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Study on Steady Flow over a Rotating Disk in Porous Medium with Heat Transfer

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Abstract : *The steady flow of an incompressible viscous fluid above an infinite rotating disk in a porous medium is studied with heat transfer. Numerical solutions of the nonlinear governing equations which govern the hydrodynamics and energy transfer are obtained and solved using Crank Nicolson method. The effect of the porosity of the medium on the velocity and temperature distributions is considered.*
Key Words: *Energy transfer, Heat transfer, Porous medium, Rotating disk and Viscous fluid.*

I. Introduction

The pioneering study of fluid flow due to an infinite rotating disk was carried by von Karman [1]. Von Karman gave a formulation of the problem and then introduced his famous transformations which reduced the governing partial differential equations to ordinary differential equations. Asymptotic solutions were obtained for the reduced system of ordinary differential equations by Cochran [2]. Their analysis was much simpler and valuable information was gained from it. This gave the problem significant theoretical value and invited many researchers to add to it new features. Benton [3] improved Cochran's solutions and solved the unsteady problem and proved that the steady state solution can be obtained via a time-dependent process.

In recent years, considerable interest has been shown in mass addition to boundary layer flows, especially in connection with the cooling of turbine blades and the skins of high speed aero-vehicles. Such a cooling process, frequently termed transpiration, might utilize a porous surface through which a coolant, either a gas or liquid, is forced. It is of interest to study the effect of the magnetic field as well as the non-Newtonian fluid behaviour on the heat transfer and, in turn, on the cooling process of such devices. These results are needed for the design of the wall and the cooling arrangements.

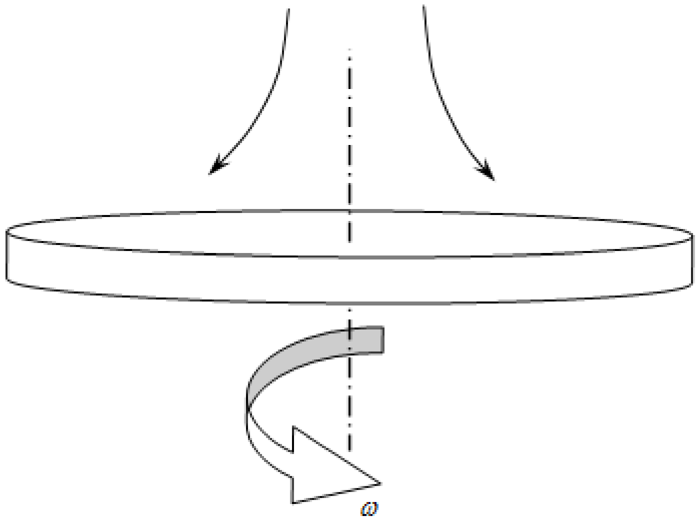
The problem of heat transfer from a rotating disk maintained at a constant temperature was first considered by Millsaps and Pohlhausen [4] for a variety of Prandtl numbers in the steady state. Sparrow and Gregg (1960) studied the steady state heat transfer from a rotating disk maintained at a constant temperature to fluids at any Prandtl number. Later Attia (1998) extended the problem discussed in (Millsaps et al. 1952, Sparrow et al. 1960) to the unsteady state in the presence of an applied uniform magnetic field where a numerical solution has been obtained. Sparrow and Gregg [5] studied thesteady state heat transfer from a rotating disk maintained at a constant temperature to fluids at any Prandtl number. The influence of an external uniform magnetic field on the flow due to a rotating disk was studied [6–8]. The effect of uniform suction or injection through a rotating porous disk on the steady hydrodynamic or hydromagnetic flow induced by the disk was investigated [9–11].

In the present work, the steady laminar flow of a viscous incompressible fluid due to the uniform rotation of a disk of infinite extent in a porous medium is studied with heat transfer. The flow in the porous media deals with the analysis in which the differential equation governing the fluid motion is based on the Darcy's law which accounts for the drag exerted by the porous medium [12-14]. The temperature of the disk is maintained at a constant value. The governing nonlinear differential equations are integrated numerically using the finite difference approximations. The effect of the porosity of the medium on the steady flow and heat transfer is presented and discussed.

