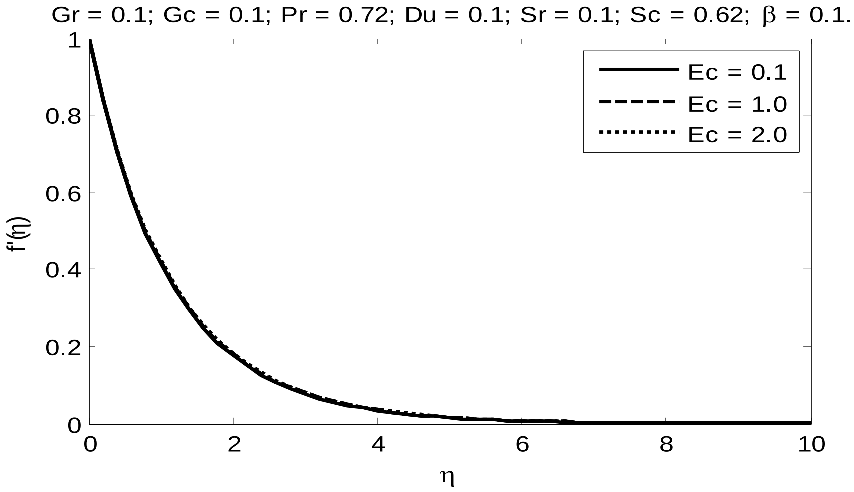

Fig. 11: Velocity profiles for different values of Ec .

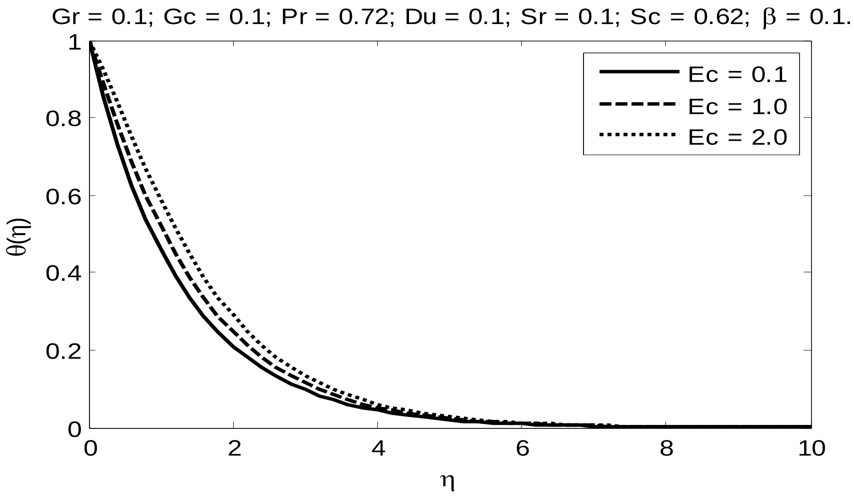

Fig. 12: Temperature profiles for different values of Ec .

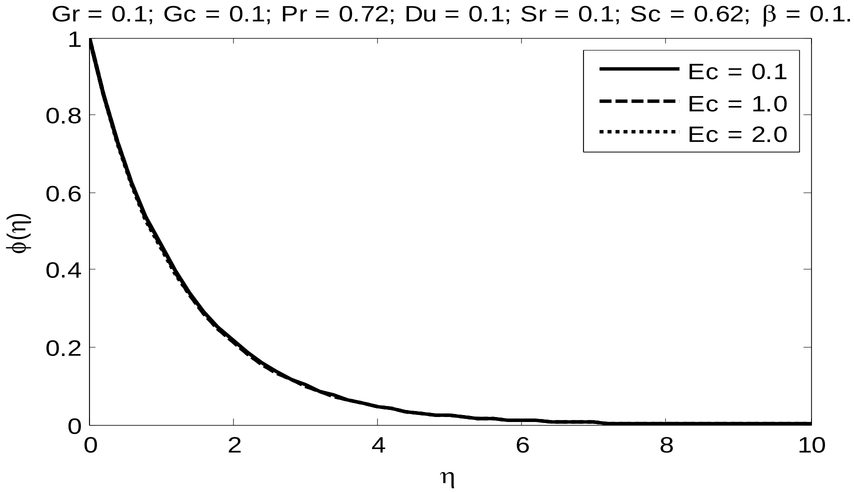

Fig. 13: Concentration profiles for different values of Ec .

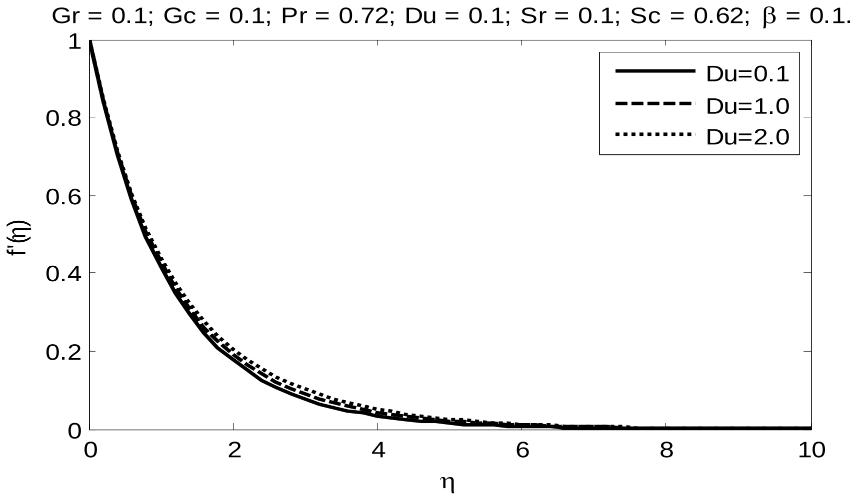

Fig. 14: Velocity profiles for different values of Du .

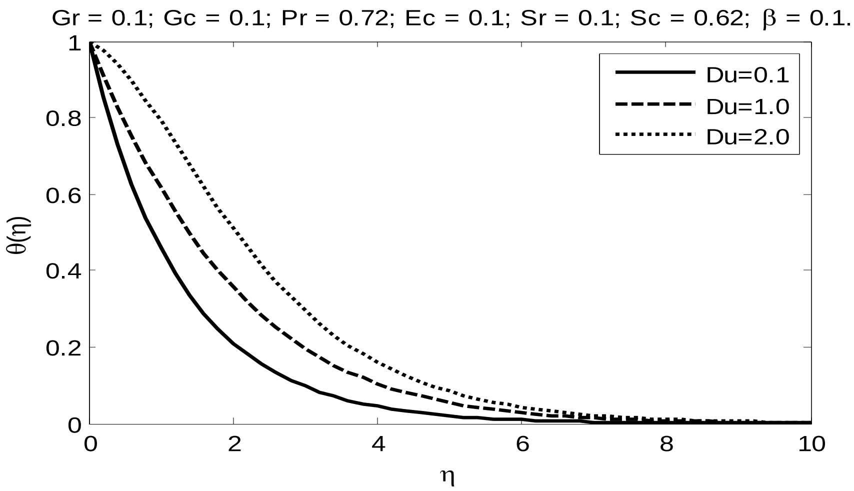

Fig. 15: Temperature profiles for different values of Du .

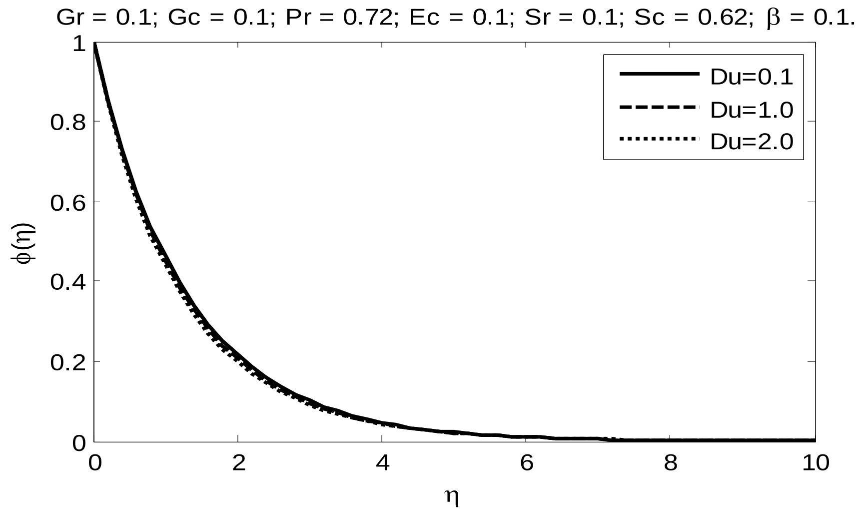

Fig. 16: Concentration profiles for different values of Du .