II. Physical Description Of The Problem

Physical model presented below consists of a rotating disk immersed in a large amount of fluid. Motions within the fluid are generated by rotating disk, which induces heat transfer phenomenon. Disk of a radius R rotates around an axis perpendicular to the surface with uniform angular velocity ω. Due to the viscous forces n a layer of fluid is carried by the disk. Fluid motion is characterized by velocity components u-radial, v-circumferential, and w-axial. Fluid is defined as single-component gas and therefore temperature is maintained constant at all points on the disk surface.

Let the disk lie in the plane z = 0 and the space z > 0 is equipped by a viscous incompressible fluid. The motion is due to the rotation of an insulated disk of infinite extent about an axis perpendicular to its plane with constant angular speed ω through a porous medium where the Darcy model [14] is assumed. Otherwise the fluid is at rest under pressure p_∞.



III. Basic Equations

Considering system of cylindrical co-ordinates:

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}$$

The equations of steady motion are given by

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \quad (3.1)$$

$$\rho\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r}\right) + \frac{\partial p}{\partial r} = \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{\mu}{k}u \quad (3.2)$$

$$\rho\left(u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} - \frac{uv}{r}\right) = \mu\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2}\right) - \frac{\mu}{k}v \quad (3.3)$$

$$\rho\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) + \frac{\partial p}{\partial z} = \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right) - \frac{\mu}{k}w \quad (3.4)$$

where u, v, w are velocity components in the directions of increasing r, φ, z respectively, P is denoting the pressure, μ is the coefficient of viscosity, ρ is the density of the fluid, and K is the Darcy permeability [12-14].

IV. Solution Of The Problem

Although equations describe behavior of fluid at some distance from an object most critical and important from physical point of view is steady flow of the layer nearest to surface of the disk z = 0. From physical and mathematical description we can determine boundary conditions considering no-slip condition at the wall of the disk:

$$z = 0: \quad u = 0,\ v = r * \omega,\ \ w = 0$$

$$z = \infty: \quad u = 0,\ v = 0$$

Successful attempt of solving similar velocity problem for an impermeable disk rotating in a single-component fluid was achieved in 1921 by T. von Karman. In order to use similarity transform to reduce the partial differential equations to ordinary differential equations new variables need to be introduced:
Independent variable is given by

$$\zeta = z\sqrt{\frac{\omega}{\nu}}$$

Dependent variables are given by

$$F(\zeta) = \frac{u}{r*\omega}\ G(\zeta) = \frac{v}{r*\omega}\ H(\zeta) = \frac{w}{\sqrt{\nu*\omega}}\ P(\zeta) = \frac{p}{\rho*\nu*\omega}$$

thus von Karman transformations [1] are given by

$$u = r\omega F(\zeta),\ v = r\omega G(\zeta),\ w = \sqrt{\omega\nu}H(\zeta),\ z = \sqrt{\nu/\omega}\zeta,\ p = \rho\nu\omega P$$

where ζ is a non-dimensional distance measured along the axis of rotation, F, G, H and P are non-dimensional functions of ζ, and ν is the kinematic viscosity of the fluid, $\nu = \mu/\rho$.
With these definitions, equations (3.1) – (3.4) take the form

$$\frac{dH}{d\zeta} + 2F = 0, \tag{3.5}$$

$$\frac{d^2F}{d\zeta^2} - H\frac{dF}{d\zeta} - F^2 + G^2 - MF = 0, \tag{3.6}$$

$$\frac{d^2G}{d\zeta^2} - H\frac{dG}{d\zeta} - 2FG - MG = 0, \tag{3.7}$$

$$\frac{d^2H}{d\zeta^2} - H\frac{dH}{d\zeta} + \frac{dP}{d\zeta} - MH = 0, \tag{3.8}$$

$M = \nu/K\omega$ is the porosity parameter. The boundary conditions for the velocity problem are given by

$$\zeta = 0,\quad F = 0,\quad G = 1,\quad H = 0, \tag{3.9a}$$

$$\zeta \to \infty,\ F \to 0,\ G \to 0,\ P \to 0, \tag{3.9b}$$

Two missing Boundary conditions are given as

$$F'(0) = 0.510,\ G'(0)\ = -0.6159$$

can be obtained by approximation method stretching of the independent variable and using least squares method to minimize the error in the differential equations.