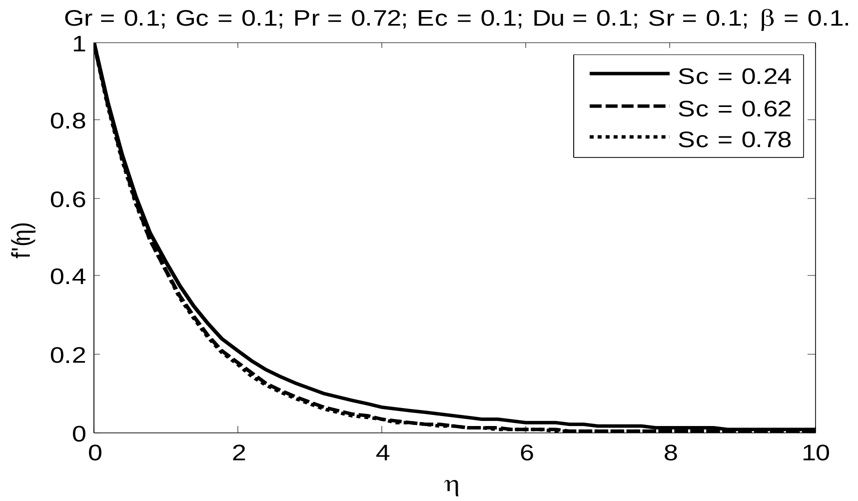

Fig. 17: Velocity profiles for different values of Sc .

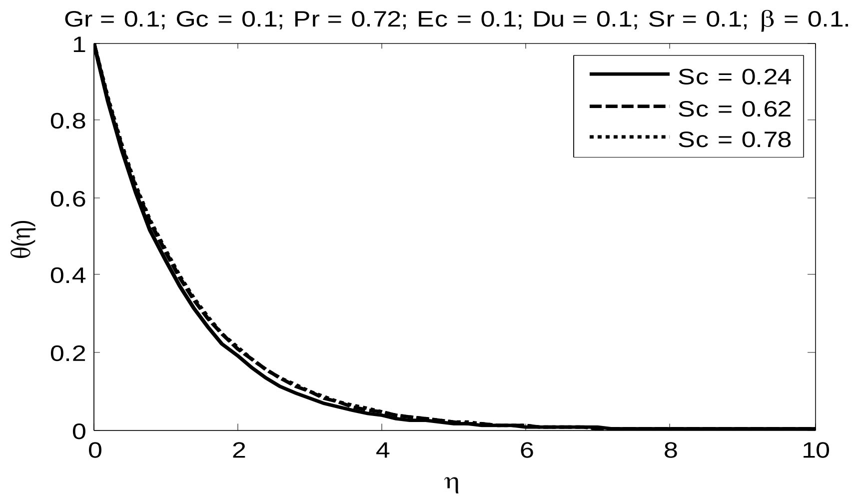

Fig. 18: Temperature profiles for different values of Sc .

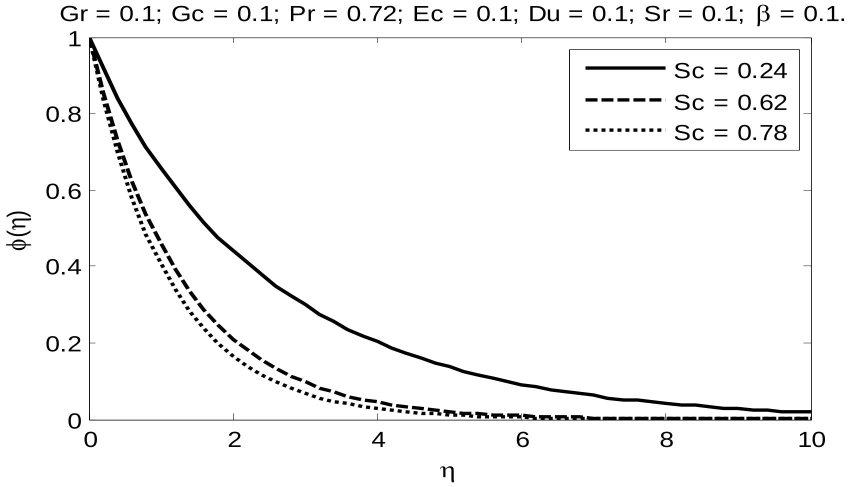

Fig. 19: Concentration profiles for different values of Sc.

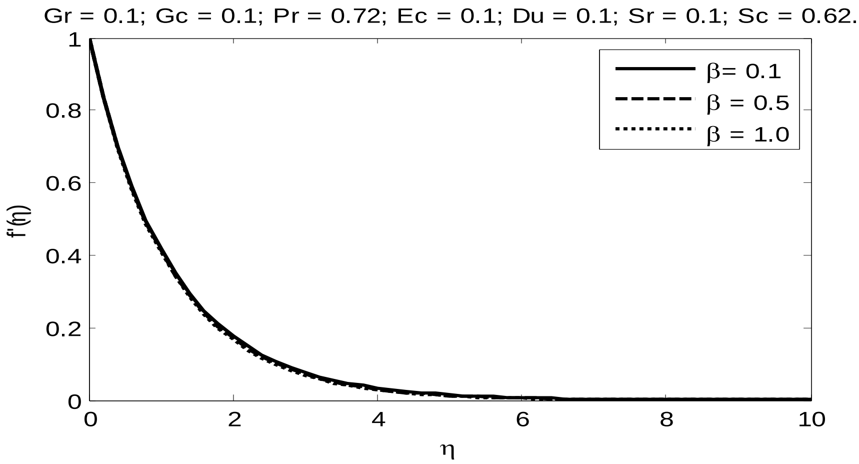

Fig. 20: Velocity profiles for different values of β.

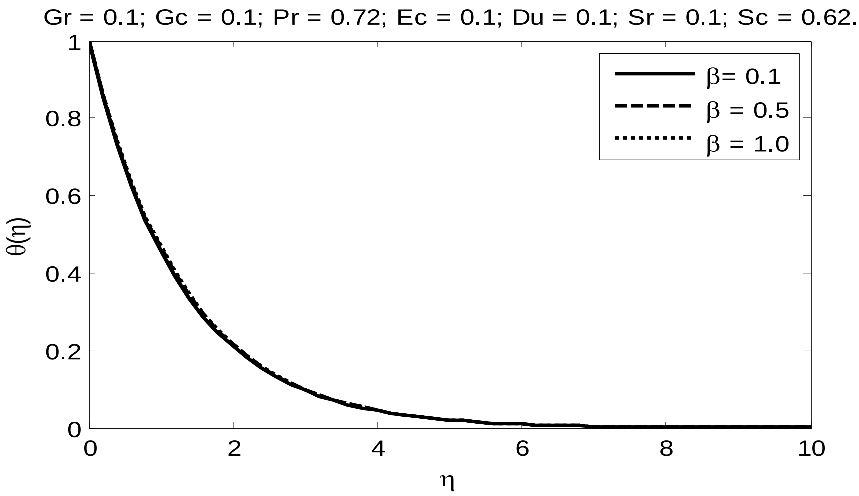

Fig. 21: Temperature profiles for different values of β.

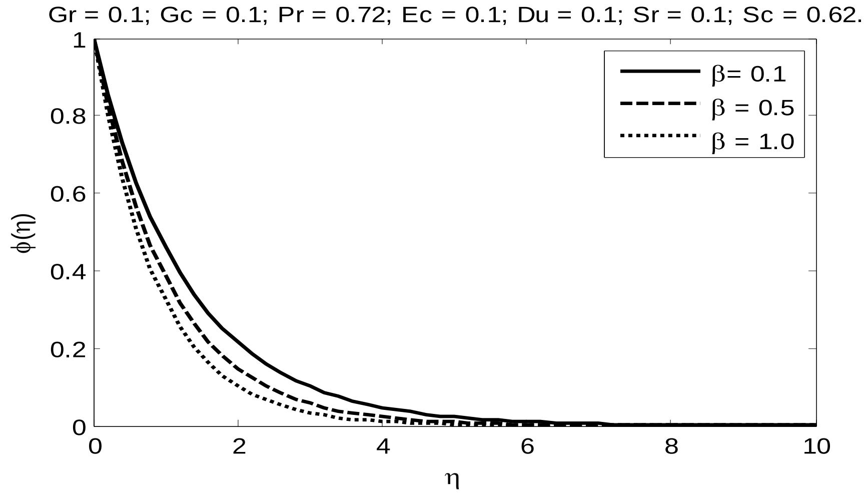

Fig. 22: Concentration profiles for different values of β .