Equation (3.9a) indicates the no-slip condition of viscous flow applied at the surface of the disk. Far from the surface of the disk, all fluid velocities must vanish aside the induced axial component as indicated in equation (3.9b). The above system of equations (3.5)–(3.7) with the prescribed boundary conditions given by equations (3.9) are sufficient to solve for the three components of the flow velocity. Equation (3.8) can be used to solve for the pressure distribution if required.

Due to the difference in temperature between the wall and the ambient fluid, heat transfer takes place. The energy equation without the dissipation terms takes the form [4,5]

$$\rho c_\rho\left(u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) - K\left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) = 0, \tag{3.10}$$

Where T is the temperature of the fluid, c_p is the specific heat at constant pressure of the fluid, and K is the thermal conductivity of the fluid. The boundary conditions for the energy problem are that, by continuity considerations, the temperature equals T_w at the surface of the disk. At large distances from the disk, T tends to T_∞ where T_∞ is the temperature of the ambient fluid. In terms of the non-dimensional variable $\theta = (T - T_\infty)/(T_w - T_\infty)$ and using von Karman transformations, equation (3.10) takes the form

$$\frac{1}{\Pr}\frac{d^2\theta}{d\zeta^2} - H\frac{d\theta}{d\zeta} = 0 \qquad (3.11)$$

Where $\Pr$ is the Prandtl number, $\Pr = c_p \mu_k / k$. The boundary conditions in terms of θ are expressed as

$$\theta(0) = 1, \;\; \theta(\infty) = 0 \qquad (3.12)$$

The system of non-linear ordinary differential equations (3.5)–(3.7) and (3.11) is solved under the conditions given by equations (3.9) and (3.12) for the three components of the flow velocity and temperature distribution, using the Crank-Nicolson method [15]. The resulting system of difference equations has to be solved in the infinite domain $0 < \zeta < \infty$. A finite domain in the ζ -direction can be used instead with ζ chosen large enough to ensure that the solutions are not affected by imposing the asymptotic conditions at a finite distance. The independence of the results from the length of the finite domain and the grid density was ensured and successfully checked by various trial and error numerical experimentations. Computations are carried out for $\zeta_\infty = 12$.

The graph is drawn for the velocity components u, v, w is easy to obtain from previously introduced equations

$$F(\xi) = \frac{u}{r*\omega} \; G(\xi) = \frac{v}{r*\omega} \; H(\xi) = \frac{w}{\sqrt{\nu*\omega}}$$

V. Results And Discussion

The nonlinear ordinary differential equations (3.5) – (3.8) subject to the boundary conditions (3.9) have been solved via crank nicolsan method for some values of the porosity parameter (M). For the present investigation, the value of the Prandtl number (Pr) is considered equal to 0.7.

Fig (1-4) represents the influence of porosity parameter, on the radial, tangential and axial velocity components as well as temperature distribution. It is noted that all velocity boundary layer thickness decrease, as porosity parameter increases. It is worth mentioning that the large resistances on the fluid particles apply as the porosity increases.