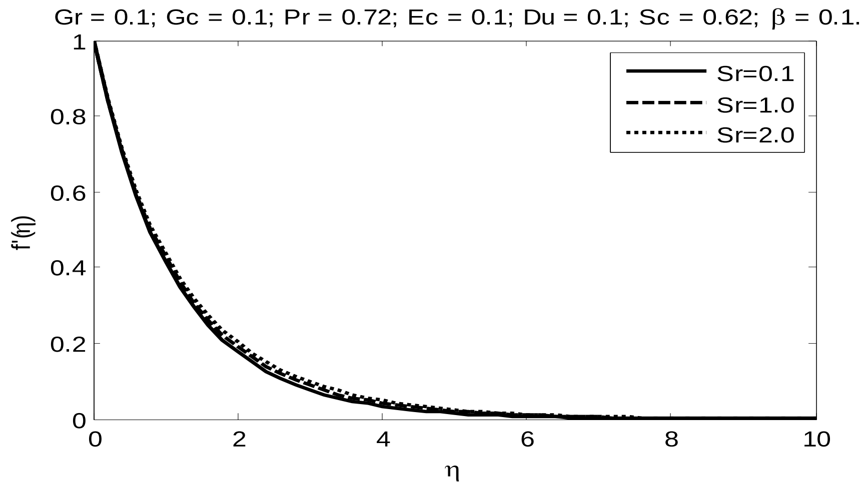

Fig. 23: Velocity profiles for different values of Sr .

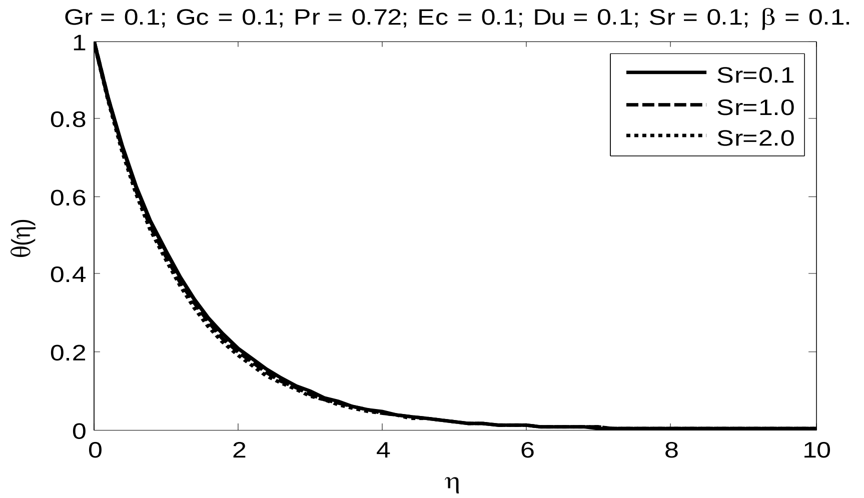

Fig. 24: Temperature profiles for different values of Sr .

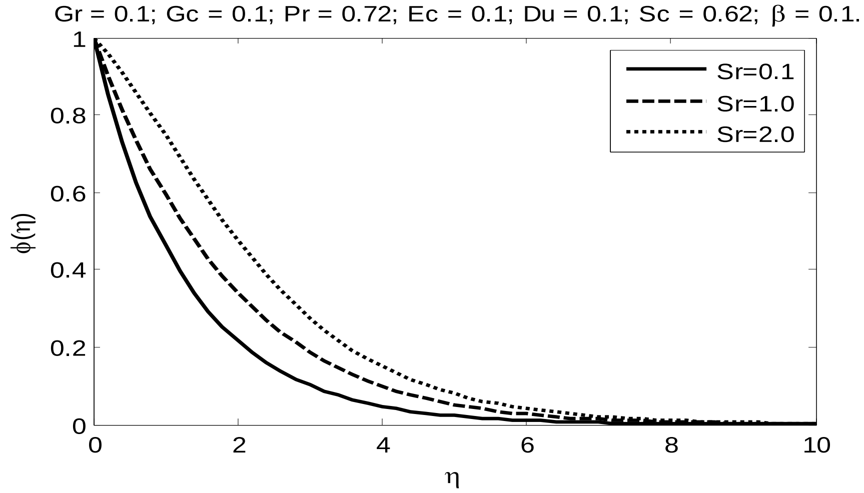

Fig. 25: Concentration profiles for different values of Sr .