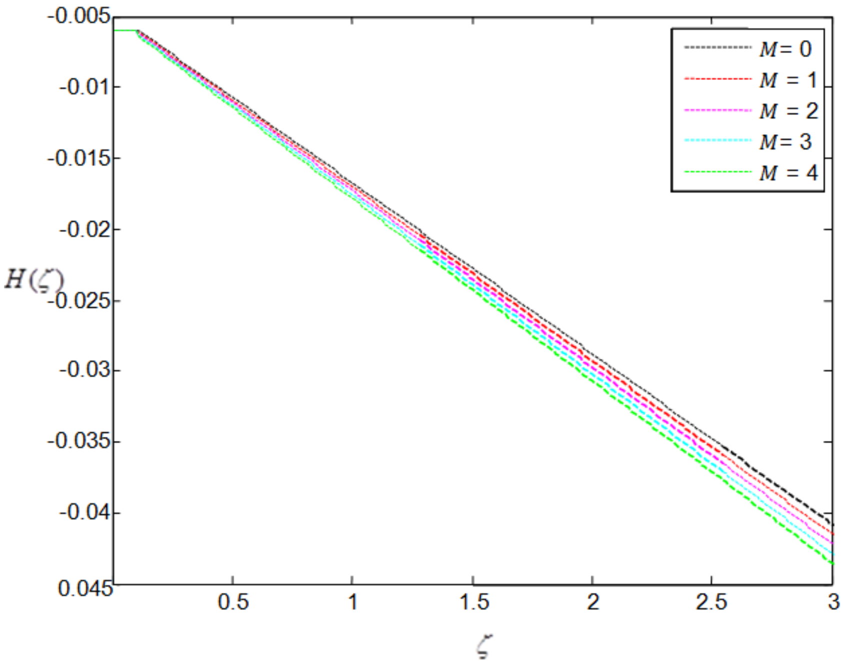


Fig. 1 Effect of porosity parameter on axial velocity components

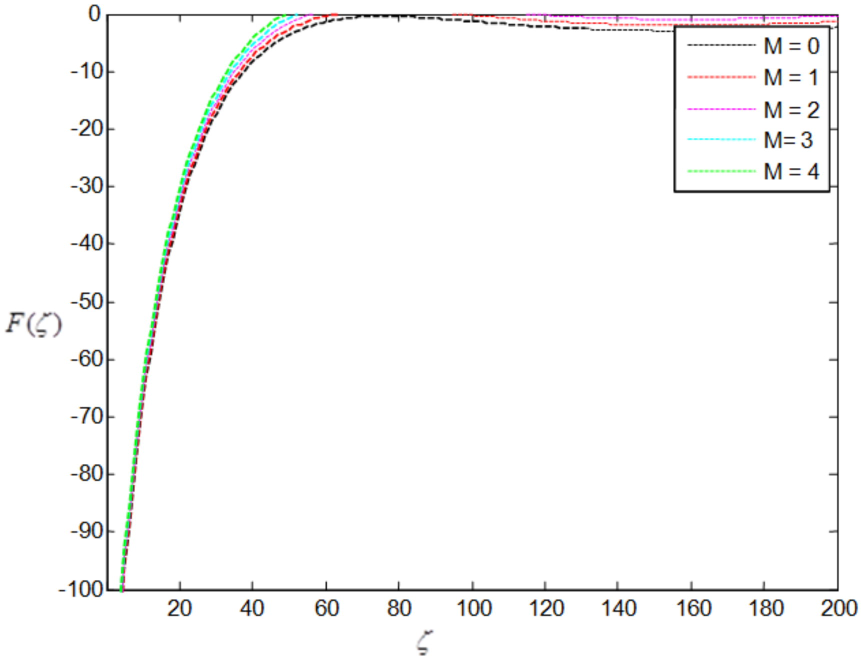


Fig. 2 Effect porosity parameter on radial velocity components

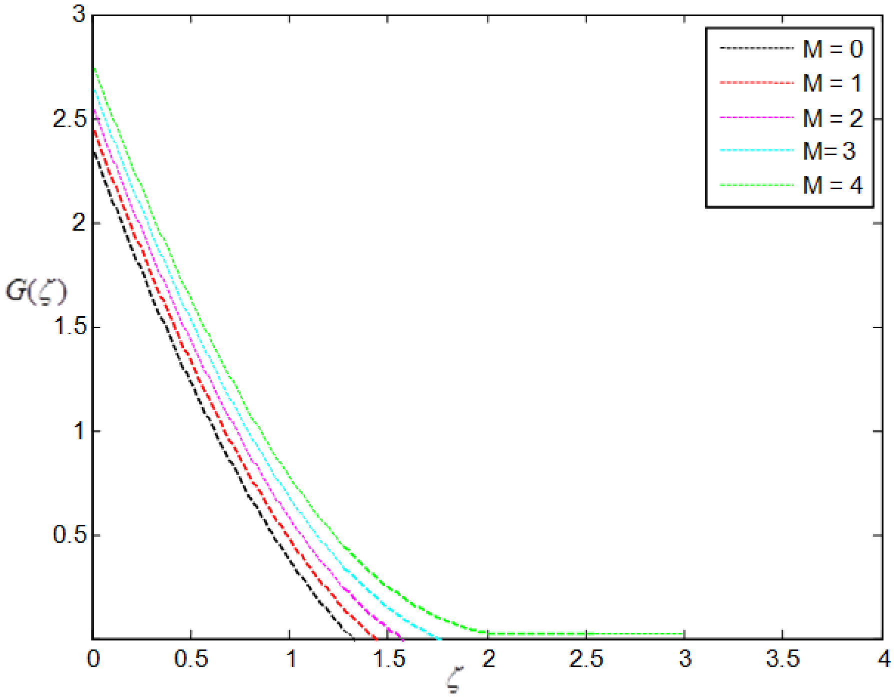


Fig. 3 Effect of porosity parameter on tangential velocity components

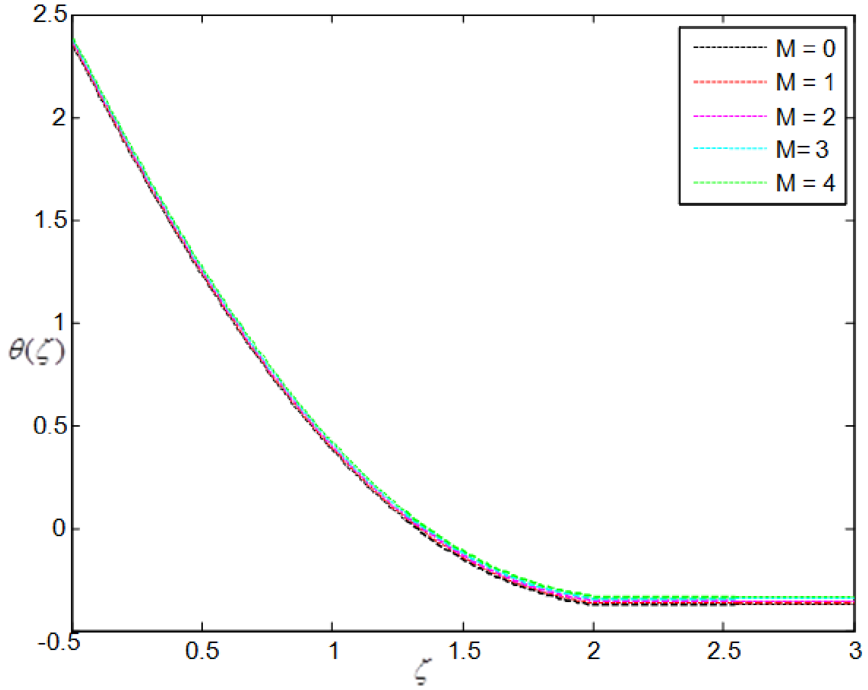


Fig. 4 Effect of porosity parameter on temperature distribution

VI. Conclusion

In this study the steady flow induced by a rotating disk with heat transfer in a porous medium was studied. The results indicate the restraining effect of the porosity on the flow velocities and the thickness of the boundary layer. On the other hand, increasing the porosity parameter increases the temperature and thickness of the thermal boundary layer.

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