Table 1: Convergence of the HAM solutions for different order of approximations when $Gr = 0.1$; $Gc = 0.1$; $\Pr = 0.72$; $Ec = 0.1$; $Du = 0.1$; $Sr = 0.1$; $\beta = 0.1$; $Sc = 0.78$; $\hbar_1 = \hbar_2 = \hbar_3 = -1.1$.

Order	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
5	0.885593	0.777757	0.785083
10	0.885349	0.782632	0.785048
15	0.85359	0.7824 94	0.784988
20	0.885356	0.782500	0.784987
25	0.885356	0.782500	0.784987
30	0.885356	0.782500	0.784987
35	0.885356	0.782500	0.784987

Table 2: Comparison of values of $-f''(0), -\theta'(0)$ and $-\phi'(0)$ for various values of $Gr,\ Gc,\ \Pr, Ec, Sr, \beta, Du = Sr = 0.$.

$\Pr$	Gr	Gc	Ec	Sc	β	Ibrahim and Makinde [23]			HAM		
						$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.72	0.0	0.0	0.0	0.0	0.0	---	0.808834	---	---	0.808647	---
1.0	0.0	0.0	0.0	0.0	0.0	---	1.0	---	---	1.0	---
0.71	0.1	0.1	0.1	0.24	0.1	0.867232	0.845372	0.453885	0.865664	0.824988	0.453521

Table 3: Numerical computations of $-f''(0), -\theta'(0)$ and $-\phi'(0)$ for various values of $Gr,\ Gc,\ \Pr, Ec, Du,\ Sr,\ Sc, \beta$.

Gr	Gc	$\Pr$	Ec	Du	Sc	β	Sr	HAM $-f''(0)$	HAM $-\theta'(0)$	HAM $-\phi'(0)$
0.1	0.1	0.72	0.1	0.1	0.62	0.1	0.1	0.885356	0.78250	0.784987
1.0	0.1	0.72	0.1	0.1	0.62	0.1	0.1	0.449390	0.871608	0.852418
0.1	1.0	0.72	0.1	0.1	0.62	0.1	0.1	0.446027	0.874197	0.854469
0.1	0.1	1.0	0.1	0.1	0.62	0.1	0.1	0.891800	0.953344	0.773716
0.1	0.1	0.72	1.0	0.1	0.62	0.1	0.1	0.880945	0.574894	0.797502
0.1	0.1	0.72	0.1	1.0	0.62	0.1	0.1	0.872309	0.463990	0.806316
0.1	0.1	0.72	0.1	0.1	0.78	0.1	0.1	0.889869	0.773158	0.900805
0.1	0.1	0.72	0.1	0.1	0.62	1.0	0.1	0.896063	0.757131	1.115300
0.1	0.1	0.72	0.1	0.1	0.62	0.1	1.0	0.874202	0.804354	0.515276

VI. Conclusion

Here, we examined the Soret and Dufour effects on two dimensional, steady MHD free convective flow of an electrically conducting viscous incompressible chemically reacting fluid past a vertical plate moving downwards in the presence of viscous dissipation. The solutions of nonlinear ordinary differential equations are obtained analytically using HAM. From our analysis the following conclusions can be drawn:

Here, we examined the Soret and Dufour effects on two dimensional, steady free convective flow of a viscous, incompressible, chemically reacting fluid past a vertical plate moving downwards in the presence of viscous dissipation. The solutions of nonlinear ordinary differential equations are obtained analytically using HAM. From our analysis the following conclusions can be drawn:

- The effect of Gr and Gc, on velocity is opposite to that on temperature and concentration profiles.
- As Pr increases velocity, temperature and concentration decrease.
- The effects of Ec and Du on velocity and concentration profiles are almost negligible but temperature increases with an increase in Ec and Du.
- The effect of Sc and K_r on temperature are opposite to that on velocity and concentration profiles.
- The effect of Sr on velocity and temperature profiles is almost negligible. But concentration increases with an increase in Sr.

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